Wave propagation in an elastic waveguide: application of the Fourier transform and finite element methods

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Introduction

Running elastic waves are widely used in the monitoring of various industrial objects for identification of damages. Piezoceramic actuators are usually used to generate high-frequency waves for non-destructive testing. Simulation of the elastic wave propagation in mechanical structures is an important task for the development of non-destructive testing and structural condition monitoring of composite materials, which are increasingly used in such fields as aircraft manufacturing, chemical industry, pipeline systems, etc.

In presented study vibrations of an isotropic waveguide that occupies the volume $D = \{(x, y, z) | -\infty < x, y < \infty; -h \le z \le 0\}$ (see Fig. 1) were considered. Lame's equations for the steady-state harmonic vibration of the considered waveguide have the form

$$L\mathbf{u} + \rho\omega^2\mathbf{u} = 0 \qquad (1)$$

where $\mathbf{u}$ is the displacement field, ρ is the density and ω is the angular vibration frequency.

The bottom surface is free of stress

$$\boldsymbol{\tau}\big|_{z=-h} = 0\,. \qquad (2)$$

The aim of this study is to determine the displacement field $\mathbf{u}$ caused by the harmonic vibrations of a piezoelectric plate bonded on the upper surface of the considered strip.

In order to calculate the unknown displacement field both Fourier transform and finite element (FE) methods were applied.

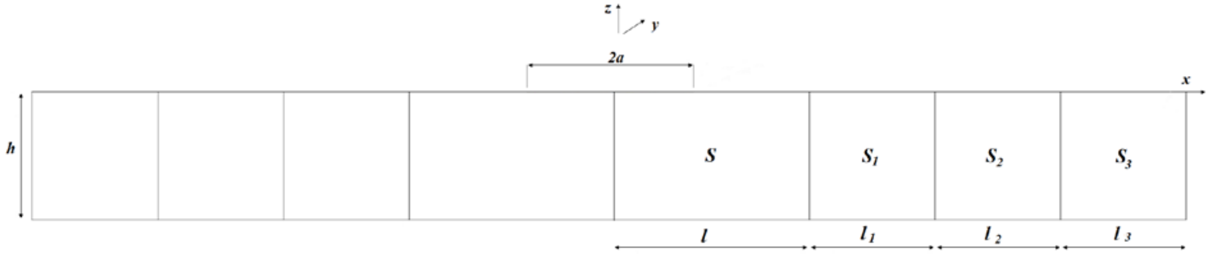


Figure 1. The scheme of the loaded structure

The Fourier transform by x was applied to Eqs. (1)-(2) with the parameter α, and the solution of the initial problem was written as follows

$$\mathbf{u}(x,z)=\frac{1}{2\pi}\int_{\Gamma}\mathbf{K}(\alpha,z)\mathbf{Q}(\alpha)e^{-i\alpha x}d\alpha\,, \tag{3}$$

where $\mathbf{K}$ and $\mathbf{Q}$ are the the Fourier transforms of the Green's matrix and of the load. The integration contour Γ in accordance with the limiting absorption principle go along the real axis, deviating when traversing the poles of Fourier transform $\mathbf{K}$ of the Green's matrix $\mathbf{k}$. The contact stresses $q(x)$ occurring under the piezoelectric actuator were described by means of a simplified model, which is commonly used for engineering calculations. According to this model, the action of the actuator has been approximated by two delta-functions applied at the boundary points $x=\pm a$ of the contact area

$$\tau_{xz}\big|_{z=0}=C(\delta(x-a)-\delta(x+a)), \tag{4}$$

where $C=\int_{-a}^{a}q(x)dx$ is a coefficient, which represents the amplitude of the applied load.

Finite element model of the considered problem was formulated and simulated in FE package Comsol Multiphysics. The steel layer of the thickness $h=0.01\ m$ (in Fig.1) was loaded according to the Eq. 4, where $a=0.01\ m$. Dimensionless angular frequencies were calculated according to the formula

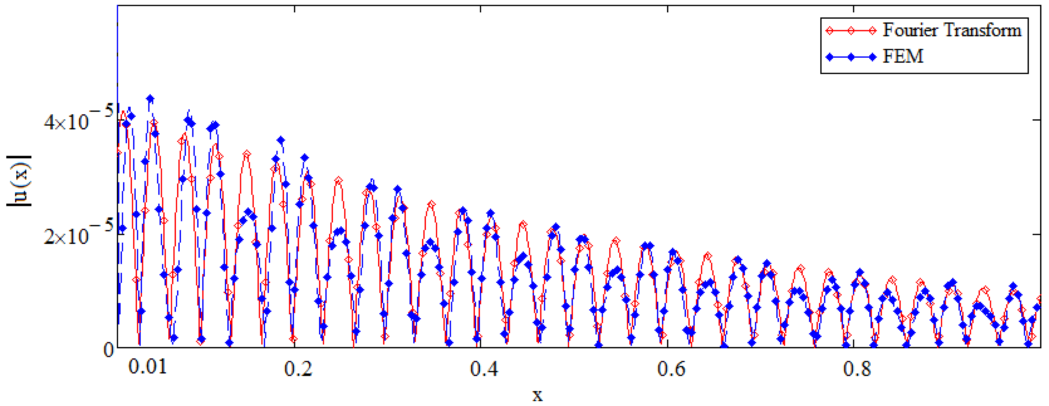
$$\omega=f_r\cdot h\cdot 2\pi/c_s\,, \tag{5}$$

where $c_s=\sqrt{G/\rho}$ is the transverse or S-wave velocity, ρ is the density, G is the shear modulus, ν is the Poisson's ratio, and f_r is the dimensional frequency in *Hz*. In order to simulate the infinite layer, three additional regions of finite length $l_i=0.1\ m,\ i=1,2,3$ were placed on the both edges of the finite region of the length $l=1\ m$. The additional regions had the same elastic properties as the considered layer, but their damping coefficients were taken to be nonzero in order to save the acoustic impedances of the regions unchanged and make the waves to smoothly attenuate when they move to the ends of the considered region. Thus there were no reflected waves appeared in the layer. Damping coefficients were chosen empirically: a wave was directed into the layer, after that the check was performed in order to ensure that there were no displacement discontinuities at the boundaries between the regions. Since the impedances of all the regions remain the same, the wavelength did not change during the propagation in the side regions. Characteristics of all the subdomains (Fig.1) are described in the Table 1.

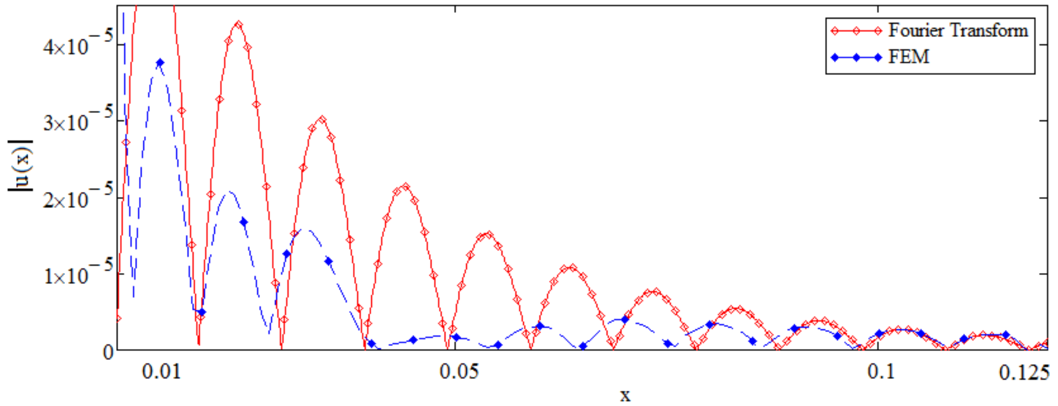
Table 1. Elastic and damping properties

Subdomain	Density, kg/m^3	Young's modulus, *GPa*	Poisson's ratio	Mass damping parameter, $1/s$	Stiffness damping parameter, s
S	7480	20	0.28	0.001×10^{-4}	0.001×10^{-4}
S_1	7480	20	0.28	0.01×10^{-4}	0.01×10^{-4}
S_2	7480	20	0.28	0.025×10^{-4}	0.025×10^{-4}
S_3	7480	20	0.28	0.5×10^{-4}	0.5×10^{-4}

In Fig.2, the absolute values of the displacement fields calculated by means of finite element method and by inverse Fourier transform are presented at different vibration frequencies: $\omega = 1.1$ (a) and $\omega = 4$ (b). It can be seen (Fig.2, a) that the compared displacements are in a better agreement for lower vibration frequency. In this case, the presented results are comparable both in the near-field and on the distance from the oscillation source. The displacements corresponding to higher frequencies (Fig.2, b) distinguish stronger in the vicinity of the actuator, but they show the good agreement on distance from the oscillation source and almost coincide in the far field.



a)



b)

Figure 2. Absolute values of displacement fields calculated by means of inverse Fourier transform and FEM at different frequencies: a) $\omega = 1.1$ **and b)** $\omega = 4$

Analysis of the displacement fields obtained by means of the aforementioned methods showed the good results agreement in far field, when the displacement amplitudes differ significantly in the vicinity of the contact area. It was also found that the efficiency of the considered numerical methods is reduced with increasing vibration frequency.

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