Flutter frequency based on bending - torsion coupling theory

***ZHENG Xin [1], LIU Yu-Bin [1], CHEN Pu[2], SHEN Feng [3], ZHANG Sheng-Jun [3], and †FU Xiang-Rong [1]**

[1] College of Water Conservancy and Civil Engineering, China Agricultural University, China.
[2] College of Engineering, Peking University, China.
[3] State Nuclear Power Research Institude, China.

*Presenting author: 476238231@qq.com
†Corresponding author: fuxr@cau.edu.cn

Abstract

This paper establishes a method which can forecast the general value range of flutter frequency by the following two steps, namely 1)Based on the theory of frequency superposition, this paper chooses the cantilever panel structure as a main object of the analysis, and controls the chord length and the root chord length as single variable separately. By the establishing and simulating of flutter procedures, this paper studies the correlation between the flutter mode and the bending - torsional coupling modes of the wing model under different geometric parameters. 2) Considering the flexible characteristics of the elastic support in the selection of node positions together with its agility to descript an intricate mechanical state of a structure, subjoined spring supports to the original model to simulated flutter statement. This approximation model simulates the flow-solid coupling state caused by the additional aerodynamic force in the flutter problem, so that provide a reference for the study of the internal mechanism of the problem.

Keywords: Bending-torsion coupling theory; Flutter frequency; Equivalent spring model;

1.Introduction

In the series of aero-elastic problems caused by the load of the aircraft, the avoidance of the flutter problem has become an extremely important part of the aircraft design due to the abruptness and destructiveness of the accident caused by it. The aero-elastic flutter damage is caused by the interaction among inertial, elastic and aerodynamic forces, thus the vibration of the structure becomes considerable significance when the mechanism of flutter is studied [1] . In the process of flutter analysis of the wing, the uncertainty and ineffectiveness of the simulation results is often caused by the geometric and physical parameters of the model, the selection of the structural and the aerodynamic mesh, the match of the Mach number and the flutter frequency.

In the process of flutter analysis, the following methods are used to predict the flutter parameters: the time-frequency domain of the flutter determinant can be used to obtain the judgment of the flutter coupling mode; or the V-g method, p-k method and other solution methods can also get the relevant paramount, such as flutter frequency, damping and flutter critical velocity; or through the frequency coupling trend and the damping zero-crossing branch to predict the flutter mode; the analysis of the matrix eigenvector and the calculation of the contribution coefficient of the flutter mode can also achieve the prediction of the flutter mode[4].

The study is based on a simplified frequency coincidence theory proposed that the critical state of the flutter concerning coupling between bending and torsional vibrations. Firstly, analyzing rectangular cantilever models with chord length as its single variable, a convenient and effective formula for calculating the flutter frequency is obtained. Secondly, by selecting the wing tip length individually, the calculation error is compared and the reliability of the formula is also verified. At last, finding out a spring stiffness value for the model that makes its first-order vibration mode frequency corresponds to the original model flutter value. The research content provides a relatively effective forecasting method for the initial selection of the flutter frequency before analysis, and also provides some reference value for the simplification of the internal mechanism of the flutter problem.

2. Theoretical development

2.1 Frequency coincidence theory

The earliest aerodynamic problem is the bending and torsional coupling vibration of the wing. The bending caused by the wing flight leads to the angle of attack to change, and causes the disturbance of the lift and the aerodynamic moment, and again causes the bending which feedback to the bending and torsional vibration[2]. It is the initial explanation of flutter. A large number of flutter analysis of the case also shows that the increase in the flow rate will cause the two branches of the frequency changes in the speed continues to increase in the process, the two branch frequencies close to each other, the coupling is strengthened, then the coupling vibration will draw energy from the airflow and reach the critical point of flutter[5]. It should be noticed that this theory is a highly simplified analysis of the mechanism of flutter, which is not an exact solution concept compared to the flutter theory defined by the excitation and damping forces. The matrix form of the simplified equation of motion for a typical binary wing is given as follows

$$\begin{bmatrix} m\lambda^2 + k_a & S_a\lambda^2 + qS\dfrac{\partial C_L}{\partial a} \\ S_a\lambda^2 & I_a\lambda^2 + k_a - qSe\dfrac{\partial C_L}{\partial a} \end{bmatrix} \begin{bmatrix} f_0 \\ a_0 \end{bmatrix} = 0 \qquad (1)$$

Where $\partial C_L / \partial a$ is the slope of the wing lift line, m is the mass of the wing, k_a is the stiffness of the torsion spring, S is the reference area of the wing, S_a is the static moment of stiffness center ($S_a = m\sigma$), I_a is the inertia of stiffness center ($I_a = I_0 + m\sigma^2$), e is the distance of aerodynamic center to stiffness center, backward for positive; q is the air flow pressure($q = \rho V^2 / 2$), ρ is the air density, f_0 is the deflection of stiffness center, down for positive, and a_0 is the rotation angle of stiffness center, counterclockwise for positive;
The characteristic equation is expressed as:

$$\begin{aligned}
&A\lambda^4 + B\lambda^2 + C = 0 \\
&A = mI_a - S_a^2 \\
&B = mk_a - I_a k_a (me + S_a) qS \frac{\partial C_L}{\partial a} \\
&C = k_a (k_a - qSe \frac{\partial C_L}{\partial a})
\end{aligned} \tag{2}$$

The latent root can be expressed as $\lambda^2 = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$.

When $q = 0, B > 0, B^2 - 4AC > 0$, it is a state of no air flow, the two natural frequencies in the vacuum are obtained, that is, the free vibration frequency; when there is air flow, $q > 0, B^2 - 4AC > 0$, there are still two vibration frequencies change with the value of q; When q increased to the point that $B^2 - 4AC < 0$, λ^2 will become a complex plural. If the real part of λ is positive, the amplitude will continue to expand, when the movement is unstable, that is flutter.

2.2 p-k method flutter equation

In this paper, the flutter frequency of the structure is calculated by the p-k method. In p-k method, the solution for the flutter problem is found by solving the eigenvalue problem.

One key advantage of using the p-k method for determining the flutter characteristics is that it allows flutter analysis to be carried out, based on any given velocity [3]. Here is the simplified equation of plate motion

$$\begin{cases} f = f_0 e(\gamma + i)\omega t = f_0 e^{pt} \\ a = a_0 e(\gamma + i)\omega t = a_0 e^{pt} \end{cases} \tag{3}$$

Where *f* is the displacement of stiffness center, *a* is the corner of stiffness center, ω is the reduced frequency, then by Eq. (3) the decay rate coefficient γ can be written as

$$\gamma = \frac{1}{2\pi} \ln \left[\frac{f(t + 2\pi/\omega)}{f(t)} \right] \tag{4}$$

The *p-k* flutter solution of two-dimensional rectangular structures can be written as

$$[\frac{V^2}{b^2}\boldsymbol{M}p^2 + \boldsymbol{K} + \frac{1}{2}\rho V^2 \mathbf{A}(K)] \begin{pmatrix} f/b \\ a \end{pmatrix} = 0 \tag{5}$$

Where, $\boldsymbol{M}$ is the mass matrix, $\boldsymbol{K}$ is the stiffness matrix, $\boldsymbol{A}$ is the aerodynamic force matrix, ρ is the free stream air density, *V* is the velocity, *b* is a reference semi-chord of the lifting surface, and *p* refers to the complex response frequency and eigenvalue. The complex response frequency and eigenvalue, *p* can be expressed as

$$p = \gamma\omega + i\omega,\ i = \sqrt{-1} \tag{6}$$

Using the Eq.(5), the flutter phenomenon of two-dimensional plate wing can be found, when γ=0. So, the flutter speed and flutter frequency can also be found, when the sign of damping value changes from negative to positive.

3. Model analysis

3.1 Flutter characteristics of rectangular cantilever wing

Selecting the aluminum alloy as the material of the model, the structure and mechanical properties are tabulated in Table 3-1. In the premise of keeping the half-wingspan l = 1m unchanged, select the reference semi-chord ***b*** as a single variable.

Table 3-1. Rectangular cantilever wing properties

Half-wingspan, l	Thickness, t	Young's Modulus, E	Poisson Ratio, μ	Density, ρ
1.0 m	0.0018 m	70.0GPa	0.3	2700 kg/m^3

When the reference semi-chord ***b*** changes, there is no effect on the bending frequency, but will have a direct effect on the torsional frequency, so that the torsion mode will "jump" between different modes, due to changes in the wing's structural geometry. The main purpose of this paper is to investigate the relationship between frequency coupling modal and the flutter modal. Based on this prerequisite, we find out seven models whose modal frequency of the first-order torsional mode is close to that of the different order bending modes separately. The corresponding values of the reference semi-chord and flutter parameters summarized as shown in Table 3-2.

Table 3-2 Corresponding parameters for 7 models

Label	1	2	3	4	5	6	7
Semi-chord, b(m)	0.3279	0.1069	0.0578	0.0535	0.0316	0.0224	0.0201
Flutter velocity , V_F(m/s)	24.3	33.2	44.4	46.0	58.0	66.0	68.5
The flutter order	2nd	3rd	4th	4th	5th	6th	6th
Coupling order	2nd &3rd	3rd &4th	4th &5th	5th &6th	6th &7th	7th &8th	8th &9th
Coupling vibration mode	2nd-bend& 1st-torsion	3th-bend& 1st-torsion	4th-bend& 1st-torsion	5th-bend& 1st-torsion	6th-bend& 1st-torsion	7th-bend& 1st-torsion	8th-bend& 1st-torsion

Conclusion: It can be seen from Table 3-2 that the flutter order is not exactly the same as the flexural and torsional coupling order, and the coupling order is hysteresis. In addition, with the increase of the reference semi-chord *b*, the flutter order increases(i.e., the frequency of participation in the coupling decreases)and the flutter velocities assume a decreasing trend and tends to be stable.

3.2 Flutter frequency prediction of rectangular cantilever wing

The first five-order bending frequency, the first-order torsional frequency and the flutter frequency of the six models are plotted in Fig.3-1. Significant trend consistency in natural frequencies can be observed from the flutter curve and the first-order torsional curve, which also meet the frequency coincidence theory. At the same time, the trend also proves that the torsional frequency is the dominant frequency of the flutter coupling in this model. In the results of some studies on the contribution coefficient of the flutter mode, it can be seen that the participation ratio of the natural frequencies is relatively disparity when the flutter occurs.

Therefore, based on the theory of frequency coincidence, using the first-order torsional modal and the first-order bending modal as the main coupling modals of the prediction formula. The main inherent frequencies, the flutter frequencies, the predicted frequencies, and the resulting error analysis of the seven modes are summarized in Table 3-3. The comparison of models' original flutter frequencies and estimated frequencies have been shown in Fig.3-2.

$$F_0 = f_{n1} + f_{t1} / 2 \quad (7)$$

Where, F_0 the estimated flutter frequency, f_{n1} is the first bending natural frequency, f_{t1} the first torsional natural frequency.

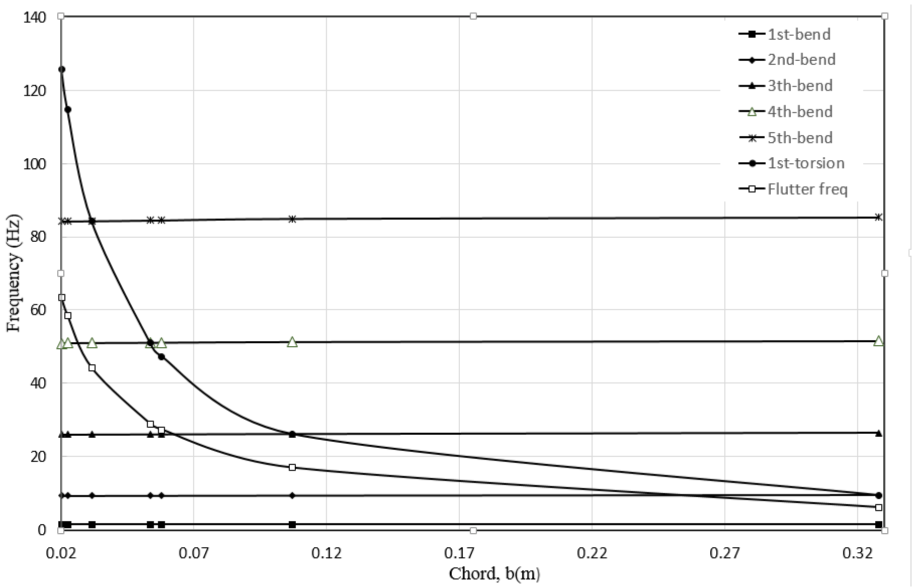

Figure 3-1 the flutter frequency and natural frequencies of seven models

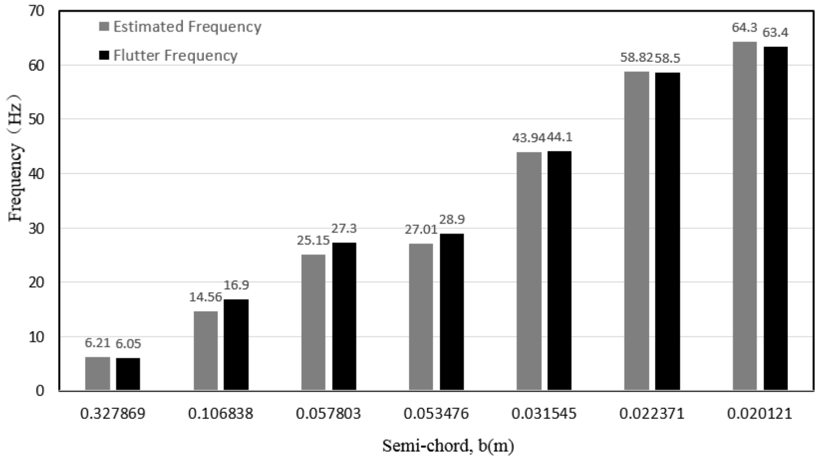

Figure 3-2 Flutter frequencies and estimated frequencies of seven models

Table 3-3 Frequencies and relative error rates of seven models

Label	1	2	3	4	5	6	7
Semi-chord, *b*(m)	0.3279	0.1069	0.0578	0.0535	0.0316	0.0224	0.0201
1st torsional freq (Hz)	1.5084	1.4911	1.4863	1.4859	1.4837	1.4829	1.4827
1st bending freq (Hz)	9.4067	26.140	47.336	51.046	84.128	114.67	125.63
Flutter frequency, *F*(Hz)	6.05	16.9	27.3	28.9	44.1	58.5	63.4
Estimated freq, F_0(Hz)	6.212	14.561	25.154	27.009	43.938	58.818	64.298
Relative error rate (%)	2.68	13.8	7.86	6.53	0.367	0.543	1.41

From Table 3-3, it can be observed that；

1) The frequency coincidence theory can be proved through the data, and there is synergy between the data of flutter frequencies and the 1st-order torsional frequencies.

2) The prediction frequencies based on the 1st-order torsional modal and the 1st-order bending modal are showing a consistency compared with the actual flutter frequencies. In addition to the first model, the relative error rate of the formula is also shrinking with the semi-chord decreases from 0.328 to 0.020

3.3 Regular validation of flutter frequency by trapezoidal cantilever wing

On the basis of the above research, the remaining parameters are kept relatively unchanged, and the symmetrical trapezoidal cantilever model is used to further verify the eq.(7), the tip chord length is chosen as a single variable. Under standard atmospheric pressure, keeping the wing root length b_1 = 0.3m unchanged and selecting the wing tip length b_2 of 0.30, 0.25, 0.20, 0.15, 0.10, 0.05, 0.00 (unit: m) respectively. A right-angle trapezoidal model is selected (R0.1) as a control model when b_2=0.1m. The grid density of the structure model is 15×50 and is 5×10 for the aerodynamic model. The structural meshing and the aerodynamic grid partition are shown in Fig. 3-3. The first four modes for T0.2, R0.1 and T0.0 are visualized in Fig. 3-4.

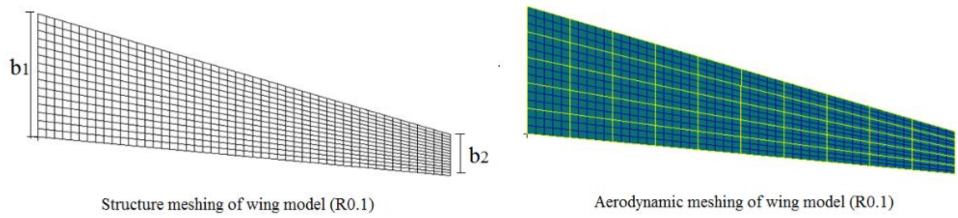

Figure 3-3　The structural meshing and the aerodynamic meshing

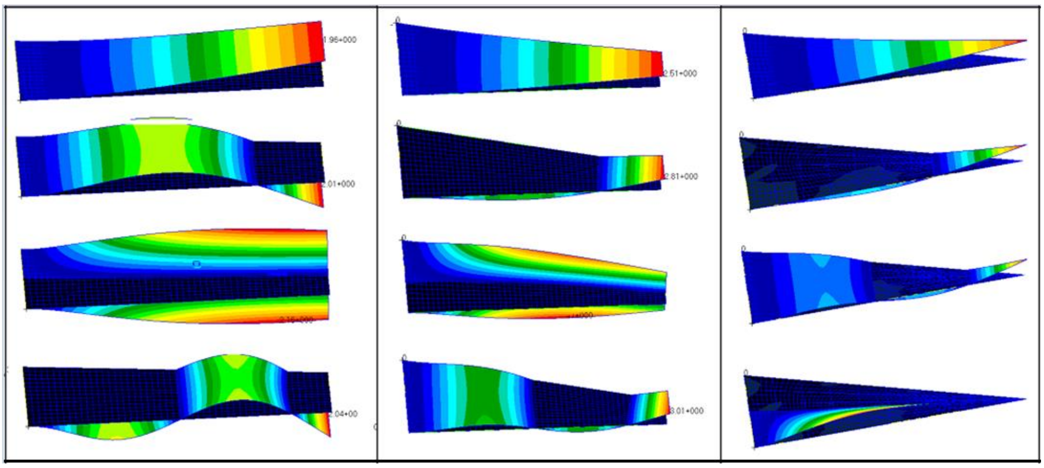

Figure 3-4　The first four - order vibration modes of T0.2、R0.1 and T0.0

According to the forecast frequency of Eq.(7), the estimated frequency can be obtained through the 1st-order torsional frequency and the 1st-order bending frequency. The necessary frequency data and the resulting error analysis of the eight models are organized in Table 3-3. And the comparison between original flutter frequencies and estimated frequencies of seven models is visualized in Fig. 3-5.

Table 3-4　Frequencies and relative error rates of eight models

Label	T0.3	T0.25	T0.2	T0.15	T0.1	T0.05	T0.0	R0.1
Wing tip length, b_2(m)	0.3	0.25	0.2	0.15	0.1	0.05	0.0	0.1
The 1st torsional frequency (Hz)	1.5067	1.5905	1.6976	1.8411	2.0473	2.3792	3.0409	2.031
The 1st bending frequency (Hz)	10.201	11.966	14.311	17.479	21.728	27.110	33.191	21.717
Flutter frequency, F(Hz)	6.5	7.3	8.1	12.8	15.4	18.8	23.1	15.3
Estimated frequency, F_0(Hz)	6.607	7.574	8.853	10.581	12.911	15.934	19.637	12.89
Error rate (%)	1.65	3.75	9.29	17.34	16.16	15.38	14.95	15.75

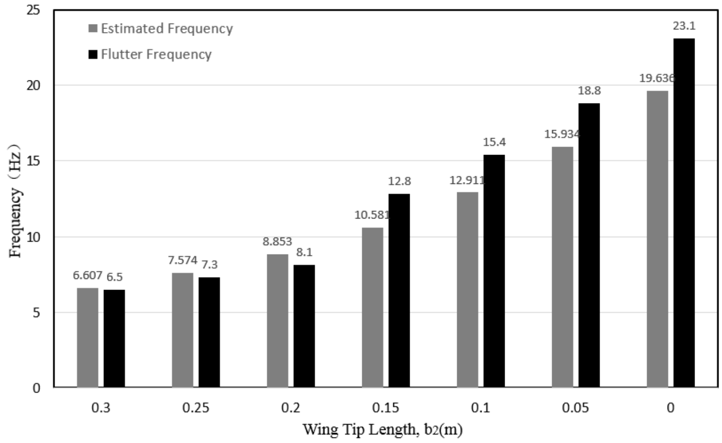

Figure 3-5 Flutter frequencies and estimated frequencies of models

Under the premise of the presumption formula, the relationship between flutter and frequency coincidence theory is analyzed as follows:

1) Through the comparison between original flutter frequencies and estimated frequencies of eight models, it can be observed that the absolute error values are controlled within 3Hz, except the triangle model (T0.0) which has a relatively obvious deviation.

2) The regularity of the Eq.(7) can be proved from Fig. 3-5 that the trend of the data obtained by the bending-torsion coupling law is basically the same as that of the flutter frequency.

3) It can be noticed that the approximation phenomenon of the previous group is different from of the second group, due to the error value does not show obvious regularity in the trapezoidal model.

4. Equivalent spring model simulation

Using the flexibility of the spring elastic support to describe a complex mechanical state of a model, it is placed in the two corresponding corner positions of the end of the cantilever model[6]. By changing the spring stiffness so that its first order vibration equal to the flutter frequency of the original model, and the modal changes after the application of the spring are compared with the flow-solid coupling states caused by the additional aerodynamic forces under the same spring model. Keep the rest of the conditions unchanged, and then add the spring at the midpoint of the end, analysis of the various modes. The inherent frequencies of two spring models are tabulated in Table 4-1.

Table 4-1 The inherent frequencies of two spring models

The flutter order	1st	2nd	3th	4th	5th
Natural frequency of 2 spring bearing model (Hz)	6.5860 (1st-bend)	20.937 (1st-torsion)	21.170 (2nd-bend)	42.724 (3th-bend)	44.817 (2nd-torsion)
Natural frequency of 3 spring bearing model (Hz)	6.6065 (1st-bend)	20.937 (1st-torsion)	21.505 (2nd-bend)	44.818 (2nd-torsion)	45.123 (3th-bend)

It can be concluded from Table 4-1 that in the 2-spring bearing model, keeping the first-order frequency equal to the flutter frequency, the first-order is still a bending model, the final spring stiffness $K = 5\times10^6$. With the same spring stiffness, the torsional frequency of the 3-spring bearing model does not change, but each bending frequency increases. The result is consistent with the flexural and torsional coupling simplified model setting.

5. Conclusion and Further work

From the results presented in this paper, it can concluded that:

1）Rectangular cantilever models with chord length as single variable: after the flutter result analysis of the cantilever structure models, consider the participation difference of the natural modals for the flutter motion, an estimated formula of the chatter frequency is deduced by the first-order torsional modal and the first-order bending modal. The relative errors between the estimated frequency and the actual flutter frequency are controlled within a relatively ideal range. In addition to the first group of values, with the reference semi-chord length decreases, the absolute error rate of the formula is also shrinking.

2) Trapezoidal cantilever models with tip chord length as single variable: analysis the relationship between the presumed formula and the flutter frequency, it can be observed that the absolute error approximation phenomenon of the first group of models does not occur in the second group of models. The main reason is that the wing tip chord changes will have an obvious disturbance to the torsional frequency, so that the value of the error does not show a clear regularity, but the frequency values remains still the same as the first group.

3) When applying the same structural analysis under the elastic support simulation, the application of the spring at the midpoint of the end of the cantilever beam does not affect the torsional frequency which conforms to the theory of plate wing analysis under the assumption of bending and the frequency coincidence theory.

4) Prospects: The speculative formula of this paper provides a convenient way to estimate the approximate range of the flutter frequency, and further confirms the simplicity of the frequency coincidence theory in dealing with some flutter cases. The model types studied in this paper are limited and the theoretical estimates are also relatively rough, thus we can refine and correct natural frequencies weights of the wing model in order to realize the more ideal prediction results in future research.

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