Backlash Computation of Harmonic Drive Based on Parametric Solid Finite Element Model

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Abstract

In order to truly investigate the backlash distribution between tooth profile of flexspline (FS) and circular spline (CS), a finite element model (FEM) of harmonic drive (HD) based on solid element is established by APDL, and backlashes are calculated with the minimum circumferential distance between engaged tooth profiles along tooth height direction. Taking involute tooth profile parameters and structural parameters of FS and CS into consideration, the FEM of planar tooth ring with involute teeth under action of four-rollers wave generator is calculated with contact analysis, and assembly status is obtained. By adjusting the radial position of the roller until the maximum radial displacement on the major axis of the deformed FS reaches the specified value. Backlashes are calculated for all engaged tooth profiles and the backlash distribution curve is obtained. The research indicates that the location of backlash alternately occurs between the addendum of FS and the addendum of CS, and their backlash curves go across twice in the engagement interval. The validity of the backlash computation based on the FEM is higher by the contrastive analysis, which provides a reliable way of backlash computation.

Keywords: Finite Element Model, Parametric Modeling, Harmonic Drive, Backlash Computation

Introduction

HD is a novel gear transmission technology which has been achieved great development in recent decades (Xie, 1979) [1]. It relies on the continuous movement of the deformation wave that caused by wave generator (WG) forcing FS deformation to achieve the movement and power transmission (Ivanov, 1987) [2]. Benefit from its large transmission ratio, high transmission accuracy, small return difference, high transmission efficiency, it is widely used in aerospace, medical equipment, multi-joint robot and other related fields (Shen, 1985) [3].

Ivanov (1987) [2] and Shen (1985) [3] have given the relatively complete research theory in establishing mathematical model of elastic deformation of FS and solving the backlash of the tooth profile. Lu et.al (2009) [4] completed simulation of the whole and local engagement of the harmonic transmission in Matlab, and conducted interference check. Pan (2010) [5] simulated the engagement interval temperature of HD by using ANSYS, and analyzed the variation law of the minimum backlash with the ambient temperature. Yin (2010) [6] made the parameter optimization design of the zero-backlash involute HD and finite element analysis. Taking zero-backlash as the objective function, the optimization of the engagement parameters was completed. Zhang (2012) [7] established the automatic optimization algorithm of tooth profile interference to calculate the minimum backlash, and realized visual simulation of the tooth profile fitting process by using math software. Chen et.al (2011, 2014)[8-9] put forward the backlash computation method of double-circular-arc tooth profile of HD, and checked interference. Liu et.al (2015) [10] made a 3D tooth profile design based on the cone deformation of FS cylinder, and solved the backlash distribution of the 3D tooth profile. Wang et.al (2017) [11] solved and fitted the conjugate tooth shape of the involute FS, and studied the backlash distribution along tooth height direction from the theoretical point of view.

These theoretical backlash computation methods are based on the assumption of small deformation and that teeth are no deformation. For backlash of tens microns, the experimental measurement is very difficult. In this paper, a parametric FEM of HD is developed. The reasonable assembly status is obtained by iteration, and the iteration can not be stopped until the maximum radial displacement on the major axis of the deformed FS reaches the specified value. Through the definition, extraction and computation of the relevant parameters in the model, the backlashes between engaged tooth profiles of FS and CS are calculated, and the backlash distribution for all engaged teeth is studied.

1 Parameter Analysis

HD has three basic components: FS, CS and WG. First of all, its initial parameters are defined, as shown in Table 1. Then its profile parameters and structural parameters are analyzed.

Table 1. Model parameters

Basic components	FS	CS	WG
Module	m	m	
Number of teeth	z_1	z_2	
Modification coefficient	x_1	x_2	
Roller location angle			β
Contact interference			η
Wall thickness	δ		
Maximum radial displacement	w_0		

(1) The parametric equation of the FS tooth profile is defined as (Shen, 1985) [3]:

$$\begin{cases} x_1 = r_1\left[-\sin\left(u_{a1}-\theta_1\right)+u_{a1}\cos\alpha_0\cos\left(u_{a1}-\theta_1+\alpha_0\right)\right] \\ y_1 = r_1\left[\cos\left(u_{a1}-\theta_1\right)+u_{a1}\cos\alpha_0\sin\left(u_{a1}-\theta_1+\alpha_0\right)\right] \end{cases} \tag{1}$$

Here, u_{a1} is the coordinate parameter of the involute FS, and θ_1 is half of the central angle corresponding to the thickness on the reference circle of FS tooth, and α_0 is the pressure angle, and r_1 is reference circle radius of FS. Among them, u_{a1} are determined by the module, the number of teeth, the pressure angle and the modification coefficient of FS. θ_1 is determined by the module and the modification coefficient of FS. α_0 is the basic constant input. r_1 is determined by the module and the number of teeth.

(2) The parametric equation of the CS tooth profile is defined as (Shen, 1985) [3]:

$$\begin{cases} x_2 = r_2\left[-\sin\left(u_{M2}-\theta_2\right)+u_{M2}\cos\alpha_0\cos\left(u_{M2}-\theta_2+\alpha_0\right)\right] \\ y_2 = r_2\left[\cos\left(u_{M2}-\theta_2\right)+u_{M2}\cos\alpha_0\sin\left(u_{M2}-\theta_2+\alpha_0\right)\right] \end{cases} \tag{2}$$

Here, u_{M2} is the coordinate parameter of the involute CS, and θ_2 is half of the central angle corresponding to the thickness of the reference circle of the CS tooth, and r_2 is reference circle radius of CS. Among them, u_{M2} is determined by the module, the number of teeth, the pressure angle and the modification coefficient of CS. θ_2 is determined by the module and the modification coefficient of CS. r_2 is determined by the module and the number of teeth.

(3) The structural parameters of FS and the four-rollers wave generator are designated. β is the roller location angle, η is radial displacement of roller (defined as contact interference). δ is the wall thickness, w_0 is maximum radial displacement.

2 Modeling Method

It is considered that HD structure is quite complex, and the design parameters are quite much. So the method of FEM is developed (Wei, 2015) [12], as shown in Fig. 1.

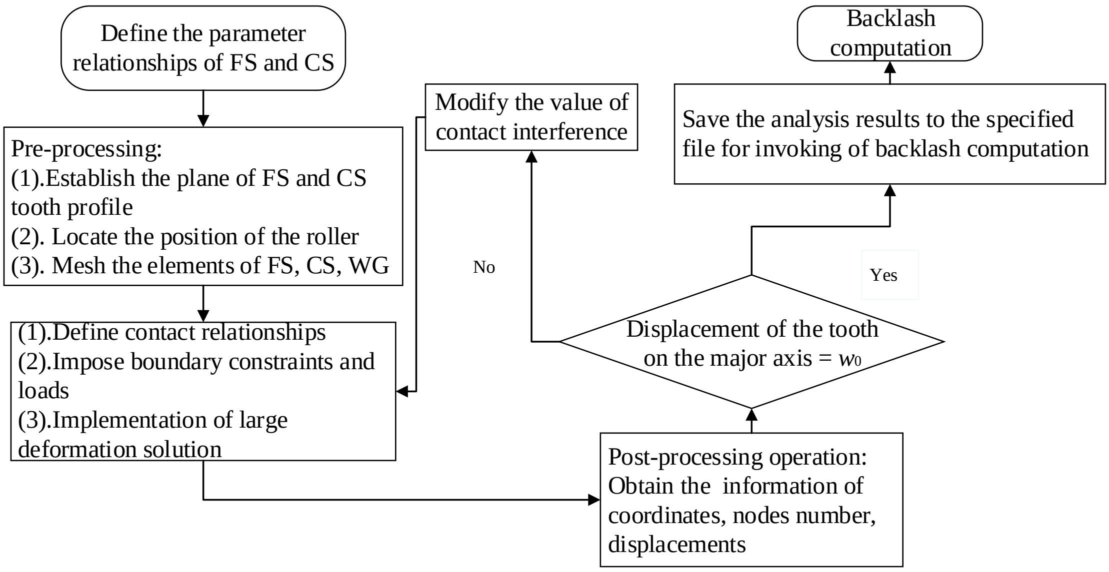

Figure 1. The flow chart of modeling method

Fig. 1 is a complete parametric analysis flow chart of modeling method, including pre-processing, solving, post-processing and iteration. The criterion of iteration is whether the tooth on the major axis of the deformed FS reaches the specified radial displacement. In consideration of the particularity and complexity of harmonic drive, the model is simplified as follows:

(1). Since the length of the ring-shaped FS cylinder is short, it is considered that the deformation along the tooth length is uniform, and it can be studied as a planar engagement problem.

(2). Considering the symmetry of structure and load of the contact model between FS and WG in the assembly status, the 1/4 model of FS and WG is analyzed.

2.1 Establishment of FEM

The establishment of FEM is as follows: (1). The structural parameters and tooth profile parameters of HD are defined before the pre-processing. (2). The bottom-up modeling method is selected. Sex keypoints of tooth profile will be selected, and its coordinates are solved by using the parametric equations (1-2). These keypoints are connected to lines, and the lines constitute a surface, and the surface is mirrored to form a tooth. The fillet is made at the root of tooth, and its radius is half of the tooth space. A tooth is arrayed into a 1/4 model of FS and CS. The coordinates of WG are determined based on the roller location angle and the contact interference. The element type of FS, CS and WG are chose as plane183. The elastic modulus is 210 GPa, and poisson's ratio is 0.3. (3). In order to analyze deformation of FS, FS is divided by the mapping mesh, which makes the grids fine and regular. CS and WG are divided by the rough free grid to reduce the computation time.

2.2 Solution of FEM

The solution of FEM is as follows: (1). The contact relationship between the FS and the WG is defined as flexible body and rigid body by the contact element. The upper semicircle of WG is defined by the Targe169 target element. The inner wall of FS is used as a flexible contact surface and is defined by the Conta172 contact element. When the contact interference is assigned, it is necessary to obtain the exact value by iterations. When the tooth on the major axis of the deformed FS reaches the specified radial displacement w_0, the iteration stopped. (2). The major axis and minor axis of the deformed FS are subjected to the symmetrical constraint, and the outer surface of WG is subjected to the fixed confinement. (3). In the process of solution, the ratio of the maximum radial deformation and the wall thickness of FS is greater than 0.2, so the large deformation solution is used. (4). During the post-processing phase, the displacement cloud is displayed and the required data is stored in the specified file (Zeng, 2010) [13].

2.3 Case study

The main parameters of HD for electronic equipment: m =0.2, z_1 =140, z_2 =142, x_1 =2.13, x_2 =1.925, η (Eta) =0.112, δ (Delta) =0.3, w_0 =0.2, β (Beta) =30 ° (Shen, 1985) [3]. In order to simplify the work of modifying the command stream file, the customization of the parameter input interface is realized. (As shown in Fig. 2)

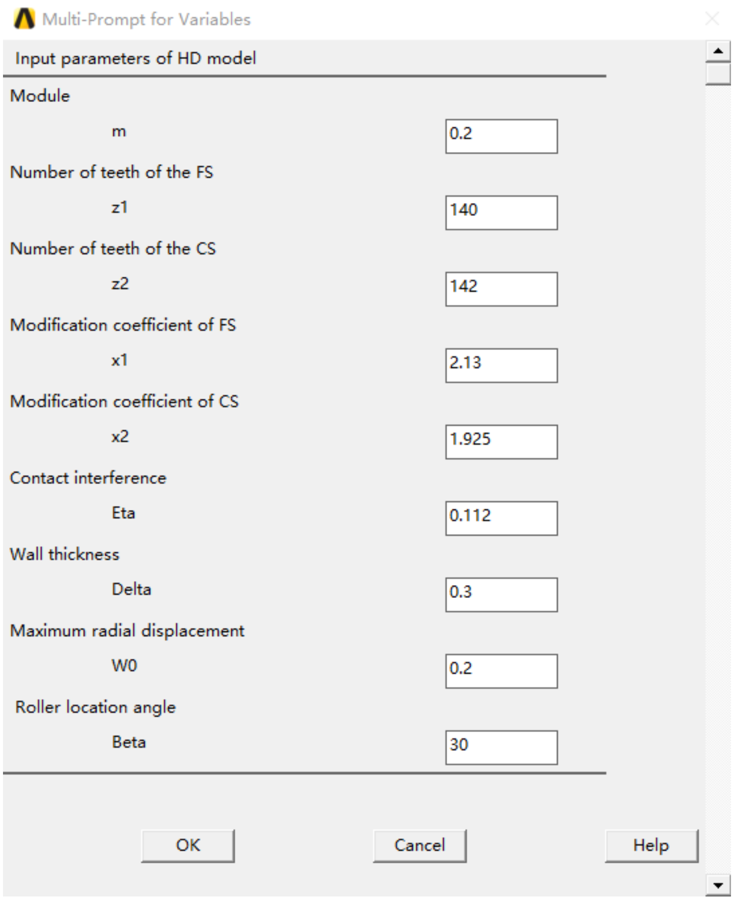

Figure 2. Parameter input interface

Fig. 3 is the FEM of undeformed FS. The nodes number of FS, CS and WG are 30168, 223 and 2808, respectively. And the percentage of the nodes number of FS in the total number is 90.87%. The elements number of FS, CS and WG are 8640, 66 and 720, respectively. And the percentage of the elements number of FS in the total number is 91.66%. Fig. 3 shows that contact relationship is defined between the FS and the four-rollers WG. When the four-rollers WG forces FS to occur deformation, the radial displacement of FEM is shown at the Fig. 4. It shows that the teeth of FS and CS are completely engaged on major axis, and are completely disengaged on minor axis, and are transient state in other areas. And it shows that radial displacement is about 0.2mm on major axis, and radial displacement is about 0.22543mm on minor axis.

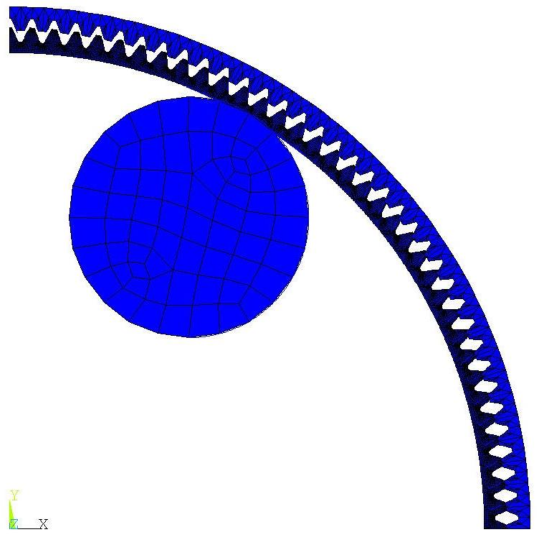

Figure 3. The FEM of the undeformed FS

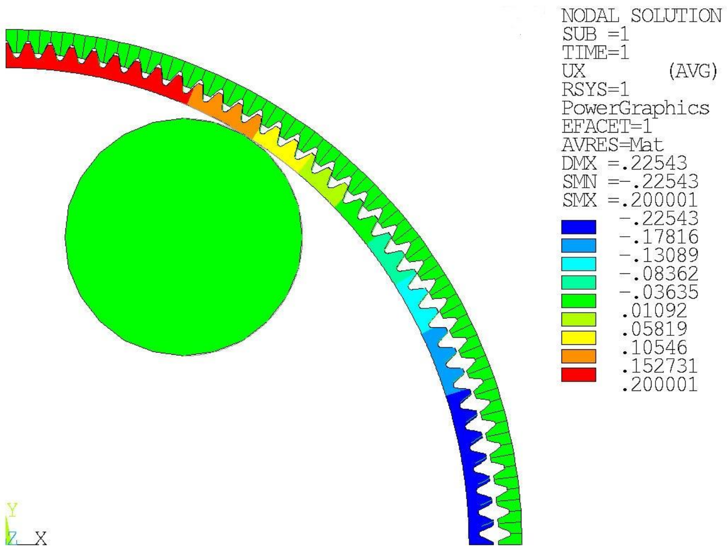

Figure 4. The radial displacement of FEM

3 Computation of backlash along tooth height direction

3.1 Backlash computation process

Most of the studies have considered that the addendum of FS is the position of the minimum distance along tooth height direction, and the backlash is defined as the circumferential distance of the addendum of FS along tooth height direction. But there are not detailed theoretical explanation and experimental data verification for the backlash distribution. In view of the above problems, this paper uses the FEM to calculate the 6 points in the FS tooth profile along tooth height direction and the addendum of CS. The FS tooth profile along tooth height direction were divided into 5 equal parts for points 1-6 (point 1 for the addendum of FS, point 6 for the dedendum of FS), point 7 for the addendum of CS, as shown in Fig. 5.

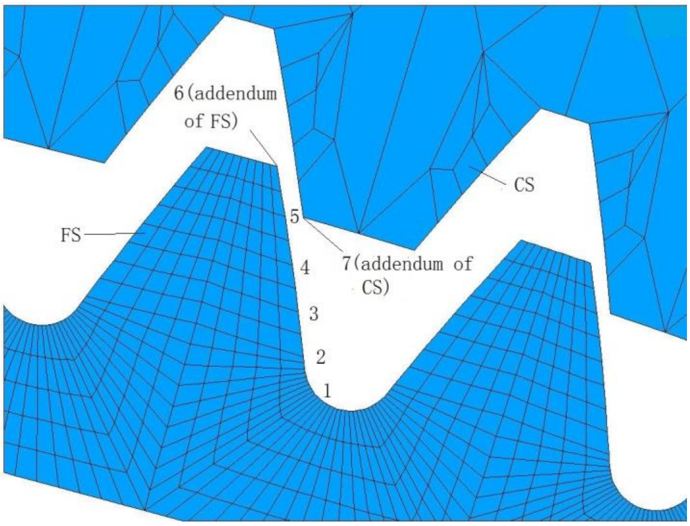

Figure 5. Local engagement status of FS and CS

The computation process of backlash is as follows in the FEM. The computation process of the addendum backlash of FS is described as an example, and the computation process of other points are similar to the addendum of FS.

(1). Definition of parameters. Because the teeth on the cylinder body of FS are reproducible and repeatable, the coordinates, the displacement and the nodes number are defined by array parameter, and cycle statement is used to assign the values. The 1/4 model has a total teeth of $N_Z=z_1/4$, so the cycle number is N_Z. The nodes number is defined as a one-dimensional array of *NZ* rows and one column. The coordinates (x, y) and displacements (x, y) of the nodes are defined as two-dimensional arrays of N_Z rows and two columns. The polar angle, the polar radius of FS nodes and the polar angle of CS nodes are defined as the three-dimensional array of N_Z rows and three columns.

(2). The information such as the coordinates, the displacements, the polar radius, and the polar angle of the addendum of FS should be solved. The coordinates (x, y), the displacements (x, y), the polar radius, and the polar angle of the addendum of FS are assigned to the array parameters by using cycle statement. The above array parameters are saved to the specified path file.

(3). The information such as the polar angle and the coordinates of the engagement point of the addendum of FS in CS involute tooth profile (this point is defined MAFC) should be solved. According to the condition that the polar radius of the addendum of FS equal to the

MAFC's, the coordinates of the MAFC are solved, and the polar angle of the MAFC is calculated. The above data is assigned to the array parameter by using cycle statement and is saved to the specified file.

(4). The circumferential distance between the addendum of FS and the MAFC is solved. The first step is to determine whether the FS teeth and the CS teeth are in the engagement interval. In this paper, we need to determine whether the polar radius of the addendum of FS is bigger than the polar radius of the addendum of CS. If the result is true, it is in the engagement interval. Then the difference between the polar angle of the addendum of FS and the MAFC's is multiplied with the polar radius of the addendum of FS, whose result is the circumferential backlash of the addendum of FS.

3.2 Analysis of the backlash distribution

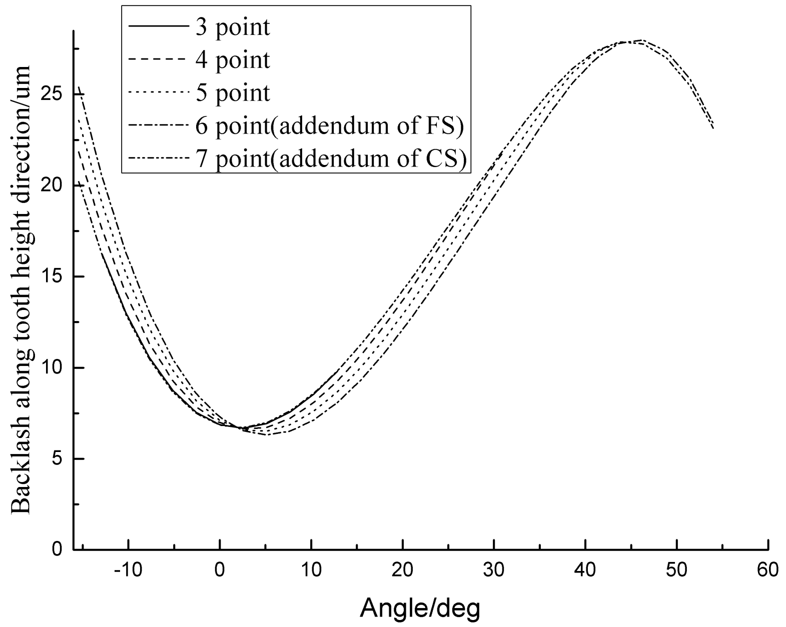

Figure 6. The backlash distribution along tooth height direction

Fig. 6 is the backlash distribution based on computation of FEM. It shows that point 1 and point 2 are close to the dedendum of FS, and do not enter the engagement interval, so there is no backlash value. Most interval of the location of backlash appears alternately between point 6 and point 7. It appears between point 3 and point 5 in $\varphi = 1°\text{-}2°$, but the interval is quite small. Therefore, it is considered that the location of backlash occurs mainly at the addendum of FS or the addendum of CS. Fig. 7 shows the backlash of the addendum of FS and the addendum of CS. At the same time, for verifying the consistency between the FEM and the theoretical algorithm, the backlash curve of the addendum of FS from the article (Shen, 1985) [3] is added in Fig. 7.

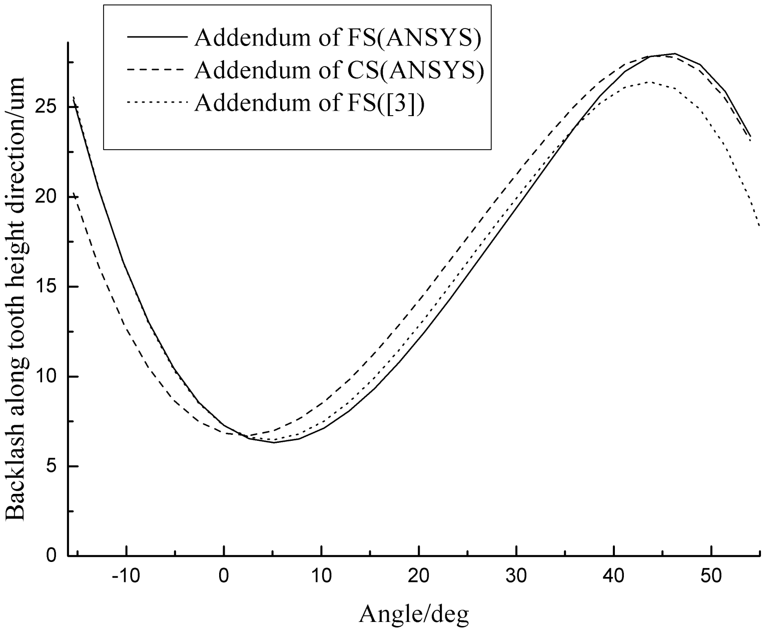

Figure 7. The backlash of the FS and the CS

Consistency analysis between the FEM and the theoretical algorithm (Shen, 1985) [3]: The backlash of addendum of FS from the FEM is basically consistent with that of the theoretical algorithm at near major axis (φ =2 °). There are differences in other areas, especially at near φ =43 °. Because the circumferential displacement is zero in the major axis, the result are consistent. This result verifies the validity of the FEM. In other areas, the circumferential displacements of the two methods are different, which causes backlash deviation. There are many reasons for the deviation, such as small deformation assumption, simplification of normal rotation formula and position formula of the deformed FS (Chen, 2014) [14].

Backlash analysis along tooth height direction: Two backlash curves go across twice at φ =2 °and φ =43 °. The backlash of the addendum of CS is smaller than the addendum of FS in φ =-15 °-2 °and φ = 43 °-55 °. The backlash of the addendum of FS is smaller than the addendum of CS in φ = 2 °-43 °. Throughout the engagement process, the location of minimum backlash appears at the addendum of FS at near φ = 5 °, and the location of maximum backlash appears at the addendum of FS at near φ =43 °.

4 Conclusions

(1) This paper presents a method of backlash computation based on the parametric FEM. Through the analysis of backlash on the major axis of the deformed FS, the validity and consistency between the FEM and the theoretical algorithm are proved. This method provides a reliable way to calculate the backlash.

(2) Two backlash curves of the addendum of FS and CS go across twice at the engagement interval. The first cross appears in the vicinity of the major axis area (φ =2 °-3 °), and the second cross appears in out of the engagement area immediately (φ =43 °-45 °). The backlash of the addendum of CS is the smallest before the first cross and after the second cross. The backlash of addendum of FS is the smallest between the first cross and the second cross. FS and CS belong to tooth surface engagement at two cross intervals, and point engagement at the rest interval.

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References

[1] Xie, J R. (1979) The application and development of harmonic drive at home and abroad, *Optical Machinery* **(4)**, 22-31.
[2] Ivanov, M. H. (1987) *The harmonic drive*, 1st edn, Defense Industry Press. Beijing, China.
[3] Shen, Y. W. and Ye, Q. T. (1985) *Theory and design of harmonic drive*. 1st edn, Mechanical Industry Press. Beijing, China.
[4] Lu, Q. H., Liang, Y., Fan, Y. X. and Wang, H. K. (2009) Research on tooth profile interference based on meshing simulation of harmonic drive, *Journal of system simulation* **21(19)**, 6317–6320.
[5] Pan, F., Dong, H. J. and Ge, W. J. (2010) Backlash analysis on the harmonic gear transmission under extremity environment, *Journal of Machine Design* **27(2)**, 58–62.
[6] Yin, Y. (2010) Parameter optimization design and finite element analysis of zero-lateral space involute harmonic gear driving, Yanshan University, Qinhuangdao, China.
[7] Zhang, J. L., Zhou, J., Zhou, F. Q. and Lin, P. (2012) Accurate analysis of harmonic gear clearance and visual simulation, *Computer Simulation* **29(7)**, 292–296.
[8] Chen, X. X., Lin, S. Z., Xing, J. Z. and Liu, Y. S. (2011) Simulation on gear backlash and interference check of harmonic drive with circular-arc teeth profile, *Computer Integrated Manufacturing Systems* **17(3)**, 643–648.
[9] Chen, X. X., Liu, Y. S., Xing, J. Z. and Xu, W. (2014) The parametric design of double-circular-arc tooth profile and its influence on the functional backlash of harmonic drive, *Mechanism and Machine Theory* **73(2)**, 1–24.
[10] Liu, D. H., Xing, J. Z. and Chen, X. X. (2015) Spatial tooth profile design and simulation analysis of harmonic drive with involute tooth profile, *Computer Integrated Manufacturing Systems* **21(3)**, 709–715.
[11] Wang, Y. Q. and Zhang, Z. Y. (2017) Harmonic gear backlash computation of involute tooth profile based on envelope accurate algorithm, *M&E Engineering Technology* **46(03)**, 96–100.
[12] Wei, L. B., Tao, Y. F. and Xu, G. N. (2015) parametric modeling and analysis of finite element based on APDL language, Design and Computation **(1)**, 27–29.
[13] Zeng, P., Lei, L. P. and Fang, G. (2010) *Finite element analysis guide: modeling and analysis of structure*, 1st edn. Mechanical Industry Press. Beijing, China.
[14] Chen, X. X., Liu, Y. S., Xing, J. Z. and Xu, W. (2014) Neutral line stretch of flexspline in harmonic driver, *Journal of Mechanical Engineering* **50(21)**, 189-196.