TUNABILITY OF SOFT PHONONIC CRYSTALS VIA LARGE DEFORMATION

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Abstract: Small-on large theory is introduced to analyze the effect of finite deformation on elastic wave propagation in periodic lattice structures made from soft material. The buckling patterns varying with loading conditions and the tunability of phononic crystals (PCs) through large deformation will be studied in details.

1. Introduction

Acoustic metamaterials have attracted a lot of attentions due to excellent properties. For phononic crystals, there might exist certain ranges of frequency (named as band gaps, BGs), in which the wave are prohibited to propagate due to the mechanisms of Bragg scattering or local resonance (LR)[1]. It is valuable to make the PCs with tunable BGs through external stimuli, such as mechanical loading [2, 5], electric fields [3], magnetic fields [4] and so on. Recently, soft PCs that can undergo large deformation provide us the opportunities to actively control the BGs through reversible finite deformation in versatile manners [3, 6].

In this work, we will study the buckling patterns and the effects of post-buckling deformation on the dynamic properties of elastic wave propagation in soft periodic lattice structures.

2. Governing equations

The small-on-large theory [6] is introduced to analyze the elastic wave propagation with initial finite deformation. Denote the coordinate of a particle within a continuum body in the reference configuration as $\mathbf{X}$, and it moves to position $\mathbf{x}=\mathbf{x}(\mathbf{X})$ in the deformed configuration, so the deformation gradient tensor is $\mathbf{F}=\partial\mathbf{x}(\mathbf{X})/\partial\mathbf{X}$ and its determinant $J=\det(\mathbf{F})>0$. The material is assumed to be hyperelastic with strain energy density as: $W=W(\mathbf{F})$, therefore, the nominal stress is: $\mathbf{S}=\partial W(\mathbf{F})/\partial\mathbf{F}$, and the Cauchy stress is: $\boldsymbol{\sigma}=J^{-1}\mathbf{F}\mathbf{S}$, provided the material is compressible. In the absence of body force, the equilibrium equations under static loading are

$$\operatorname{div}\boldsymbol{\sigma}=0, \tag{1}$$

where $\operatorname{div}(\cdot)$ denotes the divergence operator in the deformed configuration. Now we superimpose an elastic wave with small amplitude $\mathbf{u}(\mathbf{x},t)$ on the previous finite deformation, where t is the time. We note $\operatorname{grad}\mathbf{u}(\mathbf{x},t)$ is small, thus, the perturbation of nominal stress caused by elastic wave is:

$$\Delta\mathbf{S}=\mathbf{S}(\tilde{\mathbf{F}})-\mathbf{S}(\mathbf{F})\approx\mathbf{L}:\left(\mathbf{F}\cdot\operatorname{grad}\mathbf{u}(\mathbf{x},t)\right), \tag{2}$$

where $\mathbf{L}=\partial^2 W(\mathbf{F})/\partial\mathbf{F}^2$ is the fourth-order material tensor. By neglecting the high order terms and introducing the push forward of the perturbation of nominal stress, $\mathbf{T}=J^{-1}\mathbf{F}\Delta\mathbf{S}$, the governing equations of wave propagation can be obtained

$$\operatorname{div}\mathbf{T}=\rho\,\partial^2\mathbf{u}(\mathbf{x},t)/\partial t^2, \tag{3}$$

where ρ is the material mass density in the deformed configuration.

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Furthermore, the periodic boundary conditions and Bloch boundary conditions are necessary to analyze the finite deformation and waves in periodic structures [2, 5], respectively.

3. Results

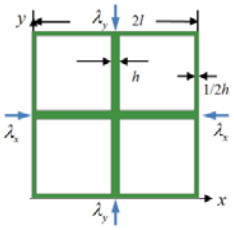


Figure 1. Lattice structure with 2x2 cells.

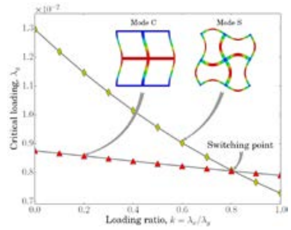
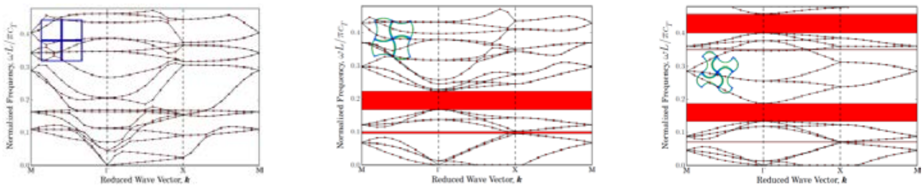


Figure 2. Ctrical loading vs. loading ratio.

Fig. 1 shows the geometry of a lattice structure with 2×2 cells2 (see [2, 5] for details), where l and h are the length and width of each beam, λ_x and λ_y are the loadings along x and y axies, respectivly. Fig. 2 plots the critical loading of λ_y for the first and second buckling patterns varying with the loading ratio k. It is evident mode 'C' shown in Fig. 2 is preferable when k is below 0.8, which is further verified to have indistinctive effects on BGs, and should be avoided in the design of tunable PCs. When k is close to unit (k=1 means the equi-biaxial loading condition), mode 'S' will be the first buckling mode. The effects of compression on BGs marked by red blocks are illustrated in Fig. 3, showing the effectiveness to tune the BGs of soft PCs through finite deformation under external mechanical loadings.



(a) Without compression; (b) Compression with 10%; (c) (b) Compression with 10%;

Fig 3. Effects of compression on BGs under equi-biaxial loading, k=1.

4. Conclusions

The buckling patterns of periodic lattices made from soft material are found to be dependent on the loading ratios, and the case with equi-biaxial loading condition are verified to be effective to tune the BGs of PCs through the control of compression mechanically.

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