Examples of applications of two-stage method in calculations of statically indeterminate trusses

***Janusz Rębielak[1]**

[1]Laboratory of Building Structures, Faculty of Architecture, Cracow University of Technology, ul. Warszawska 24, 31-155 Kraków, Poland.

*Presenting author: j.rebielak@wp.pl

Abstract

The two-stage method belongs to the group of approximate methods of calculation of statically indeterminate systems because differences of the stiffness of component parts, joined in the same node, are not considered in this method. The paper presents a comparison between the values of forces calculated in the members of a selected type of statically indeterminate plane truss under load by forces applied in a symmetrical way and in an asymmetrical way. The comparison is drawn between values of forces calculated for the same trusses by the application of the two-stage method and by the application of suitable computer software. The values of the forces determined by the computer software are considered as exact results because in this case the stiffness differences of the component parts are taken into consideration. The point of the two-stage method is to remove, from the area of the statically indeterminate truss, a certain number of members, which number equals the statically indeterminacy of the basic truss. Appropriate statically determinate truss is calculated in each stage what implies, that is one of very simple methods, like for instance Cremona's method, can be used for this purpose. The trusses calculated in each stage are of the same clear span and they have identical construction depth like the basic indeterminate truss, however they are loaded by forces of half values compared to forces applied to the basic one. The final forces in the members of the statically indeterminate truss are calculated as resultants of forces defined in each stage for members having appropriate positions in the area of the truss system.

Keywords: Truss system, Calculus of vectors, Cremona's method, Superposition method, Statically indeterminate system, Approximate solution.

Introduction

Statically indeterminate systems are calculated by the application of suitable methods like for instance, the force method, the displacement method, the iterations methods, the method of successive approximations, and the finite elements method etc., which nowadays are adapted to the requirements of numerical technology and are applied in various types of computer software [1-4]. Values of forces determined by the means of these methods are considered as exact because in their calculation procedures are taken into consideration, among numerous others, the stiffness differences between members connected to the same nodes. It implies that these methods are of a very complex computational structure what further implies, that procedures developed for suitable computer calculation software have to be very complex. The two-stage method of approximate calculation of statically indeterminate trusses has been developed during initial static analysis of certain types of tension-strut structures. More comprehensive analyses of calculated forces in members of the basic geometry of the truss system are presented in papers [5,6]. In the paper are calculated trusses of similar geometry, however devoid of vertical members. Values of forces determined by means of the two-stage method are compared with the results obtained for the same structural conditions by application of suitable computer software.

Subject of static calculations

Schemes of the initial static calculations have been carried out, according to the principles of the two-stage method, for the truss systems shown in Fig. 1a-c. The basic truss system, see Fig. 1a, is a plane statically indeterminate truss with vertical members. It is loaded by concentrated loads of equal values (F) applied to each node of its upper chord. Figure 1b presents the geometry of the statically determined truss recommended for the first stage of this method together with the way of its loading. The number of members excluded from the geometry of the basic truss equals the degree of statically indeterminacy of the basic system. The scheme of the truss and the way of the loading assumed in the second stage of the proposed method is shown in Fig. 1c. Analyses of results obtained for this type of the truss system are presented in paper [7].

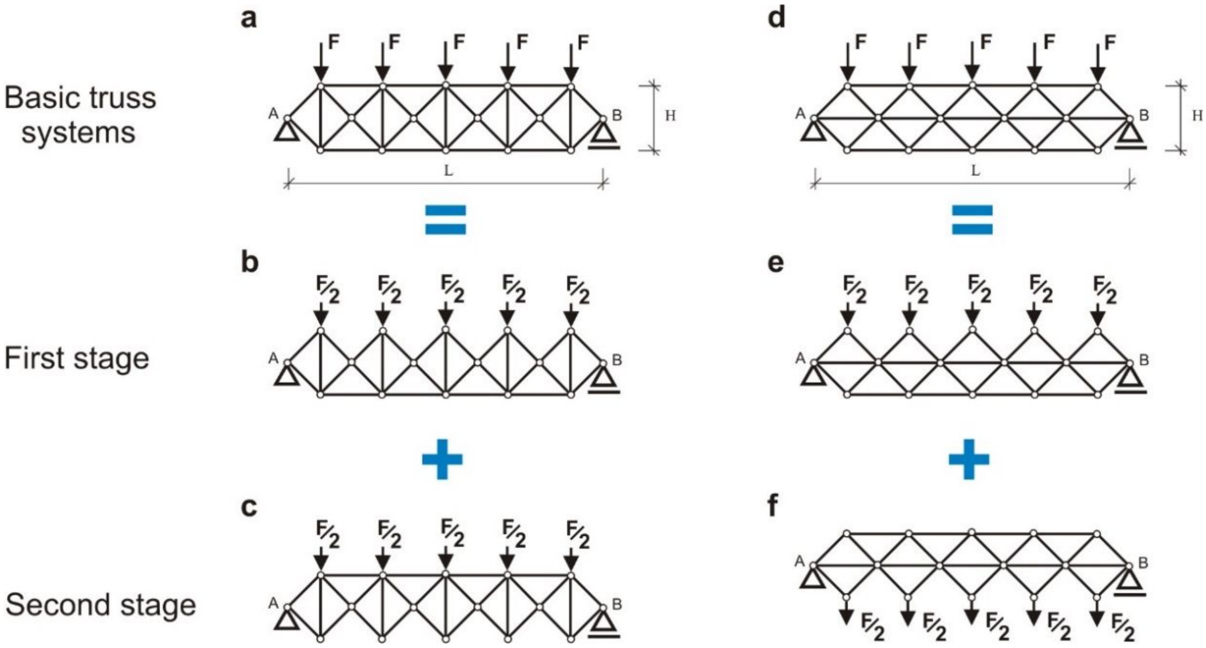


Figure 1. General schemes of calculation procedures applied for, a-c) truss system with vertical members, d-f) truss system not having vertical members

In order to reveal more characteristic features of the two-stage method a series of static calculations have been carried out for the truss system shown in Fig. 1d. The calculated truss has no vertical members but most other geometrical parameters are almost the same like of the truss shown in Fig. 1a, except horizontal members located in the middle chord. Schemes of the appropriate trusses considered in the first stage, Fig. 1e, and in the second stage, Fig. 2f, are defined according to the basic rules of the two-stage method. It was assumed that the number of nodes is defined by symbol "w", while symbol "p" defines number of members. The condition for the inner statically determinacy of the plane truss is defined as follows:

$$p = 2 \cdot w - 3 \qquad (1)$$

The considered truss system shown in Fig.1d is built by a number of nodes, w = 16, what implies that the statically determinate truss created by means of this number of nodes has to be built by the help of the following number of members:

$$29 = 2 \cdot 16 - 3 \qquad (2)$$

Because the truss shown in Fig.1d is built by the number of members, p = 33, it indicates that the considered structure is the fourfold the statically indeterminate system. It implies that in order to make the basic truss a statically determined system one should exclude 4 suitable members. Proceeding according to rules of the two-stage method in its first sage one should exclude 4 members of upper chord, see Fig 1e, and to apply half values of the unit forces to the same nodes like in the basic truss, see Fig. 1d. In the second stage it is necessary to exclude also

4 members from the lower chord, see Fig. 1f, but in this case suitable load forces have to be applied also to nodes of the lower layer. In each stage the statically determinate truss is calculated, due to which one can apply one of the simple methods like e.g. Cremona's method, to determine values of the forces in its members. Final values of the forces in the basic system will be resultants of forces calculated in each stage for members of appropriate positions. All the assumptions are congruent with rules of calculus of vectors, principle of superposition and with the three fundamental conditions of equilibrium presented below:

$$\sum_{i=1}^{n} F_{ix} = 0 \tag{3}$$

$$\sum_{i=1}^{n} F_{iy} = 0 \tag{4}$$

$$\sum_{i=1}^{n} M_{i} = 0 \tag{5}$$

The statically indeterminate truss system, of geometry of members shown in Fig. 1d, is subject of the static calculations. It was assumed that the basic truss is subjected to two types of loading. In the first one the unit load forces F of value equal to 1,00 kN, are symmetrically applied to all nodes of upper chord. In the second type of loading these unit forces F are applied in an asymmetric way only along a part of its clear span. In this case it means that these loads are applied only to two successive nodes of the upper chord, which are adjacent e.g. to support node B. Basic structures are of 5,00 m clear span and their construction depth is equal to 1,00 m.

Values of forces calculated for symmetrically loaded truss

The first static calculations have been made for symmetrical loading of the basic truss. In the first stage the concentrated forces, each of value equal to 0,50 kN, are uniformly distributed to all nodes of the upper chord of a suitably assumed scheme of the statically determinate truss. Values of forces defined in this stage for all members of the truss, together with appropriate Cremona's polygon of forces, are shown in Fig. 2. In the horizontal member located, for example, between nodes 12 and 13 acts as a tension force of value equal to +3,00 kN. Because in this stage the truss member is not present between node number 3 and node number 4, that is why for this area the force value is estimated as equal to zero. In this stage the value of the compression force calculated for cross brace placed between nodes 3 and 8 equals -0,353 kN. The horizontal member placed between e.g. node 8 and node 9 acts compression force of value equal to -2,250 kN.

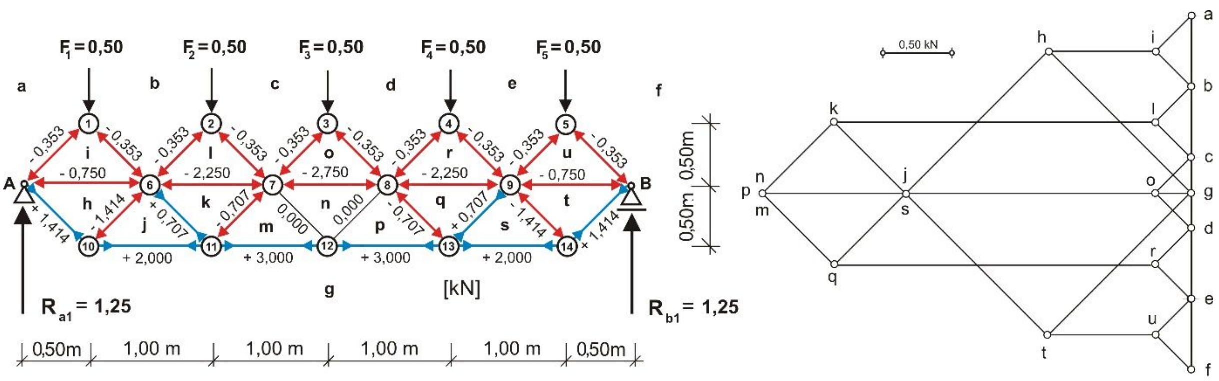


Figure 2. Values of forces determined in the first stage of calculations for symmetrically loaded truss together with Cremona's polygon of forces

Static scheme of the investigated truss considered in the second stage of calculations, together with their results and Cremona's polygon of forces, are shown in Fig. 3.

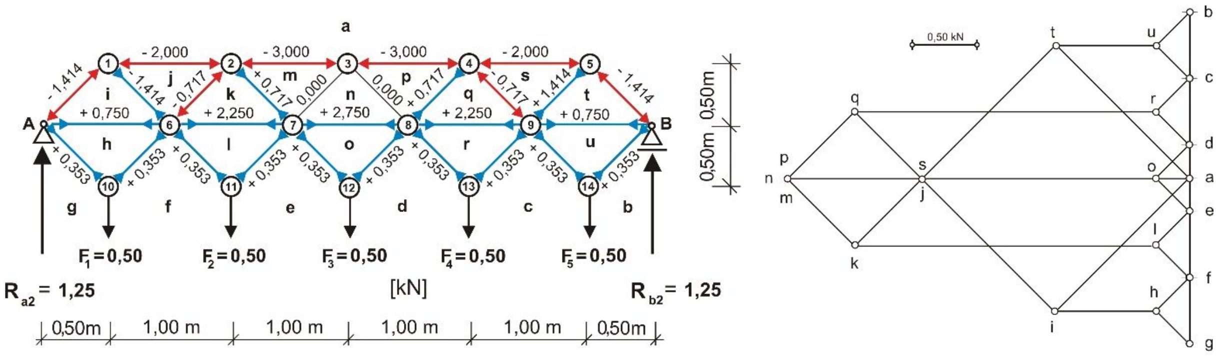
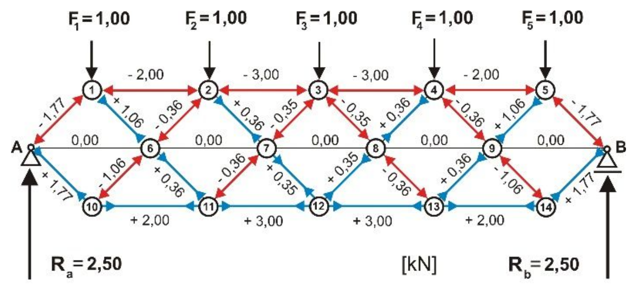


Figure 3. Values of forces calculated in the second stage of calculations for symmetrically loaded truss together with Cremona's polygon of forces

The values of the forces calculated in the second stage for some counterpart members are following. In the cross brace located between nodes 3 and 8 acts the force of zero value, a horizontal member placed between nodes 8 and 9 is subjected to an act of tension force having the value +2,250 kN, see Fig. 3. Moreover in the member of the upper chord located between nodes 3 and 4 acts compression force of the value equal to -3,000 kN. The final values of the forces calculated by the means of the two-stage method in members of the basic form of statically indeterminate truss are presented in Fig. 4a. For instance the final value of force acting in the cross brace placed between nodes 3 and 8 is defined as -0,35 kN and it is the resultant of the force value calculated for the same member in the first stage, (-0,353 kN) and the force value calculated in the second stage (0,000 kN). Similar procedures have been done for all members of the calculated truss.

a



b

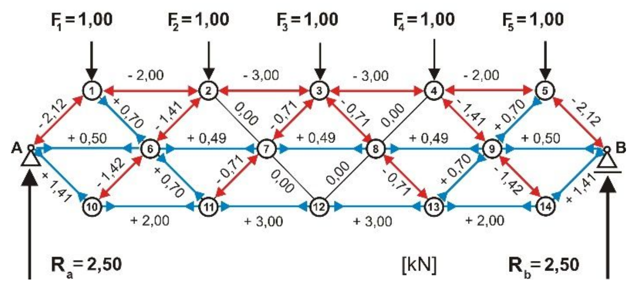


Figure 4. Values of forces in members of symmetrically loaded truss calculated, a) in the two-stage method, b) by means of suitable computer software

The same basic truss has been subjected to static calculation carried out by the application of the Autodesk Robot Structural Analysis Professional 2016, for which the software is designed for the precise calculation of force values acting in members of the statically indeterminate systems. Static calculations were made by assumption that the truss consists of steel tubular members having diameter of 30.00 mm, the thickness of the section equals to 4.00 mm and the steel material has the Young's modulus equal to 210 GPa. Results achieved in this way are presented in Fig. 4b. From the analysis of results obtained from both compared methods for the same members follows, that the biggest forces calculated for the same members are of the same values and of the same sense. This remark refers to the forces acting in the members of the outer chords of the truss. One can notice bigger differences in values of the forces calculated in the cross braces. For instance the value of the compression force defined in the two-stage method in members located, for example, between node 5 and support node B is equal to -1,77 kN, while by application of computer software the compression force acting in the same cross brace is estimated as -2,12 kN, which constitutes about 16,5 % of the biggest value. Much bigger relative differences are noticeable in other members, however the absolute values of these forces are rather very small.

Values of forces calculated for asymmetrically loaded truss

The second group of calculations have been made for the same basic form of the statically indeterminate truss but this time loaded in an asymmetric way. In this case the load forces are applied only to two nodes of upper chord, which are located in the close vicinity to the support node B. In the first stage of the two-stage method there are removed four members of the upper chord and then to suitable nodes of this chord are applied load forces of half unit, which are of value equal to 0,50 kN, see Fig. 5.

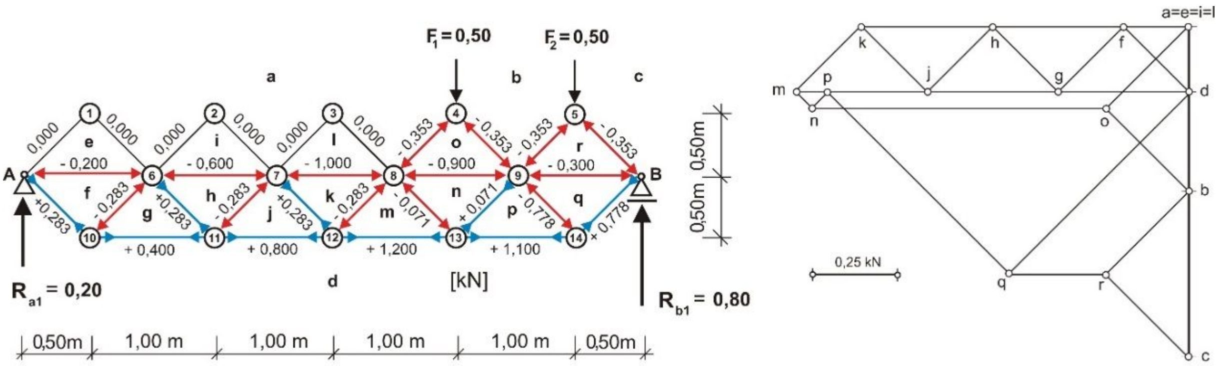


Figure 5. Values of forces determined in the first stage of calculations for asymmetrically loaded truss together with Cremona's polygon of forces

According to the rules of the two-stage method in its second stage it is considered a statically determinate truss, the pattern of which is defined by removing four members from the bottom chord, see Fig. 6. Then to appropriate nodes of this chord are applied load forces having a value of half of the unit load. In this case values of the load forces are also equal to 0,50 kN. The final values of the forces calculated by the means of the two-stage method in members of the investigated truss are sown in Fig. 7a.

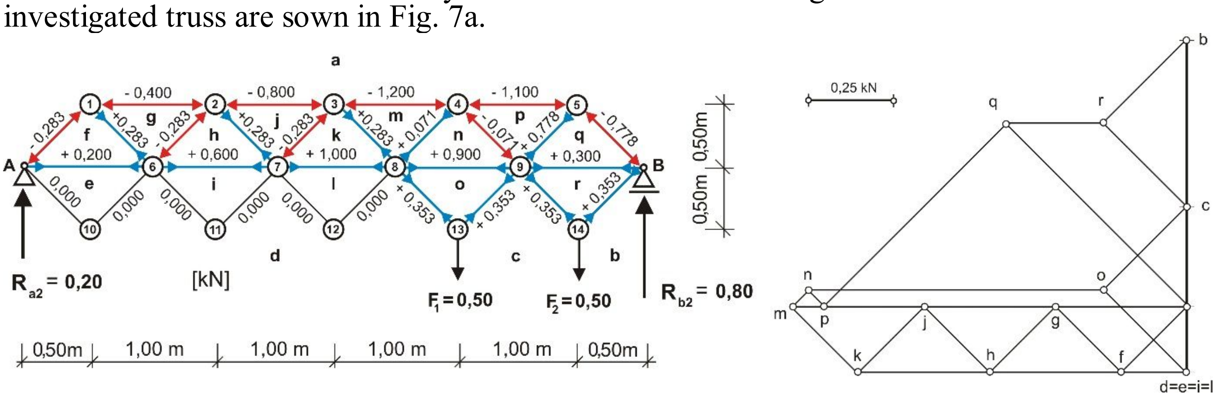


Figure 6. Values of forces defined in the second stage of calculations for asymmetrically loaded truss together with Cremona's polygon of forces

A statically indeterminate truss of the same static scheme was calculated also by the application of the Autodesk Robot Structural Analysis Professional 2016 for identical material parameters of component parts as it was previously listed. The results of these calculations are presented in Fig. 7b. The values of the forces acting in the members of the outer chords calculated in both methods are identical. Differentiation of force values defined for other members of the basic statically indeterminate structure is bigger. For instance the value of the tension force calculated in the two-stage method for the cross brace placed between node 8 and 13 equals +0,28 kN, see Fig. 7a, while by application of computer software in the same member is defined a compression force of value equals -0,07 kN, see Fig. 7b. In this case forces calculated for the same member in two compared methods are of different values as well as of various senses. In spite of this relatively big differences one should notice that they appear only in members subjected to act of the smallest values of forces.

a

b

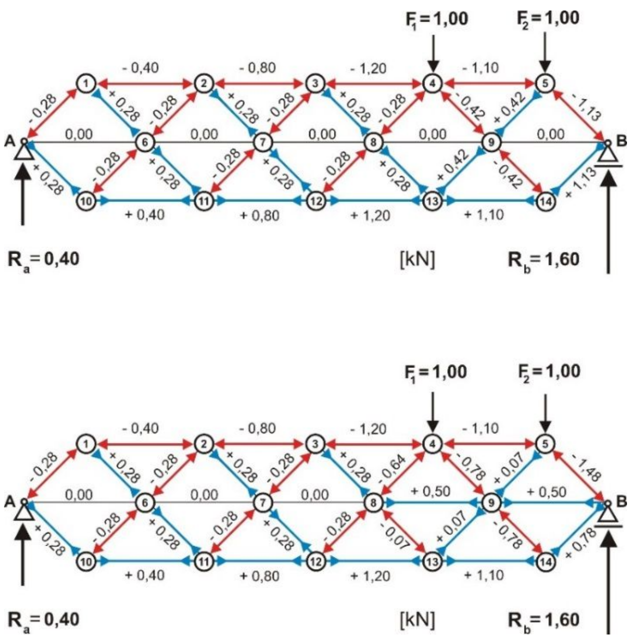


Figure 7. Values of forces in members of asymmetrically loaded truss calculated, a) in the two-stage method, b) by means of suitable computer software

Conclusions

The two-stage method for the calculation of the statically indeterminate trusses easy produces results which closely approximate the values of forces determined by the application of the exact methods of calculations. Accuracy of the results obtained by the two-stage method can be significantly enhanced by the application of suitable sets of coefficients determined individually for each node, due to which the stiffness differences between members connected in a particular node can be taken into consideration during definition of the final values in the calculation processes.

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