Numerical simulation of stress oscillation in a functionally graded piezoelectric thin plate

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Abstract

When a functionally graded material (FGM) structure is subjected to an impact loading, the stress induced by the elastic wave propagation changes complicatedly as time advances. Maximizations of the tensile and compressive stresses due to the stress focusing effect may cause a serious damage to an FGM structure, research on the behavior of dynamic stress in it is needed.

Researches on functionally graded materials have attracted considerable attention since around 1990. Numerous papers related to dynamic elastic problems in FGM structures were published. Most of them dealt with dynamic responses to harmonic excitations, while there were a few works which investigated dynamic stresses induced in FGM structures by the elastic wave propagations.

The first author, et al. mathematically analyzed a one-dimensional dynamic elastic problem in an infinite FGM thin plate subjected to an impact loading. Material properties were then assumed to vary exponentially in the thickness direction in order to obtain the analytical solution. It was seen from the numerical results that the stress changed alternately between compression and tension with time progress and the maximum amplitude of the stress oscillation in the FGM thin plate was about twice as large as it in the homogeneous thin plate.

The present paper extends the previous study to the case of a functionally graded piezoelectric (FGP) thin plate. Material properties are considered to vary in the thickness direction according to a power low distribution. It is assumed that all response quantities are initially zero, one surface of the thin plate is subjected to uniform impact pressure, and the other surface is fixed to a flat rigid body. For the electric field, both surfaces of the FGP thin plate are considered to be free of electric charge. In the case of the one dimensional dynamic elastic problem, the dielectric flux density becomes zero throughout the FGP thin plate. Applying the electric constitutive equation, the electric field intensity is eliminated from the elastic constitutive equation. Introducing the particle velocity which is the partial differentiation of the displacement with respect to the time variable, the elastic constitutive equation and the equation of motion governing the particle velocity and stress are reduced to two first-order coupled partial differential equations. The method of characteristics is applied to solving the two partial differential equations, because it is a very powerful technique for solving wave propagation problems and has the advantage of determining the wave front exactly. The characteristic lines as well as the characteristic equations governing the particle velocity and stress are derived from the two partial differential equations. Integrating the characteristic equations along the characteristic lines, a system of simultaneous linear equations is derived and thus the particle velocity and stress are computed. From the obtained numerical results, it is found that the time history of dynamic stress is complicated and the amplitude of the stress oscillation changes intensively with time progress even in the case of the FGP thin plate.

Keywords: Functionally graded piezoelectric thin plate, Dynamic elastic problem, Stress oscillation, Method of characteristics