Linear Dynamic Reanalysis Using Frequency-Shift Combined Approximations

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Abstract

In the structural dynamic optimization procedure, many repeated analyses are conducted to evaluate dynamic performance of successively modified structural designs. It is noted that, the reduction of degrees of freedom is more important for computational effort in dynamic problems than in static problems. This paper focuses on the reanalysis for structural dynamic problem in the framework of combined approximations (CA) method. A new procedure for structural dynamic reanalysis is developed based on iteration and inverse iteration method with frequency-shift in mode superposition method, and linear combination acceleration is also used to reduce the high computational cost of structural reanalysis. Frequency-shift factor is calculated first, and then combined approximations method corrected by the given factor allows calculating higher modes accurately. After higher modes are obtained, mode expansion and dynamic response can be described accurately. Numerical example is presented to demonstrate the accuracy of the proposed method. Excellent results can be obtained when large modifications are made.

Keywords: Frequency-Shift, Combined Approximations, Mode Superposition, Dynamic Reanalysis.

Introduction

In order to make a design structure satisfy the predetermined demands, such as the structural dynamic design procedure, usually the designer will modify the structure repeatedly. The dynamic responses changed by the modifications of parameters on the structure. In the structural optimization, dynamic analyses are repeated in successively modified structure design procedure. Research of how to reduce the computational cost has made sense.

Reanalysis technology was established to evaluate responses of changed structures without complete analysis in process of design and optimization[1]. Reanalysis of structure for displacements and stresses have been discussed since the 20th century[2]. Combined Approximations (CA) approach is one of the most effective methods for solving static displacement equations[3]. After CA method was founded, extended CA methods were proposed[4]. IFU method is proposed for general low-rank local modifications, including boundary modifications and non-boundary modifications[5].

Reanalysis methods for vibration problems have been presented since the early 21st century[6]. Kirsch grafted the CA approach to solve eigenproblems [7]. Combining CA and Rayleigh quotient, an extended CA method of eigenproblem for large changes was presented by Chen[8]. A Modified Combined Approximations(MCA) method for solving large-scale structure dynamic problem was discussed[9]. With a suitable frequency shift coefficient, FSCA approach allowed to calculate higher modes accurately[10].

Some studies in the literature have approached the structural dynamic reanalysis problem for large perturbations in the structural parameters. A method for the dynamic reanalysis of structures subjected to deterministic or stochastic loads is presented by Cacciola [11]. The CA approach, developed originally for linear static reanalysis, is also used for dynamic reanalysis of structures by Kirsch[12, 13]. The approach is based on the integration of several concepts and methods, including series expansion, reduced basis, matrix factorization, and Gram-Schmidt orthogonalizations. Based on epsilon-algorithm, the dynamic response reanalysis method has been developed by Chen[14]. In his computational process, the Neumann series expansion was used to construct the vector sequence for epsilon-algorithm iterative form.

In this study, a linear dynamic reanalysis process using FSCA method is proposed. The formulations of mode superstition based on CA method with frequency shift are expressed, and then the application of this algorithm to a truck body finite element analysis is described. Conclusions are discussed at last.

Linear dynamic analysis by mode superposition

Linear dynamic analysis consider the equations of motion for a system subjected to external dynamic forces

$$\mathbf{M}\,{}^t\ddot{\mathbf{u}} + \mathbf{C}\,{}^t\dot{\mathbf{u}} + \mathbf{K}\,{}^t\mathbf{u} = {}^t\mathbf{R} \tag{1}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is the damping matrix and $\mathbf{K}$ is the stiffness matrix. The displacement vector ${}^t\mathbf{u}$, the velocity vector ${}^t\dot{\mathbf{u}}$, the acceleration vector ${}^t\ddot{\mathbf{u}}$ and the load vector ${}^t\mathbf{R}$ are functions of the time t.

In practical analysis, the common procedures can be solved by mode superposition method, where the equilibrium equations are transformed in to a form in which only limited modes are considered. In this approach a change of basis from the finite element nodal displacements to the eigenvectors of the generalized eigenproblem is preformed prior to the time integration. The following transformation is used.

$${}^t\mathbf{u} = \mathbf{\Phi}\,{}^t\mathbf{X} \tag{2}$$

where $\mathbf{\Phi}$ is $n \times n$ transformation matrix and components of ${}^t\mathbf{X}$ are the modal coordinates. With the transformation, new system stiffness, damping and mass matrices are obtained, which are much smaller scale than those in the original system. And then, a new dynamic equation is obtained.

$$\mathbf{\Phi}^T\mathbf{M}\mathbf{\Phi}\,{}^t\ddot{\mathbf{X}} + \mathbf{\Phi}^T\mathbf{C}\mathbf{\Phi}\,{}^t\dot{\mathbf{X}} + \mathbf{\Phi}^T\mathbf{K}\mathbf{\Phi}\,{}^t\mathbf{X} = \mathbf{\Phi}^T\,{}^t\mathbf{R} \tag{3}$$

where

$$\tilde{\mathbf{M}} = \mathbf{\Phi}^T\mathbf{M}\mathbf{\Phi} \qquad \tilde{\mathbf{C}} = \mathbf{\Phi}^T\mathbf{C}\mathbf{\Phi} \qquad \tilde{\mathbf{K}} = \mathbf{\Phi}^T\mathbf{K}\mathbf{\Phi} \qquad {}^t\tilde{\mathbf{R}} = \mathbf{\Phi}^T\,{}^t\mathbf{R} \tag{4}$$

The transformation matrix $\mathbf{\Phi}$ is the orthogonalized modal displacement solution of the free vibration.

$$\mathbf{I} = \mathbf{\Phi}^T \mathbf{M} \mathbf{\Phi} \qquad\qquad \mathbf{\Lambda} = \mathbf{\Phi}^T \mathbf{K} \mathbf{\Phi} \tag{5}$$

where $\mathbf{I}$ is the identity matrix and $\mathbf{\Lambda}$ is the spectral matrix. Then the equilibrium equation correspond to the orthogonalized modal displacements is described.

$${}^t\ddot{\mathbf{X}} + \mathbf{\Phi}^T \mathbf{C} \mathbf{\Phi}\, {}^t\dot{\mathbf{X}} + \mathbf{\Lambda}\, {}^t\mathbf{X} = \mathbf{\Phi}^T\, {}^t\mathbf{R} \tag{6}$$

When damping effects are not considered, Eq.(6) becomes

$${}^t\ddot{\mathbf{X}} + \mathbf{\Lambda}\, {}^t\mathbf{X} = \mathbf{\Phi}^T\, {}^t\mathbf{R} \tag{7}$$

The individual equation is of the form

$${}^t\ddot{\mathbf{X}}_i + \lambda_i\, {}^t\mathbf{X}_i = \boldsymbol{\varphi}_i^T\, {}^t\mathbf{R} \tag{8}$$

The initial conditions at time 0 are obtained by Eq.(9).

$${}^0\mathbf{X} = \mathbf{\Phi}^T \mathbf{M}^{(0)} \mathbf{u} \qquad\qquad {}^0\dot{\mathbf{X}} = \mathbf{\Phi}^T \mathbf{M}^{(0)} \dot{\mathbf{u}} \tag{9}$$

Mode reanalysis by FSCA method

In practice, natural frequency has the same mean with eigenvalue in mathematics. The equation of the first m eigenvalues and eigenvectors can be expressed:

$$\underset{n\times n}{\mathbf{K}^{(0)}} \underset{n\times m}{\mathbf{\Phi}^{(0)}} = \underset{n\times n}{\mathbf{M}^{(0)}} \underset{n\times m}{\mathbf{\Phi}^{(0)}} \underset{m\times m}{\mathbf{\Lambda}^{(0)}} \tag{10}$$

where $\mathbf{\Lambda}^{(0)}$ denotes the matrix of the first m eigenvalues and $\mathbf{\Phi}^{(0)}$ is the corresponding matrix of first m eigenvectors, *n* is DoFs for the initial system. Assuming there are changes in the stiffness and mass matrices, respectively.

$$\mathbf{K} = \mathbf{K}^{(0)} + \Delta\mathbf{K} \qquad\qquad \mathbf{M} = \mathbf{M}^{(0)} + \Delta\mathbf{M} \tag{11}$$

The eigenproblem of the changed structure can be rearranged:

$$\underset{n\times n}{\mathbf{K}} \underset{n\times m}{\mathbf{\Phi}} \underset{m\times m}{\mathbf{\Lambda}^{-1}} = \underset{n\times n}{\mathbf{M}} \underset{n\times m}{\mathbf{\Phi}} \tag{12}$$

where $\mathbf{\Lambda}$ denotes the matrix of the first m eigenvalues and $\mathbf{\Phi}$ is the corresponding matrix of first m eigenvectors for the changed structure.

Eq.(12) is rearranged using a frequency-shift factor:

$$\underset{n\times m}{\mathbf{\Phi}} = [(\mathbf{M} - \mu^{-1}\mathbf{K})^{-1} \underset{n\times n}{\mathbf{K}}]\, \underset{n\times m}{\mathbf{\Phi}} [\mathbf{\Lambda}^{-1} - \mu^{-1} \underset{m\times m}{\mathbf{I}}\,] \tag{13}$$

Given an initial $\mathbf{\Phi}^{(i)}$, we can compute $\mathbf{\Phi}^{(i+1)}$ by solving iterative formula as Eq.(14).

$$\underset{n\times m}{\mathbf{\Phi}^{(i+1)}} = (\mathbf{M} - \mu^{-1}\mathbf{K})^{-1} \underset{n\times n}{\mathbf{K}} \underset{n\times m}{\mathbf{\Phi}^{(i)}} (\mathbf{\Lambda}^{-1} - \mu^{-1}\mathbf{I}) \tag{14}$$

Assuming that a linear expression of $\mathbf{\Phi}^{(i)}$, where $i = 0,1,\cdots,s-1$, can be close to the exact solutions, the linear expression is given:

$$\begin{aligned}
\underset{n\times m}{\mathbf{\Phi}_c} &= a_0 \underset{n\times m}{\mathbf{\Phi}^{(0)}} + a_1 \underset{n\times m}{\mathbf{\Phi}^{(1)}} + a_2 \underset{n\times m}{\mathbf{\Phi}^{(2)}} + \cdots + a_{s-1} \underset{n\times m}{\mathbf{\Phi}^{(s-1)}} \\
&= a_0 \underset{n\times m}{\mathbf{\Phi}^{(0)}} + a_1 (\mathbf{M} - \mu^{-1}\mathbf{K})^{-1} \underset{n\times n}{\mathbf{K}} \underset{n\times m}{\mathbf{\Phi}^{(0)}} (\mathbf{\Lambda}^{-1} - \mu^{-1}\mathbf{I}) + a_2 ((\mathbf{M} - \mu^{-1}\mathbf{K})^{-1} \underset{n\times n}{\mathbf{K}})^2 \underset{n\times m}{\mathbf{\Phi}^{(0)}} (\mathbf{\Lambda}^{-1} - \mu^{-1}\mathbf{I})^2 \\
&\quad + \cdots + a_{s-1} ((\mathbf{M} - \mu^{-1}\mathbf{K})^{-1} \underset{n\times n}{\mathbf{K}})^{s-1} \underset{n\times m}{\mathbf{\Phi}^{(0)}} (\mathbf{\Lambda}^{-1} - \mu^{-1}\mathbf{I})^{s-1} \\
&= [\underset{n\times m}{\mathbf{\Phi}^{(0)}}, (\mathbf{M} - \mu^{-1}\mathbf{K})^{-1} \underset{n\times n}{\mathbf{K}} \underset{n\times m}{\mathbf{\Phi}^{(0)}}, ((\mathbf{M} - \mu^{-1}\mathbf{K})^{-1} \underset{n\times n}{\mathbf{K}})^2 \underset{n\times m}{\mathbf{\Phi}^{(0)}}, \cdots, ((\mathbf{M} - \mu^{-1}\mathbf{K})^{-1} \underset{n\times n}{\mathbf{K}})^{s-1} \underset{n\times m}{\mathbf{\Phi}^{(0)}}] \\
&\quad \cdot \left[a_0 \mathbf{I}, a_1 (\mathbf{\Lambda}^{-1} - \mu^{-1}\mathbf{I}), \quad a_2 (\mathbf{\Lambda}^{-1} - \mu^{-1}\mathbf{I})^2 \quad \cdots \quad a_{s-1} (\mathbf{\Lambda}^{-1} - \mu^{-1}\mathbf{I})^{s-1} \right]^T \\
&= \underset{n\times ms}{\mathbf{R}} \underset{ms\times m}{\mathbf{X}}
\end{aligned} \tag{15}$$

Premultiplying Eq.(12) by $\mathbf{R}^T$, a condensed equation is got and expressed in the following form:

$$[\mathbf{R}^T \underset{ms\times ms}{\mathbf{K}} \mathbf{R}] \underset{ms\times m}{\mathbf{X}} = [\mathbf{R}^T \underset{ms\times ms}{\mathbf{M}} \mathbf{R}] \underset{ms\times m}{\mathbf{X}} \underset{m\times m}{\mathbf{\Lambda}} \tag{16}$$

The matrices $[\mathbf{R}^T \mathbf{K} \mathbf{R}]$ and $[\mathbf{R}^T \mathbf{M} \mathbf{R}]$ of the condensed system are much smaller than those in the initial system. So we can calculate a new $ms \times ms$ system in Eq.(16) instead. The computing time can be greatly reduced.

Frequency shift consideration

For the purpose of improving the accuracy of the higher modes calculation and eliminate the numerical errors, the approximate modes and basis vectors are recalculated using Gram-Schmidt orthogonalizations in FSCA method

The advantage of the shift factor is that more accuracy results are obtained. In FSCA method, to improve the accuracy of higher modes calculation, the highest mode vector is chosen to generate the frequency shift factor.

$$\mu^{(i+1)} = \frac{\underset{n\times 1}{\boldsymbol{\varphi}_m^{(i)}}{}^T \underset{n\times n}{\mathbf{K}} \underset{n\times 1}{\boldsymbol{\varphi}_m^{(i)}}}{\underset{n\times 1}{\boldsymbol{\varphi}_m^{(i)}}{}^T \underset{n\times n}{\mathbf{M}} \underset{n\times 1}{\boldsymbol{\varphi}_m^{(i)}}} \tag{17}$$

where $\underset{n\times 1}{\varphi_m^{(i)}}, i = 0,\cdots,s-1$ is the highest mode in the i th iteration. Considering the increasing computational cost for $\mu^{(i+1)}$ calculations, the Rayleigh quotient Eq.(18) is chosen for the frequency-shift factor in FSCA method instead of Eq.(17). The numerical example demonstrates that the frequency-shift factor is effective.

$$\mu = \frac{\underset{n\times1}{\boldsymbol{\varphi}_m}^{(0)T} \underset{n\times n}{\mathbf{K}} \underset{n\times1}{\boldsymbol{\varphi}_m}^{(0)}}{\underset{n\times1}{\boldsymbol{\varphi}_m}^{(0)T} \underset{n\times n}{\mathbf{M}} \underset{n\times1}{\boldsymbol{\varphi}_m}^{(0)}} \tag{18}$$

Numerical example

A truck body dynamic reanalysis numerical example, as shown in Fig.1, is given to demonstrate the accuracy of the FSCA method for large scale dynamic reanalysis. The objective is to evaluate the response of the seat for loading on the left rear wheel. The truck body contains 1896 shell and solid elements, 1944 nodes and 11664 degrees of freedom. The time step is 0.1s. The Young's modulus of the material is $E = 2.1\times10^{11} Pa$; the mass density is $\rho = 7.8\times10^3 kg/m^3$; the Poisson's ratio is 0.3. Assuming that the response of the initial structure is known, the modified response has been evaluated by the FSCA approach with only 3 basis vectors.

The resulting vertical displacement, velocity and acceleration at the seat installation point are shown in Fig.2-4. It is observed that good agreement is obtained between solutions of the FSCA formulations and Lanczos formulations of the modified structure.

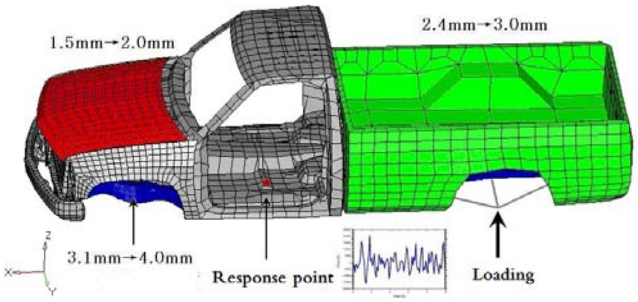

Figure 1. Modifications of truck body

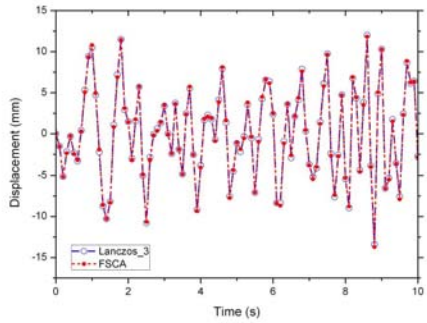

Figure 2. Comparison of displacement responses

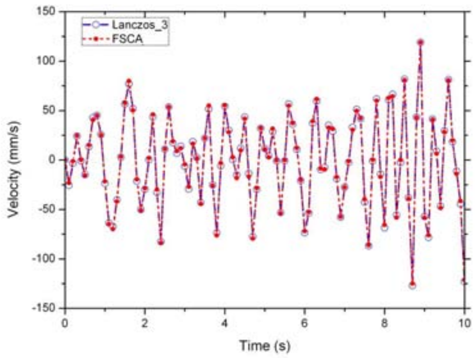

Figure 3. Comparison of velocity responses

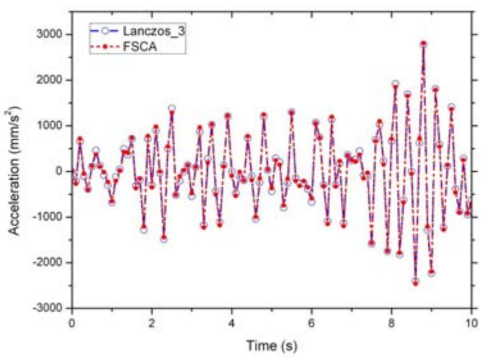

Figure 4. Comparison of acceleration responses

Conclusions

In this study, a new reanalysis technique, the FSCA method has been developed for dynamic reanalysis with respect to improve the solution accuracy in case where global large modifications are made. Numerical example is shown for the demonstrations of accuracy in this work. It can be seen that the accuracy approximate solutions were achieved with FSCA method with large changes.

When general optimization problems are considered, a lot of research has been performed to reduce the computational cost in repeated analysis of modified structures. It is expected that the FSCA method could reduce the overall computational cost in problems where repeated analyses are needed.

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References

[1] Kirsch U. (2010) Reanalysis and sensitivity reanalysis by combined approximations, *Struct Multidiscip Optim* **40**, 1-15.

[2] Phansalkar SR. (1974) Matrix iterative methods for structural reanalysis, *Comput Struct* **4**, 779-800.

[3] Kirsch U. (2003) A unified reanalysis approach for structural analysis, design, and optimization, *Struct Multidiscip Optim* **2003**, 67-85.

[4] Zuo W, Yu Z, Zhao S, Zhang W. (2012) A hybrid Fox and Kirsch's reduced basis method for structural static reanalysis, *Struct Multidiscip Optim* **46**, 261-272.

[5] Huang GX, Wang H, Li GY. (2016) An exact reanalysis method for structures with local modifications, *Struct Multidiscip Optim* **54**, 499-509.

[6] Chen SH, Yang XW, Lian HD. (2000) Comparison of several eigenvalue reanalysis methods for modified structures, *Struct Multidiscip Optim* **20**, 253-259.

[7] Kirsch U. (2003) Approximate vibration reanalysis of structures, *AIAA J* **41**, 504-511.

[8] Chen SH, Yang XW. (2000) Extended Kirsch combined method for eigenvalue reanalysis, *AIAA J* **38**, 927-930.

[9] Zhang G, Nikolaidis E, Mourelatos ZP. (2009) An efficient re-analysis methodology for probabilistic vibration of large-scale structures, *J Mech Design* **131**,

[10] Xu T, Guo G, Zhang H. (2011) Vibration reanalysis using frequency-shift combined approximations, *Struct Multidiscip Optim* **44**, 235-246.

[11] Cacciola P, Impollonia N, Muscolino G. (2005) A dynamic reanalysis technique for general structural modifications under deterministic or stochastic input, *Comput Struct* **83**, 1076-1085.

[12] Kirsch U, Bogomolni M, Sheinman I. (2007) Efficient Dynamic Reanalysis of Structures, *Journal of Structural Engineering* **133**, 440-448.

[13] Kirsch U, Bogomolni M, Sheinman I. (2006) Nonlinear dynamic reanalysis of structures by combined approximations, *Computer Methods in Applied Mechanics & Engineering* **195**, 4420-4432.

[14] Chen SH, Ma L, Meng GW. (2008) Dynamic response reanalysis for modified structures under arbitrary excitation using epsilon-algorithm, *Comput Struct* **86**, 2095-2101.