Analysis of Modal Characteristics and Reconstructions of Rotating Flow Field Using POD Method

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Abstract

The discrete POD method is used to analyze the rotating flow to further understand and accurately model the details of the flow physics in this paper. The low-order flow modes are obtained using modified discrete POD method proposed by Sirovich, and are used for the reconstruction of the unsteady flow field. The unsteady rotational flow of a simplified impeller is used as numerical example to show the effectiveness of the POD method for the analysis of rotating flow. The numerical results show that the maximal reconstruction deviation is reduced to a very low level with a small increase in the number of low order modes. Furthermore, the regions at relatively high values of deviation shrink distinctly to impeller tip nearby in more accurate approximations.

Keywords: POD method, Flow mode, RSM model, Rotating flow, Flow field reconstruction.

1. Introduction

Rotating flow is critically important across a wide range of scientific, engineering, and product design applications. The better modeling and understanding of the rotating flow could improve the design capability for products such as jet engines, pumps, and vacuum cleaners [1]-[3]. With the rapid development of computer hardware and software, the computational fluid dynamics (CFD) has become standard procedure for academic and engineering applications. However, duo to the complexity of the flow phenomena, one would have to use millions of grids to discretize the solution domain, and thus it becomes very time-consuming for flow field analysis, especially for the turbulent flows [4]. Therefore, the high-fidelity reduced-order models are developed to improve the numerical efficiency of CFD applications.

Among the CFD reduced-order models, the one based on the proper orthogonal decomposition (POD) has become the most popular approach. The essential idea of the POD-based method is to extract characteristics for dominant energy modes, so that the complex flow field can be approximated using a much lower dimensional form but with optimal capture of the system energy. Sirovich [5] made a significant modification to the POD method to improve the numerical stability and efficiency.

The POD method has been widely applied for the model reduction of large-scale CFD problems. For the rotating flow, Christensen et al [6] studied the unsteady flow with rotating cap inside cylindrical vessel using the POD method. The results showed that the first transition from steady to oscillatory motion was detected as a supercritical Hopf bifurcation. The numerical solution obtained using POD method agreed very well with the experimental data, and thus showed the potential of POD method for the analysis of rotating flow field. However, related research on the POD method for rotating flows has been seldomly reported.

In this paper, the application of POD method for modal analysis and reconstruction of rotating flow field is presented. The theories and application of the method are introduced in Section 2, and a numerical example of a simplified impeller is used to show the applicability of the POD method in Section 3. Some conclusions are drawn in the last section.

2. POD method for rotating flow field

2.1 Discrete POD method

In general, the POD method can be divided into two categories [7], namely the continuous POD and the discrete POD method. For the discrete POD method which is used in this paper, the sample vectors of transient numerical solution at time t_m of velocity and static pressure (or the other parameters) can be expressed as,

$$vel(\mathbf{x},t_m)=\left[vel(x_1,t_m) \quad vel(x_2,t_m) \quad \cdots \quad vel(x_N,t_m)\right]^{\mathrm{T}} \tag{1}$$

and

$$sp(\mathbf{x},t_m)=\left[sp(x_1,t_m) \quad sp(x_2,t_m) \quad \cdots \quad sp(x_N,t_m)\right]^{\mathrm{T}}. \tag{2}$$

where, vel and sp represent sample vectors of velocity and static pressure, respectively, and N refers to the total number of discrete points in spatial domain.

The important sample matrix for POD method is comprised of appropriate sample vectors,

$$\begin{aligned}\mathbf{V}&=\left[vel(\mathbf{x},t_1) \quad vel(\mathbf{x},t_2) \quad \cdots \quad vel(\mathbf{x},t_M)\right]\\ \mathbf{P}&=\left[sp(\mathbf{x},t_1) \quad sp(\mathbf{x},t_2) \quad \cdots \quad sp(\mathbf{x},t_M)\right].\end{aligned} \tag{3}$$

where **V** and **P** denote the velocity and static pressure sample matrices, respectively, and M refers to total number of instantaneous time, also namely, total number of snapshots.

To calculate the POD modes for the rotating flow field, the covariance matrix should be calculated at first,

$$\mathbf{C}^v=\mathbf{V}^T\mathbf{V},\quad \mathbf{C}^s=\mathbf{P}^T\mathbf{P}. \tag{4}$$

In the above equation, $\mathbf{C}^v$ and $\mathbf{C}^s$ are the covariance matrices calculated from sample matrix of velocity and static pressure, respectively. It is noted that $\mathbf{C}^v$ and $\mathbf{C}^s$ are both real symmetric matrix, and thus have non-negative eigenvalues. Therefore, both matrices can be transformed into the following eigenvalue problem,

$$\mathbf{C}^v\mathbf{G}_v^{[r]}=\lambda_r^v\mathbf{G}_v^{[r]},\quad \mathbf{C}^s\mathbf{G}_s^{[r]}=\lambda_r^s\mathbf{G}_s^{[r]}. \tag{5}$$

where $\mathbf{G}_v^{[r]}$ and $\mathbf{G}_s^{[r]}$ are eigenvectors for velocity and static pressure, namely, time-varying coefficient matrix of rotating flow modes.

$$\begin{aligned}\mathbf{G}_v^{[r]}&=\left[g_v^r(t_1) \quad g_v^r(t_2) \quad \cdots \quad g_v^r(t_M)\right]^{\mathrm{T}}\\ \mathbf{G}_s^{[r]}&=\left[g_s^r(t_1) \quad g_s^r(t_2) \quad \cdots \quad g_s^r(t_M)\right]^{\mathrm{T}}\end{aligned} \tag{6}$$

The r-th order of POD mode for the swirling flow field are defined as,

$$vel_r(\mathbf{x}) = \frac{1}{M\lambda_r^v}\sum_{m=1}^{M} g_v^{[r]}(t_m)\, vel(\mathbf{x},t_m)$$
$$sp_r(\mathbf{x}) = \frac{1}{M\lambda_r^s}\sum_{m=1}^{M} g_s^{[r]}(t_m)\, sp(\mathbf{x},t_m). \tag{7}$$

where λ_r^v and λ_r^s are eigenvalues corresponding to POD modes of velocity and static pressure. All the eigenvalues can be listed in descending order according to energy contribution of flow modes. Therefore, extraction of dominant flow modes in the rotating flow field can be achieved.

The number of discrete nodes in the space N is usually much higher than number of snapshots M in most CFD problems. Therefore, it is worth noting that POD-Sirovich method has significantly improved computational efficiency since the dimension of the matrices which needs to calculate eigen-parameters is $M \times M$ not $N \times M$.

2.2 Reconstruction of the rotating flow field using POD method

For the reconstruction of rotating flow field, the low-order approximation can be described using the flow modes and corresponding time coefficients as,

$$vel(\mathbf{x},t_m) = \sum_{r=1}^{n} coe_r^v(t_m) vel_r(\mathbf{x})$$
$$sp(\mathbf{x},t_m) = \sum_{r=1}^{n} coe_r^s(t_m) sp_r(\mathbf{x}). \tag{8}$$

where $coe_r^v(t_m)$ and $coe_r^s(t_m)$ are the time coefficients for the corresponding POD modes, and are calculated as the inner product of assumed flow field and the POD modes. The parameter n in the above equation denotes the number of POD modes used for the reconstruction. The relative error for the reconstruction is defined as,

$$\vec{e}_{vel,t_m} = 100 \cdot \left|\overrightarrow{vel}(x_{POD},t_m) - \overrightarrow{vel}(x_r,t_m)\right| \Big/ \left(\sum_{r=1}^{N} \left\|\overrightarrow{vel}(x_r,t_m)\right\|_2 \Big/ N \right)$$
$$\vec{e}_{sp,t_m} = 100 \cdot \left|\overrightarrow{sp}(x_{POD},t_m) - \overrightarrow{sp}(x_r,t_m)\right| \Big/ \left(\sum_{r=1}^{N} \left\|\overrightarrow{sp}(x_r,t_m)\right\|_2 \Big/ N \right). \tag{9}$$

2.3 Implementation of POD method for rotating flow using CFD

The implementation of the POD method for flow modal analysis and reconstruction is shown in Fig. 1, and can be described as following:

1. Carry out CFD simulation for unsteady flows and assemble the CFD results for snapshot at time in the following format,

$$\begin{bmatrix} \text{Node number} & \text{X coordinate} & \text{Y coordinate} & \text{Velocity} & \text{Static pressure} \\ 1 & x_1(t_m) & y_1(t_m) & v_1(t_m) & p_1(t_m) \\ 2 & x_2(t_m) & y_2(t_m) & v_2(t_m) & p_2(t_m) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ l & x_l(t_m) & y_l(t_m) & v_l(t_m) & p_l(t_m) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ N & x_N(t_m) & y_N(t_m) & v_N(t_m) & p_N(t_m) \end{bmatrix}. \tag{10}$$

2. Generate the sample matrix from the snapshots, and calculate the POD modes and corresponding time coefficients using Eq. (7).
3. The reconstruction is realized using the calculated POD modes and corresponding time coefficients using Eq. (8).

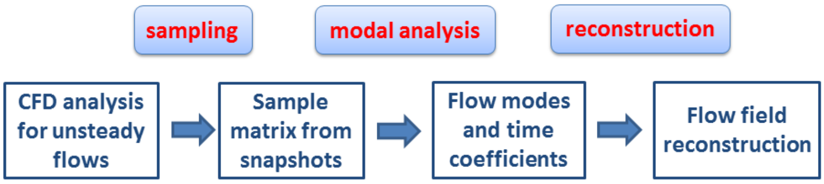


Figure 1. Implementation of POD method for modal analysis and flow field reconstruction

3. Numerical example

A two dimensional simplified impeller as shown in Fig. 2 is used as example to verify the proposed method. The rotating velocity is 305 rpm, and the unsteady turbulent flow fields are simulated. The interface between non-rotating region and internal swirling flow area is modeled by sliding mesh method. For the POD analysis, the total number of snap shots 180, corresponding to 0.5 snap per degree. In this paper, only the velocity filed is considered, and similar conclusions can be drawn for the pressure field.

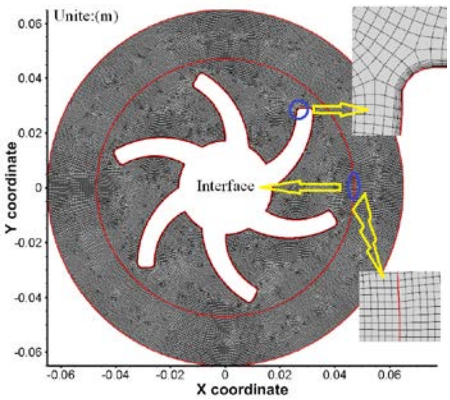


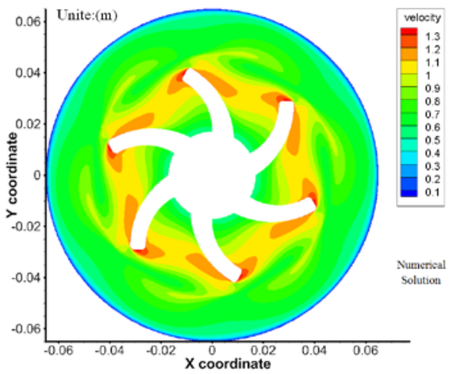
Figure 2. Layout and grids division for impeller rotating flow field

3.1 Modal analysis of rotating flow

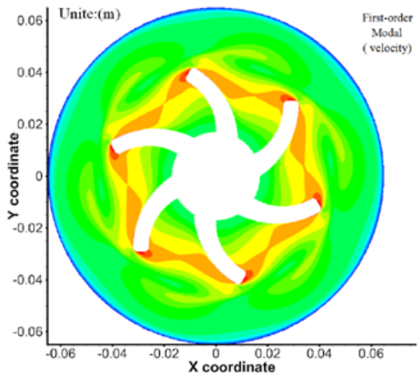
The first 5 POD modes of velocity and pressure are shown in Fig. 3 and 4, respectively. From the results, it can be concluded the following:

1. The 1st order POD mode of velocity contains the largest proportion of the system energy. The mode shape is extremely similar to the temporal results, including the tip vortex region and the boundary layer region, as shown in Fig. 3(a) and (b).

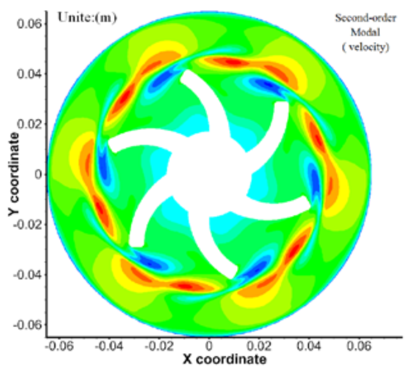
2. Circumferentially symmetrical distribution of the low-order POD modes is observed for both velocity and pressure field. However, due to energy reduction in higher order modes, random fluctuations appear around the blade tip regions.



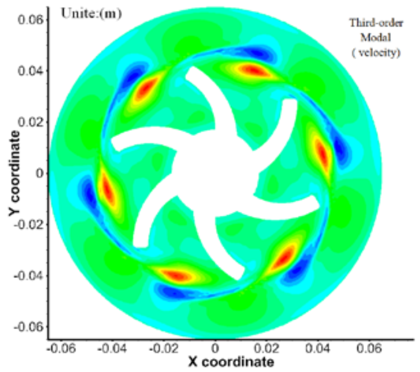
(a) Velocity distribution at t=1.089 s



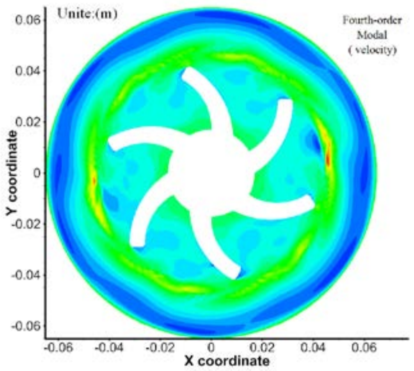
(b) The 1st order POD mode



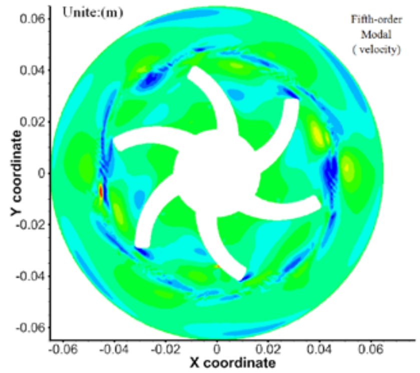
(c) The 2nd order POD mode



(d) The 3rd order POD mode



(e) The 4th order POD mode

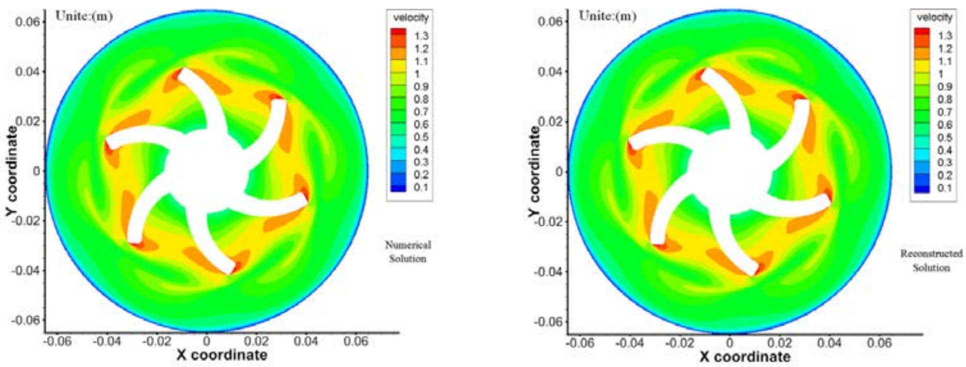


(f) The 5th-order POD mode

Figure 3. The first 5 POD modes of velocity field

3.2 Reconstruction of the rotating flow field

In this section, the flow field is reconstructed using different numbers of POD modes in order to verify the effectiveness for accuracy of the reconstruction for rotating flow field by POD method. The reconstructed field and their deviations are shown in Fig. 5 and 6, respectively.

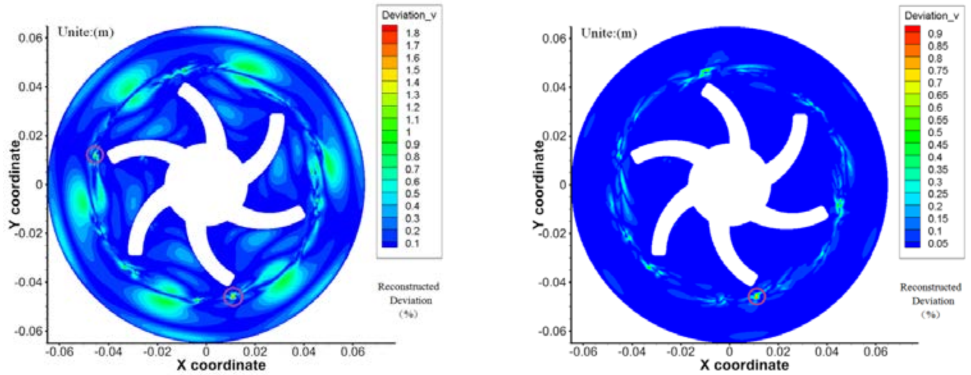


(a) Original field **(b) Reconstructed field**

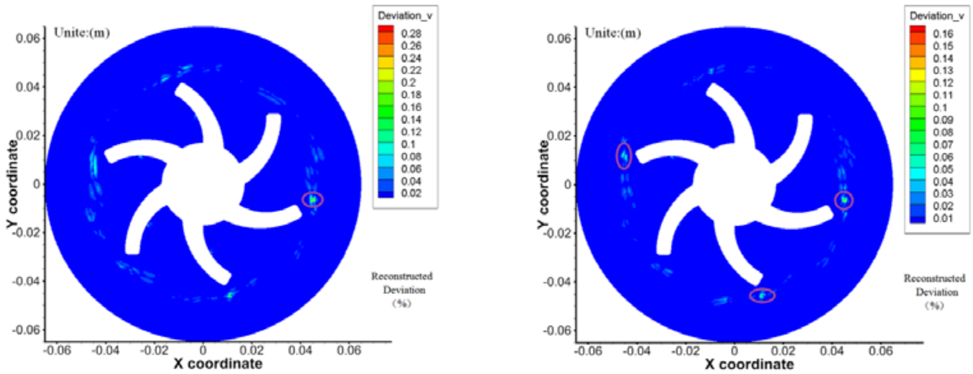
Fig. 4 Reconstructed velocity field from the first 3 POD modes

From the results, we can conclude the following:

1. Very good agreement can be achieved between the original and reconstructed velocity field using only the first 3 POD modes, as shown in Fig. 4.
2. For the reconstructed velocity field using the first 3 POD modes, relative larger error (around 2%) is observed around the blade tip, or the sliding mesh interface. With the increase of POD modes, the reconstruction error is significantly reduced from 0.9% with 6 POD modes to 0.16% with 18 POD modes.



(a) First 3 POD modes **(b) First 6 POD modes**



(c) First 12 POD modes **(d) First 18 order deviation**

Fig. 5 Deviation (relative error) of the reconstructed velocity fields with different number of POD modes

4. Conclusions

The theory and implementation of discrete POD method for modal analysis and reconstruction of rotating flow field is introduced in this paper. For POD analysis of the unsteady flows, the major computational cost is the computation of snapshots and the formulation of POD modes. The computational efforts for flow field reconstruction are negligible since only linear combinations of POD vectors.

Numerical example indicates that the lower order modes of rotating flow fields are circumferentially symmetrical and contain the most part of the system energy. Therefore, the rotating flow field can be accurately reconstructed using the first several POD modes, and the error can be less than 0.2%. Although only a two-dimensional flow problem is analyzed here, the method proposed in this paper is general and can also be applicable to three-dimensional rotating flows.

Acknowledgments

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Biography

Jianyao Yao received his Ph.D. from Beihang University, China, in 2011. He was a Postdoctoral Fellow at University of Cincinnati, USA, from 2011 to 2014, supervised by Dr. G. R. Liu. He is currently a research fellow at College of Aerospace Engineering, Chongqing University, Chongqing, China. He services as the Assistant Dean and Department Head of the College. His research interests include computational mechanics, structural safety and reliability of aeroengine, probabilistic design and analysis method, biomechanics, etc.