A Non-standard Finite-difference Scheme for a Burgers-Fisher Partial Differential Equation with Bounded Travelling-wave solutions

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Abstract

We present a non-standard finite difference method to approximate solutions of Burgers-Fisher equation with generalized advection factor and logistic reaction. The equation possesses bounded travelling-wave solutions that are monotone functions of both time and space. This paper presents a theoretical study on the stability and convergence issues of such a nonstandard-FDM approach, with rigorous proofs. Theoretical conditions for both the convergence and stability of bounded solutions are successfully found and examined in detail. A number of numerical examples are then presented to demonstrate the effectiveness of the theoretical predictions. All these examples have showed that our nonstandard-FDM can ensure the solution automatically bounded, as long as the theoretical conditions are satisfied. We provide simulations that show that, indeed, our technique can preserves the positivity, the boundedness, the temporal and spatial monotonicity of solutions.

Keywords: Convergence and stability, Burgers-Fisher equation; Finite-difference method; Positivity and boundedness solutions.