Fluid-structure interaction eigenvalue analysis by using a coupled FE-BE solver

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Abstract

The vibration behavior of thin elastic structures is noticeably influenced by the surrounding water, which represents a heavy fluid. In this case, the feedback of the fluid pressure onto the structures cannot be neglected and a strong coupling scheme between the structural domain and the fluid domain is required. In this paper, a coupled finite element and boundary element (FE-BE) solver is developed for the modal analysis of three-dimensional submerged elastic structures. The structures are modeled by means of the finite element method (FEM). The compressibility of the surrounding fluid is taken into consideration, and thus the Helmholtz equation is used as the governing equation and solved by using the boundary element method (BEM). The resulting nonlinear eigenvalue problem (NEVP) is converted into a small linear one by using a contour integral method. A numerical example is finally given to demonstrate the effectiveness and applicability of the developed method.

Keywords: Fluid-structure interaction, Modal analysis, Coupled FE-BE method, Nonlinear eigenvalue problem, Contour integral method

Introduction

Finding resonances allows the designers to anticipate the structural vibration and to ensure that the resonance frequencies are distinct from those of the vibrating sources. In engineering applications, it is common to apply the FEM to perform the modal analysis of a structure *in vacuo* due to the high flexibility and applicability of the FEM to large-scale models. However, when the structure is submerged in a heavy fluid, e.g., water, a strong interaction between the structural domain and the fluid domain occurs and noticeably alters the resonance frequencies, especially for thin elastic structures [1]. Numerical simulations of the vibro-acoustic behavior of submerged structures usually require dealing with the fluid-structure interaction (FSI) since the feedback of the fluid pressure onto the structures can not be neglected for a heavy fluid. Thus, a scheme which takes the effect of the fluid on the structure into account should be used. Although some techniques such as the perfectly matched layer (PML) can be used to simulate the infinite fluid domains, the FEM still has some troubles in solving exterior problems, for instance the questions related to the position, size and parameter settings of the PML, and also the consequently big discretized model. By contrast, the BEM is much more favorable for the numerical solution of exterior problems since only the boundary of the structural domain has to be discretized and the Sommerfeld radiation condition can be satisfied automatically by the choice of the fundamental solution [2]. As a result, the coupled FE-BE methods are usually preferred for the numerical solution of the FSI problems [3][4].

In the numerical modal analysis of submerged structures, the fluid is sometimes assumed to be incompressible and hence modeled by the Laplace equation for simplicity [5]. The effect of the fluid on the structure can be regarded as adding mass and then a generalized eigenvalue problem (GEVP) which is easy to solve can be obtained. However, when the compressibility of the fluid is taken into account and thus the fluid is modeled by the Helmholtz equation, the resulting eigenvalue problem is nonlinear since the frequency parameter appears nonlinearly in the boundary integral formulations of the Helmholtz equation. To solve such a NEVP, one scheme is to set up a GEVP by treating the term involving wave number in the Helmholtz equation as a non-homogeneous term. The fundamental solution of the Laplace equation is applied in the boundary integral formulations instead of that of the Helmholtz equation. The volume integrals caused by the non-homogeneous term can be transformed into boundary integrals by means of various methods, such as the dual reciprocity method [6] and the radial integration method [7]. Some other schemes which are presented for instance in [8][9] are based on the polynomial approximations of the coupled FE-BE coefficient matrix.

In addition to the approaches mentioned above, a group of methods based on contour integrals [10]-[12] have been recently developed. Through the use of these methods, a NEVP can be easily converted into a GEVP whose dimension is much smaller than the original NEVP. The eigenvalues lying inside a domain enclosed by a prescribed contour path can then be extracted by solving the small GEVP. The conversion is achieved directly by solving a series of linear systems of equations along the contour path. Since these systems of equations are independent and in the similar form as the ones arising in the response analysis, the big advantages of the contour integral methods are that they are very easy to be implemented and more suitable to be parallelized effectively. So far some of these methods have already been applied to solve some NEVPs in engineering applications. For instance, the method proposed by Asakura *et al.* [10] has been applied to solve the acoustic eigenvalue problems in [13] and to conduct the band structure analysis of phononic crystals in [14]. Kimeswenger *et al.* [15] analyzed the approximation of an FSI eigenvalue problem and used Beyn's method [11] for the numerical solution of the discretized NEVP. In this paper, a coupled FE-BE solver is developed for the modal analysis of three-dimensional submerged structures. The resulting NEVP is converted into a small linear one by using the contour integral method proposed by Asakura *et al.* [10]. Numerical implementation of the method in the FSI eigenvalue problems is given and some discussions are also given to further improve the efficiency and effectiveness of the method. A numerical example is employed finally to demonstrate the applicability and effectiveness of the developed FSI modal analysis method.

Formulation

In this section, a NEVP is first formulated for the modal analysis of an elastic structure which is submerged in an infinite fluid domain. The NEVP is then converted into a small GEVP by using a contour integral method proposed in [10]. Numerical implementation of the method in the modal analysis of submerged elastic structures is given in detail and some discussions are also given to improve the efficiency and avoid missing the resonance frequencies of interest.

FSI eigenvalue problems

Modal analysis of an elastic structure which is submerged in an infinite compressible inviscid fluid domain is discussed in this paper. If the structure is subjected to a time-harmonic load, we can derive an FEM system of equations in the frequency domain as

$$\left(\mathbf{K}-\omega^2\mathbf{M}\right)\mathbf{u}=\mathbf{f}_s+\mathbf{f}_f \tag{1}$$

where $\mathbf{K}$ and $\mathbf{M}$ are the global stiffness and mass matrices of the structure, ω is the circular frequency, $\mathbf{u}$ is the nodal displacement vector, $\mathbf{f}_s$ and $\mathbf{f}_f$ are the vectors with respect to the nodal values for the structural excitation force and fluid interaction force, respectively.

Because the compressibility of the fluid is taken into account in this paper, the propagation of time-harmonic acoustic waves in the fluid is described by the Helmholtz equation, which can be recast into a Kirchhoff-Helmholtz boundary integral equation (HBIE). It is widely known that the BEM based on the HBIE suffers from the fictitious eigenfrequency problem or the non-unique solution difficulty at the eigenfrequencies of the associated interior problems [16]. To overcome this difficulty, the Burton-Miller formulation [16] which is a linear combination of the HBIE and its normal derivative is adopted in this paper. Discretizing the HBIE or the Burton-Miller formulation and collecting the equations for all collocation points allow us to obtain a BEM system of equations as

$$\mathbf{Hp} = i\rho\omega\mathbf{Gv} + \mathbf{p}_i \tag{2}$$

where **p** and **v** are the vectors with respect to the nodal values for the sound pressure and the normal velocity on the fluid-structure interface, **H** and **G** are the BEM coefficient matrices corresponding to **p** and **v**, *i* is the imaginary unit, ρ is the mass density of the fluid, and $\mathbf{p}_i$ is the vector for the incident wave on the fluid boundary.

In the coupled FE-BE method for the numerical analysis of the FSI problems, Eqs. (1) and (2) have to be linked up via the coupling conditions across the fluid-structure interface to obtain a fully coupled system of equations. Firstly, considering the continuity of the normal surface velocity on the interface, we obtain

$$\mathbf{v} = -i\omega\mathbf{L}^{-1}\mathbf{T}_{fs}\mathbf{u} \tag{3}$$

where $\mathbf{L} = \int_{\Gamma_I} \mathbf{N}_f^T \mathbf{N}_f \, \mathrm{d}\Gamma$ and $\mathbf{T}_{fs} = \int_{\Gamma_I} \mathbf{N}_f^T \mathbf{n} \mathbf{N}_s \, \mathrm{d}\Gamma$, $\mathbf{N}_f$ and $\mathbf{N}_s$ are the BEM and FEM interpolation functions for the fluid and structural domains, respectively. $\mathbf{N}_f^T$ is the transpose of $\mathbf{N}_f$ and **n** is the unit normal vector on the fluid-structure interface Γ_I. In addition, the interaction force vector $\mathbf{f}_f$ represents the effect of the sound pressure on the structure and can be calculated by

$$\mathbf{f}_f = \mathbf{T}_{sf}\mathbf{p} \tag{4}$$

where $\mathbf{T}_{sf} = \mathbf{T}_{fs}^T = \int_{\Gamma_I} \mathbf{N}_s^T \mathbf{n} \mathbf{N}_f \, \mathrm{d}\Gamma$.

An appropriate scheme to generate a fully coupled system of equations is to substitute the FEM system into the BEM system with the use of Eqs. (3) and (4) to generate

$$\mathbf{Ap} = \mathbf{Bf}_s + \mathbf{p}_i \tag{5}$$

where $\mathbf{A} = \mathbf{H} - \mathbf{BT}_{sf}$, $\mathbf{B} = \mathbf{GW}$, $\mathbf{W} = \rho\omega^2\mathbf{L}^{-1}\mathbf{T}_{fs}\mathbf{A}_s^{-1}$ and $\mathbf{A}_s = \mathbf{K} + i\omega\mathbf{C} - \omega^2\mathbf{M}$.

The coupled system, i.e., Eq. (5) is fundamental in the numerical analysis of the FSI problems. However, as its coefficient matrix $\mathbf{A}$ involves the frequency parameter implicitly, we obtain a NEVP in the FSI modal analysis for finding the eigenpairs (λ_j, ϕ_j^p) that satisfy

$$\mathbf{A}(\lambda_j)\phi_j^p = 0 \tag{6}$$

where λ_j is the eigenvalue and ϕ_j^p represents the corresponding eigenvector with respect to the sound pressure on the fluid-structure interface.

Eq. (6) has non-trivial solutions when the determinant of $\mathbf{A}(\lambda_j)$ is equal to zero. In general, it is not an easy task to solve such a problem directly, therefore, a contour integral method is employed next to convert such a NEVP into a small GEVP which is much easier to deal with.

Contour integral method

The contour integral method proposed by Asakura *et al*. [10] and usually referred to as the block Sakurai-Sugiura (bSS) method is introduced here. It is a projection method which can extract eigenvalues while preserving their multiplicities in a domain enclosed by a positively oriented Jordan curve. In this method, the projection is performed through two Hankel matrices $\mathbf{H}_1, \mathbf{H}_2 \in \mathbb{C}^{KL\times KL}$, which are formed by

$$\mathbf{H}_1 = \left[\mathbf{M}_{j+l-2}\right]_{j,l=1}^{K} \text{ and } \mathbf{H}_2 = \left[\mathbf{M}_{j+l-1}\right]_{j,l=1}^{K} \tag{7}$$

where the moments $\mathbf{M}_l \in \mathbb{C}^{L\times L}$ are defined by

$$\mathbf{M}_l = \frac{1}{2\pi i}\oint_C z^l \mathbf{V}^H \mathbf{A}^{-1}(z)\mathbf{V}\,\mathrm{d}z, \quad l = 0,1,\cdots,2K-1, \tag{8}$$

and C is a positively oriented closed Jordan curve in the complex plane, $\mathbf{V}$ is a nonzero matrix chosen as random, $\mathbf{V}^H$ is the conjugate transpose of $\mathbf{V}$, $\mathbf{A}$ is the coefficient matrix of Eq. (5), K and L are positive integers.

It has been proved mathematically in [10] that the eigenvalues of the linear matrix pencil ($\mathbf{H}_2$, $\mathbf{H}_1$) are identical to those of the original NEVP lying inside C. After obtaining the eigenpairs (λ_j, ψ_j) of the matrix pencil, we can calculate the eigenvectors for the original problem by

$$\phi_j^p = \mathbf{S}\psi_j \tag{9}$$

where $\mathbf{S} = [\mathbf{S}_0, \mathbf{S}_1, \cdots, \mathbf{S}_{K-1}]$, and

$$\mathbf{S}_l = \frac{1}{2\pi i}\oint_C z^l \mathbf{A}^{-1}(z)\mathbf{V}\,\mathrm{d}z, \quad l = 0,1,\cdots,K-1, \tag{10}$$

It is found that the original NEVP has been converted into a GEVP whose dimension is much smaller than the original problem through the bSS method. The conversion can be achieved readily and directly by solving a series of linear system of equations, i.e., **AX**=**V**, which are independent and similar to the one used in the normal FSI analysis. Thus, the method is very easy to be implemented and suitable to be parallelized effectively. Next, the implementation of the bSS method in the numerical modal analysis of submerged structures is presented.

Numerical modal analysis of submerged structures

It is found from Eqs. (8) and (10) that two sets of contour integrals in the form of

$$\mathbf{I}_l = \frac{1}{2\pi i}\oint_C z^l f(z)\,\mathrm{d}z \tag{11}$$

have to be evaluated in numerical computation. In Eq. (11), $f(z) = \mathbf{V}^H\mathbf{A}^{-1}(z)\mathbf{V}$ for $\mathbf{M}_l$ and $f(z) = \mathbf{A}^{-1}(z)\mathbf{V}$ for $\mathbf{S}_l$. When the eigenvalues of interest are located in an interval of $[\lambda_{\min}, \lambda_{\max}]$ and the contour path C is chosen from a family of ellipse of $z = \gamma + \rho(\cos\theta + i\zeta\sin\theta),\ \theta\in[0,2\pi]$, Eq. (11) can be shifted, scaled and approximated by the N-point trapezoidal rule to produce

$$\hat{\mathbf{I}}_l = \frac{1}{N}\sum_{j=1}^{N}\rho\left(\frac{z_j-\gamma}{\rho}\right)^l\left(\zeta\cos\theta_j + i\sin\theta_j\right)f\left(z_j\right) \tag{12}$$

where $z_j = \gamma + \rho(\cos\theta_j + i\zeta\sin\theta_j)$, $\theta_j = (2\pi/N)(j-1/2)$, $\gamma = (\lambda_{\max}+\lambda_{\min})/2$, $\rho = (\lambda_{\max}-\lambda_{\min})/2$ and ζ is a scaling factor. When ζ is set to 1, the contour path C turns into a circle.

As the results of using Eq. (12), $\hat{\mathbf{H}}_1, \hat{\mathbf{H}}_2$ which are the shifted and scaled approximations of the two Hankel matrices $\mathbf{H}_1$, $\mathbf{H}_2$ can be obtained. In order to calculate the eigenpairs of the matrix pencil $(\hat{\mathbf{H}}_2, \hat{\mathbf{H}}_1)$, a singular value decomposition (SVD) is performed on $\hat{\mathbf{H}}_1$ to obtain

$$\hat{\mathbf{H}}_1 = \mathbf{P}\mathbf{\Sigma}\mathbf{Q}^H \tag{13}$$

where $\mathbf{P}$ and $\mathbf{Q}$ are unitary matrices, $\mathbf{\Sigma} = \mathrm{diag}(\sigma_1, \sigma_2, \cdots, \sigma_{KL})$ and $\sigma_1, \sigma_2, \cdots, \sigma_{KL}$ are nonnegative real numbers in descending order.

The original NEVP then can be converted into an ordinary linear eigenvalue problem to find the eigenpairs $(\hat{\lambda}_j, \hat{\psi}_j)$ of the following matrix:

$$\hat{\mathbf{H}}_3 = \mathbf{P}^H \hat{\mathbf{H}}_2 \mathbf{Q}\mathbf{\Sigma}^{-1} \tag{14}$$

After obtaining the eigenpairs of $\hat{\mathbf{H}}_3$, the original eigenvalue λ_j can be recovered by

$$\lambda_j = \gamma + \rho\hat{\lambda}_j \tag{15}$$

and the corresponding eigenvector ϕ_j^p with respect to the sound pressure on the fluid-structure interface can be calculate by

$$\phi_j^p = \mathbf{S}\mathbf{Q}\mathbf{\Sigma}^{-1}\hat{\psi}_j \tag{16}$$

The eigenvector ϕ_j^u with respect to the structural nodal displacements can then be obtained by solving

$$\mathbf{A}_s \phi_j^u = \mathbf{T}_{sf} \phi_j^p \tag{17}$$

In order to omit small singular values which bring irrelevant results, a threshold δ is used in the original bSS method [10] to truncate the SVD of $\hat{\mathbf{H}}_1$. However, this brings a difficulty to specify a proper value for δ. A large δ may cause a possibility of missing some eigenvalues of interest, while a small δ may not filter out irrelevant results totally. Moreover, it is pointed out by Sakurai *et al.* [17] that it is not necessary to take a large N to reduce the quadrature error in calculating contour integrals. However, it is found from our numerical experiments that irrelevant results cannot be filtered out by using a small δ, like $\delta = 10^{-12}$ used in [10], even with a large N. The reason is that not only the quadrature error of calculating contour integrals exists in engineering applications but also some other errors, e.g., the errors from the modeling, mesh discretization and solution of systems of equations. To filter out all irrelevant results, a large N (i.e., more integration points) is usually required to achieve a better performance of the filter, but unfortunately this makes the computational cost increase quickly.

In this paper, in order to truncate the SVD effectively and efficiently, the gaps between the singular values of $\hat{\mathbf{H}}_1$, i.e., $\sigma_1, \sigma_2, \cdots, \sigma_{KL}$ are tested and the SVD of $\hat{\mathbf{H}}_1$ is truncated first at the biggest gap. $\hat{\mathbf{H}}_3$ in Eq. (14) is then truncated according to the truncation of $\hat{\mathbf{H}}_1$, and as a result a set of eigenvalues denoted by Λ_{gap} can be separated out by finding the eigenvalues of the truncated matrix of $\hat{\mathbf{H}}_3$. At the same time, the SVD of $\hat{\mathbf{H}}_1$ is also truncated by a threshold δ and another set of eigenvalues denoted by Λ_{thr} can be obtained. The components in the two sets, i.e., Λ_{gap} and Λ_{thr}, are then checked. If every number in Λ_{gap} is inside the contour and every number in the difference set $\Lambda_{thr} \setminus \Lambda_{gap}$ (i.e., $\{\lambda \in \Lambda_{thr} \mid \lambda \notin \Lambda_{gap}\}$) is outside the contour, the numbers in Λ_{gap} are taken as the final numerical solutions. Otherwise, a large N is required. Thus, it is found that the threshold δ is now used only to check if the truncation at the biggest gap is reasonable or not, and to make sure that no eigenvalue of interest is missed. Moreover, the present truncation scheme can be treated as a stopping criterion, and thus a small N can be given initially in the bSS method and increased gradually until the final solutions are obtained. To further improve the efficiency, the integration points of the trapezoidal rule are evenly distributed with respect to the angle θ and doubled for the increase of *N*. As a result only the systems of equations at the new integration points need to be solved and the rest have already been solved at the previous steps.

Numerical example

An elastic spherical shell structure is employed in this section as a numerical example to show the effectiveness and applicability of the present numerical tool for the modal analysis of three-dimensional submerged structures. The shell structure is made of steel, and the material properties for the structure and the surrounding water are listed in Table 1. The structure has the outer radius of a = 5.0m and the thickness of h = 0.05m.

Table 1. Material properties for the structures and water

Density (structures)	ρ_s	7800	kg/m^3
Young's modulus (structures)	E	210	GPa
Poisson's ratio (structures)	v	0.3	-
Density (water)	ρ	1000	kg/m^3
Speed of sound (water)	C_f	1482	m/s

In the numerical analysis, the structure is modeled into a finite element mesh with 600 shell elements, which corresponds to 6492 DOFs. The fluid-structure interface is discretized into a boundary element mesh with 600 discontinuous quadratic elements, which corresponds to 4800 DOFs. Resonance frequencies in an interval of [34.0, 82.0]Hz are calculated, so that an elliptical path with $\gamma = (58.0, 0)$, $\rho = 24.0$ and $\zeta = 0.05$ can be employed as the contour path. The parameters used in the modified block SS method are set as K = 4, L = 15 and $\delta = 10^{-12}$. The computation terminates automatically at N = 16, and the computed eigenfrequencies are listed in the left part of Table 2. It is observed that the eigenfrequencies whose multiplicities are equal to or larger than one can both be extracted by using the present numerical tool. In addition, it is observed that the imaginary parts of these numerical eigenfrequencies are all negative, which implies that they are physically related to the radiation damping. This example is also analyzed numerically in [9], where a polynomial approximation method is used to solve the underlying NEVP. The calculated eigenfrequencies therein are 55.84-1.18*i*, 70.48-0.31*i* and 80.59-0.042*i*, and the multiplicities for them are 5, 7 and 9, respectively. It can be found that the numerical eigenfrequencies obtained by the present numerical tool are very close to the numerical results presented in [9].

The fluid-loaded modes (sometimes also called the wet modes) of the spherical shell structure, which are obtained by the developed FSI eigensolver are illustrated in Fig. 1. In Table 2, the computed eigenfrequencies of the *in vacuo* structure are also given, where the finite element based eigenvalue problem is solved by the bSS method (indicated by SS-FEM) and ANSYS, respectively. Another elliptical path with $\gamma = (137.0, 0)$, $\rho = 17.0$ and $\zeta = 0.05$ is used in the SS-FEM, and the parameters utilized in the bSS method are set the same as the ones used above. The computation terminates automatically at N=32. The computed eigenfrequencies are listed in the right part of Table 2, and their mode shapes are similar to the fluid-loaded modes of the eigenfrequencies listed in the left part of Table 2. It can be observed that the real parts of the numerical results obtained by the SS-FEM are equal to the numerical results obtained by ANSYS, and the imaginary parts are very small and can be neglected directly. Furthermore, as can be seen, the fluid has a significant influence on the eigenfrequencies of the submerged elastic structure. All frequencies are lowered due to the fluid, that is because the surrounding fluid acts like an adding mass to the submerged structure. Accordingly, such variations of the eigenfrequencies necessitate the solution of the coupled eigenvalue problem.

Table 2. Eigenfrequencies of the spherical shell structure

i	Frequencies (Hz, with fluid)	Frequencies (Hz, no fluid)	
	SS-FEM-BEM	SS-FEM	ANSYS
1	55.84 – 1.18i	120.91 – 3.07 $\times$ 10^{-14}i	120.91
2	55.84 – 1.18i	120.91 – 2.36 $\times$ 10^{-14}i	120.91
3	55.84 – 1.18i	120.91 – 4.16 $\times$ 10^{-14}i	120.91
4	55.84 – 1.18i	120.91 + 2.74 $\times$ 10^{-14}i	120.91
5	55.84 – 1.18i	120.91 + 1.30 $\times$ 10^{-13}i	120.91
6	70.48 – 0.31i	143.22 – 2.41 $\times$ 10^{-13}i	143.22
7	70.48 – 0.31i	143.22 + 4.57 $\times$ 10^{-13}i	143.22
8	70.48 – 0.31i	143.22 + 1.91 $\times$ 10^{-13}i	143.22
9	70.48 – 0.31i	143.22 + 9.27 $\times$ 10^{-14}i	143.22
10	70.48 – 0.31i	143.22 + 2.32 $\times$ 10^{-13}i	143.22
11	70.48 – 0.31i	143.22 + 9.42 $\times$ 10^{-13}i	143.22
12	70.48 – 0.31i	143.22 + 5.34 $\times$ 10^{-13}i	143.22
13	80.59 – 0.042i	152.10 – 6.35 $\times$ 10^{-13}i	152.10
14	80.59 – 0.042i	152.11 + 1.26 $\times$ 10^{-13}i	152.11
15	80.59 – 0.042i	152.11 + 3.58 $\times$ 10^{-14}i	152.11
16	80.59 – 0.042i	152.11 – 6.37 $\times$ 10^{-14}i	152.11
17	80.60 – 0.042i	152.12 + 1.76 $\times$ 10^{-13}i	152.12
18	80.60 – 0.042i	152.12 + 1.69 $\times$ 10^{-13}i	152.12
19	80.60 – 0.042i	152.12 – 6.41 $\times$ 10^{-14}i	152.12
20	80.61 – 0.042i	152.13 – 2.45 $\times$ 10^{-14}i	152.13
21	80.61 – 0.042i	152.13 – 5.88 $\times$ 10^{-15}i	152.13

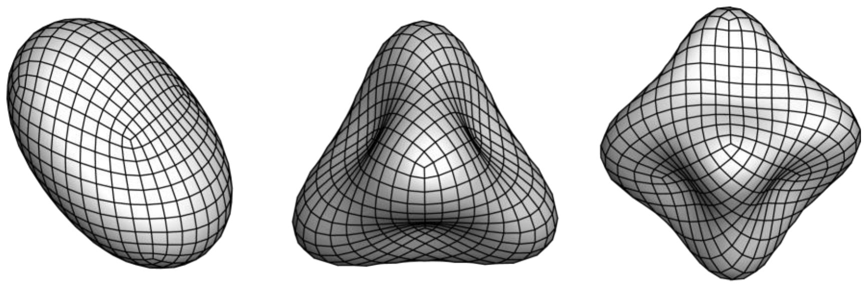

Figure 1. Mode shapes of the submerged spherical shell structure

Conclusions

In this paper, the numerical modal analysis of three-dimensional submerged elastic structures is carried out by using a coupled FE-BE solver. The submerged structure is modeled by the FEM. The compressibility of the surrounding infinite fluid domain is taken into account and hence the Helmholtz equation serves as the governing equation and is solved by the BEM. A contour integral method proposed by Asakura *et al.* [10] is employed to convert the resulting NEVP into a small GEVP. Numerical implementation of the method in the FSI eigenvalue problems is given in detail. In order to improve the efficiency of the method and also avoid missing the eigenvalues of interest, a novel scheme for the truncation of the small singular values is presented. This scheme is then used as a stopping criterion, and thus a small number of integration points for the numerical quadrature of contour integrals can be given initially and then increased gradually until the final solutions are obtained. The effectiveness and applicability of the present numerical tool for the modal analysis of submerged structures are shown by a numerical example of an elastic spherical shell structure. All eigenfrequencies of the spherical shell structure are lowered due to the surrounding fluid, and the variations of the eigenfrequencies necessitate the solution of the coupled eigenvalue problem.

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