A truss section size optimization design method based on constraint variation principle

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Abstract

In the actual structure, rational optimization of truss structure section size, can make each bar bear maximum load and play a role. By this way, we can not only make the truss material give full play to its own performance, but also reduce weight of truss, and cost savings. At present, there are two methods for section size optimization: Mechanical criterion method and mathematical programming method. This paper combines the two methods and analyze a truss-braced structure. In this paper, the truss node displacement is used as the constraint condition. It takes the cross section area of the bar in the truss as the independent variable and takes the steel weight of the truss as objective function. It uses constrained variation principle and force method equation to create a hybrid numerical method. Through concrete examples, the results of this algorithm are compared with the simulation results, and the reliability and correctness of truss section size optimization are verified.

Keywords: Truss; Section size; Optimization; Constrained variation principle; Software simulation;

1. Introduction

In the building structure, supporting truss is one of the important structure form which consists of a number of bar through some articulated points. Truss structure is widely used in homes, factories and other buildings because of its great Wholeness, large stiffness, great seismic ability. But in the actual structure, the force of the bar in supporting truss is not the same in affected by external force, so that it leads to the different bar stress. Rational design of truss structure, can meet the requirements of the actual working conditions, so that the bar can bear the maximum load and play a role. In this way, we can not only make the truss material give full play to its own performance, but also achieved the goal that reduce weight of truss, materials and save cost.

In the theoretical calculation stage, this paper firstly analyze the static-truss structure, discusses the case of single-independent and multivariable ones, deduces the iterative criterion. And then it explains the application of the optimization method by an example of cantilever statically indeterminate structure. At last it is compared with the software simulation results in order to obtain a more secure form of structure, so that the truss structure tends to rationalize the force.

2. Theoretical analysis

2.1 Statically determinate truss single variable optimization theoretical derivation

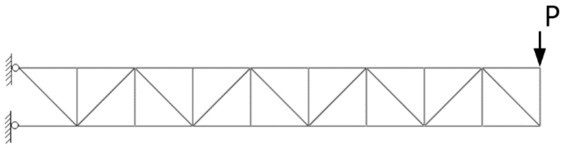

Figure 3-1.statically determinate truss single variable optimization

As is shown in the picture, the distance between two nodes of a truss is 0.5m.The truss is made in NO.45 steel：Density ρ =7800kg/m3，E=210GPa，μ=0.3。It has the same cross-sectional area A in chord member and web member. There is each 70000N static-state load which is downward as we can see in the picture.
Nodal displacement calculation formula:

$$\Delta = \sum \int \frac{\bar{F}_N F_P}{EA} ds \tag{1}$$

Truss quality calculation formula:

$$M(A) = k_1 \sum_i \rho l A + k_2 \sum_j \rho l A \tag{2}$$

Make $\Delta < 0.1m$, $g(A) = \Delta - 0.1$, The above questions can be expressed as:

$$\begin{array}{ll} \min & A \\ s.t. & A \leq g(\end{array} \tag{3}$$

The problem is an extreme one including an unknown value. It takes the cross section area of the bar in the truss as the independent variable, takes the points of force application as constraint condition and takes the steel weight of the truss as objective function by finite element software. We make $\Delta < 0.1m$, determine the optimum case when the independent variable is approaching.

2.2 Statically determinate truss double variables optimization theoretical derivation

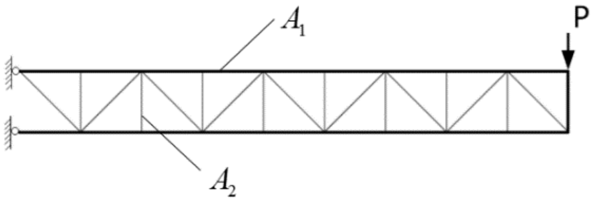

Figure 3-2.statically determinate truss two variables optimization

The parameter information is the same as we can see in the first example. The cross-sectional area in chord member is A_1 and the cross-sectional in web member is A_2 .

Nodal displacement calculation formula:

$$\Delta = \sum_i \int \frac{\overline{F}_N F_P}{EA_i} ds \tag{4}$$

Truss quality calculation formula:

$$M(A_1, A_2) = k_1 \sum_1 \rho l A_1 + k_2 \sum_2 \rho l A_2 \tag{5}$$

Make $\Delta < 0.1m$, $g(A) = \Delta - 0.1$.The above questions can be expressed as:

$$\begin{aligned} &\min \quad && \mathrm{M}(A_1, A_2) \\ &s.t. \quad && g(A_1, A_2) \le 0 \end{aligned} \tag{6}$$

It makes A_1, A_2 as design variables.

Vector form:

$$\boldsymbol{A} = [A_1, A_2]^{\mathrm{T}} \tag{7}$$

First we construct an internal punishment function:

$$\phi(A_1, A_2, r) = M(A_1, A_2) - r^k \frac{1}{g(A_1, A_2)} \tag{8}$$

Use analytical method to seek extreme value.

$$\begin{cases} \dfrac{\partial \phi}{\partial A_1} = 0 \\ \dfrac{\partial \phi}{\partial A_2} = 0 \end{cases} \tag{9}$$

Solve it simultaneously

$$\begin{cases} A_1(\mathrm{r}^k) \\ A_2(\mathrm{r}^k) \end{cases}, \begin{cases} A_1'(\mathrm{r}^k) \\ A_2'(\mathrm{r}^k) \end{cases} \tag{10}$$

Meet

$$\begin{cases} A_1(\mathrm{r}^k) > 0 \\ A_2(\mathrm{r}^k) > 0 \\ \mathrm{g}(A_1, A_2) \le 0 \end{cases} \begin{cases} A_1(\mathrm{r}^k) > 0 \\ A_2(\mathrm{r}^k) > 0 \\ \mathrm{g}(A_1, A_2) \le 0 \end{cases} \tag{11}$$

Ignore solutions that don't meet the requirement. $r^{(k)}$ is the penalty factor in the formula.it is descending positive sequence:

$r^{(0)} > r^{(1)} > r^{(2)} > \cdots > r^{(k)} > r^{(k+1)} > \cdots > 0, r^{(k-1)} \cdot c = r^{(k)}\ r^{(k-1)} \cdot c = r^{(k)}$

Reduction coefficient c:

$$0 < c < 1, \lim_{k\to\infty} r^{(k)} = 0, \phi(A_1, A_2, r) \to M(A_1, A_2) \tag{12}$$

Flow diagram:

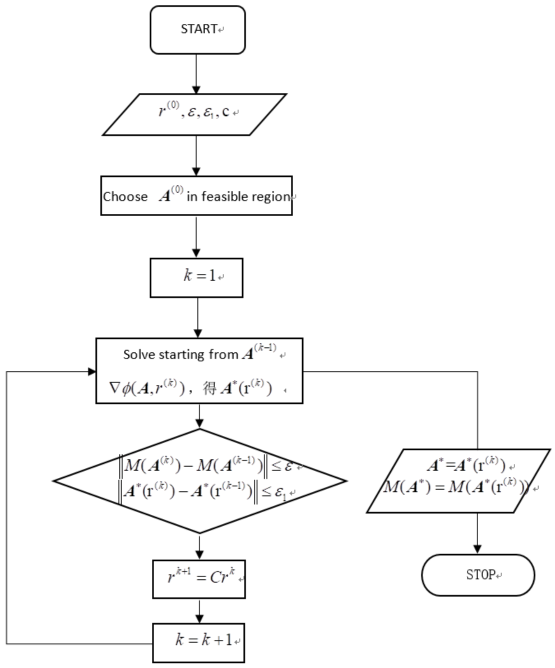

Figure 3-3.Flow diagram

2.3 Cantilever statically indeterminate structure single variable optimization theoretical derivation.

A project used support truss structure. The distance between two nodes of a truss is 0.5m.The truss is made in NO.45 steel：Density ρ =7800kg/m3，E=210GPa，μ=0.3。It has the same cross-sectional area A in chord member and web member. Each cross-sectional area is 350mm2 before optimized. The mechanical model is shown below:

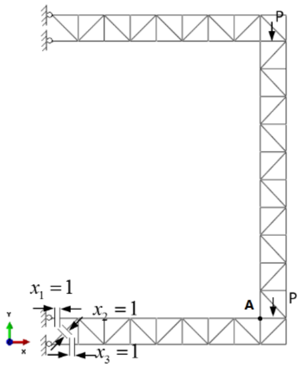

Figure 3-3.statically indeterminate structure single variable optimization

The truss itself is subjected to gravity, and there is each 70000N static-state load which is downward as we can see in the picture.

Establish force method equation:

$$\begin{cases} \delta_{11}x_1 + \delta_{12}x_2 + \delta_{13}x_3 + \Delta_{1p} = 0 \\ \delta_{21}x_1 + \delta_{22}x_2 + \delta_{23}x_3 + \Delta_{2p} = 0 \\ \delta_{31}x_1 + \delta_{32}x_2 + \delta_{33}x_3 + \Delta_{3p} = 0 \end{cases} \tag{13}$$

Get unknown coefficients and free term coefficients in force method equation by graphic multiplication method, then put it into formula to solve out $[x_1, x_2, x_3]^T$, which is the fundamental unknown force vector. Put the vector into formula to get internal force each in the statically indeterminate structure.

Transform mechanical problem into optimization problem:

$$\begin{array}{ll} \min & \mathrm{M}(A) \\ s.t. & \mathrm{g}(A) \le 0 \end{array} \tag{14}$$

Determine the optimum case by finite element software when the independent variable is approaching.

3.4 Cantilever statically indeterminate structure double variables optimization theoretical derivation.

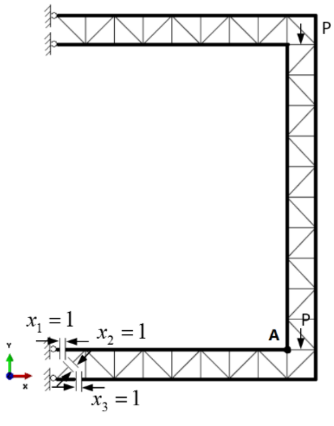

Figure 3-4.statically indeterminate structure double variables optimization

The parameter information is the same as we can see in the third example. The calculation method is the same as we can see in the second example. Then we get a series of solutions by finite element software.

Table 1. Comparison between optimization results

example		I	II	III	IV
before optimization	A1/m2	1e-3	1e-3	4e-4	4e-4
	A1/m2	1e-3	1e-3	4e-4	4e-4
	M/kg	158.83	158.83	191.25	191.25
algorithm optimization	A1/m2	8.59e-4	1.02e-3	3.43e-4	4.29e-4
	A1/m2	8.59e-4	2.2e-4	3.43e-4	2.26e-4
	M/kg	136.46	94.17	164.00	152.3
simulation optimization	A1/m2	8.62e-4	1.04e-3	3.25e-4	4.34e-4
	A1/m2	8.62e-4	2.0e-4	3.25e-4	1.95e-4
	M/kg	136.46	94.012	155.39	145.4

Conclusions

Through the comparison and research of the derivation and numerical simulation of the truss structure, the following conclusions can be drawn:
(1) After the cross-sectional size optimization, the cross-sectional area of the structural bar is reduced, but it has made full use of the material properties, that is, the amount of steel is reduced, the cost is less, and the material performance can be fully utilized. It is suitable for the scientific development concept.
(2) Based on the previous research, the cross section area of the structural chord and the web are optimized as the optimization variables. The optimized effect is better than the effect of using the cross-sectional area as a single optimization variable. As the cross-sectional area of the chord increases, the total amount of steel is reduced and the strength of the material is brought into full play.
(3) Compared with the numerical results and the results of the algorithm, the results show that the numerical simulation results are slightly smaller than the algorithm optimization results, and there exist about 5% errors.

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