## A novel alpha smoothed finite elements method for ultra-accurate solution using quadrilateral elements

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## Abstract

Smoothed finite element method (S-FEM) based on triangular element meshes has recently been widely used for solving solid and fluid mechanics problems. In this paper, a novel  $\alpha$  smoothed finite element method using quadrilateral elements ( $\alpha$ -SFEM-Q4) is proposed for obtaining ultra-accurate solutions in the displacement and strain energy for solid mechanics problems. This method combines node-based S-FEM (NS-FEM), edge-based S-FEM (ES-FEM) and cell-based S-FEM (CS-FEM) using a scale factor  $\alpha$  that controls the contribution from each of these three different S-FEMs. This novel combination makes the best use of the upper bound property of the NS-FEM and the lower bound property of the ES-FEM and CS-FEM, and establishes a continuous strain-energy function of a scale factor  $\alpha$  for obtaining close-to-exact solutions. Our  $\alpha$ -SFEM-Q4 ensures the variational consistence and the compatibility of the displacement field, and hence guarantees reproduction linear field exactly. Various solid mechanics problems are presented to validate the stability, effectiveness and ultra-accuracy of the proposed method.

**Keywords:** *a*-SFEM-Q4, *a*FEM, upper bound, ultra-accurate solution, NS-FEM, ES-FEM, CS-FEM, FEM.