Analysis and Experimental validation of free vibration of the Reissner-Mindlin Plates based on ES-FEM.

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Abstract

Recently, the edge-based smoothed finite element method (ES-FEM) is proposed based on the Reissner-Mindlin theory, holding the advantage of higher accuracy. In this work, a simple and reproducible experiment is carefully designed and conducted, in which the mode values and shape of a rectangular steel plate is tested by the LMS equipment, to further examine the performance of ES-FME. For comparison, mathematics model of the rectangular steel plate is built, and the mode values and shapes are calculated using FEM、ES-FEM (based on the Matlab) and the business software (Hypermesh). It is found that excellent agreement was achieved between the ES-FEM results and test result. The comparison demonstrates that the ES-FEM improves accuracy of the free vibration analysis of the plate structures.

Keywords: ES-FEM, Reissner-Mindlin Plates, Free Vibration, FEM.

1.Introduction

In the past several decades, Finite element method (FEM)[1] is one of the mostly successful method applied to model dynamic behavior of plates systems. However, in practical level, the conventional FEM often entails some inherent drawbacks, which is closely associated with the well-known “over-stiff” feature [2], which leads to inaccuracies and sensitivity to mesh distortion. In another aspect, FEM constructed based on the Reissner-Mindlin theory often suffered from the so-called “shear locking” problem [3]. Therefore, the precision and reliability of the response of plate system depend on the reasonable elimination of “shear locking” as well as the advanced and high accuracy technique.

In order to eliminate “over-stiff” feature problems, numerical techniques such as strain smoothing was applied in the conventional FEM, thus a series of novel smoothed FEM were proposed. The NS-FEM [4] was first developed using strain smoothing based on nodes, and has an “overly-soft” behavior. However, it was found unstable in the dynamic analysis of Reissner-Mindlin plates. Then FS-FEM[5]、CS-FEM[6] and ES-FEM[7] were successively developed, Compared with NS-FEM and the standard FEM, the edge-based strain smoothing techniques bring “proper softening effects” into the discrete model. And these effects enable the ES-FEM model to show neither “overly soft” nor “overly stiff” features. So, ES-FEM model is capable of yielding much more accurate numerical solutions for the dynamical analysis [7]. Given the superior performance of the ES-FEM, in this work, ES-FEM is chosen to simulate the dynamic property of the standard rectangle plate.

On the other hand, in order to eliminate “shear locking” problems, numerical techniques and effective formulations had been proposed, such as the selective reduced integration scheme [8] ， the enhanced assumed strain (EAS) methods[9] and assumed natural strain (ANS) methods [9]. The stability and accuracy were also further improved with the development the discrete shear gap (DSG) method[11]. In this work, the DSG and ES-FEM is combined to give a better solution.

In the previous work, the advantage of the higher accuracy of ES-FEM is analyzed and compared using the numerical methods, in this work; a standard test is designed to further examine the property of the ES-FEM. As it will be shown in the examples, the present ES-FEM is affords a “suitable” stiffness of the whole system, and hence it is more accurate than other existing techniques FEM. It is a good competitor to test method.

The paper is organized as follows: in section 2, we begin with a basic theory of ES-FEM for Reissner-Mindlin plates, in section 3, Numerical analysis and Experimental validation of ES-FEM are presented to demonstrate the performance of the ES-FEM for free vibration analyses of Reissner-Mindlin plates. Finally, a summary is given in section 4 to conclude this work.

2.Basic Theory of ES-FEM for Reissner-Mindlin plates

In this work, the edge-based smoothing operation is applied to the standard FEM to give a so-called ES-FEM for the plate elements. The dynamic variation equation for Reissner-Minlin plate elements can be described as follow:

For free vibration analysis, we have:

$$(\mathbf{K}-\omega^2\mathbf{M})\mathbf{u}=0 \tag{1}$$

in which ω is defined as the natural frequency, $\mathbf{K}$ and $\mathbf{M}$ are defined as the global stiffness matrix and mass matrix, respectively . Two matrix expressions are written as follows in detail:

$$\mathbf{K}=\int_\Omega \mathbf{B_b}^{\mathbf{T}}\mathbf{D_b}\mathbf{B_b}d\Omega+\int_\Omega \mathbf{B_s}^{\mathbf{T}}\mathbf{D_s}\mathbf{B_s}d\Omega \tag{2}$$

Where $\mathbf{B_b}$ the strain deflection matrix for bending, $\mathbf{B_s}$ is the strain deflection matrix for shearing.

$$\mathbf{M}=\int_\Omega \rho\mathbf{N_s}^T diag[\frac{t^3}{12}\quad \frac{t^3}{12}\quad t]\mathbf{N_s}d\Omega \tag{3}$$

Where t is the thickness of the plate.

For the application of smoothing technique, the smoothing domains should be first constructed based on the standard domain discretization in the conventional FEM. As shown in the Figure 1, the smoothing domain Ω_k is constructed by connecting the centroids of the neighboring triangles and the end-points of edge *k*. and is highlighted in the black color, which is also served as the integral domain.

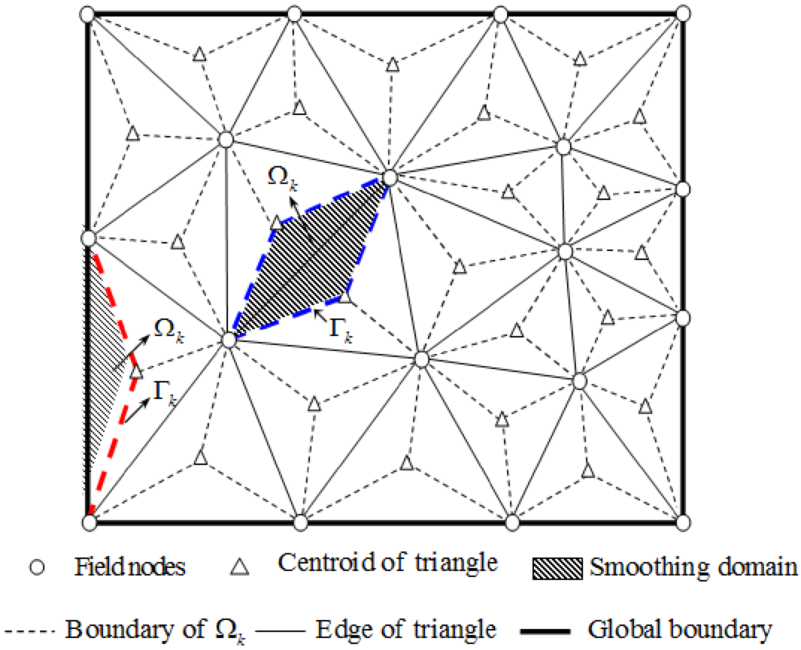


Figure 1. 2D edge-based smoothing domains constructed by connecting the centroid of cell *i* to end-nodes of the edge *k* of triangles.

Based on the smoothing domain, the smoothing operation is applied to the strain deflection matrix for bending $\mathbf{B}_b$ and the strain deflection matrix for shearing $\mathbf{B}_s$.

$$\bar{\mathbf{B}}_{bi}(\mathbf{x}_k)=\frac{1}{A_k}\sum_{i=1}^{N_k^e}\frac{1}{3}A_e^i\mathbf{B}_{bi}(\mathbf{x}_k),\bar{\mathbf{B}}_{si}(\mathbf{x}_k)=\frac{1}{A_k}\sum_{i=1}^{N_k^e}\frac{1}{3}A_e^i\mathbf{B}_{si}(\mathbf{x}_k) \tag{4}$$

where A_k is the area of the constructed smoothing domain. Then, the smoothed $\bar{\mathbf{K}}$ can be assembled based on the smoothing domain.

$$\bar{\mathbf{K}}_b^{(k)}=\int_{\Omega_k}\bar{\mathbf{B}}_b^{\mathrm{T}}\mathbf{D}_b\bar{\mathbf{B}}_b\mathrm{d}\Omega=\sum_{k=1}^{N_s}A_k\bar{\mathbf{B}}_b^{\mathrm{T}}\mathbf{D}_b\bar{\mathbf{B}}_b, \tag{5}$$

$$\bar{\mathbf{K}}_s^{(k)}=\int_{\Omega_k}\bar{\mathbf{B}}_s^{\mathrm{T}}\mathbf{D}_s\bar{\mathbf{B}}_s\mathrm{d}\Omega=\sum_{k=1}^{N_s}A_k\bar{\mathbf{B}}_s^{\mathrm{T}}\mathbf{D}_s\bar{\mathbf{B}}_s, \tag{6}$$

The global smoothed bending stiffness $\bar{\mathbf{K}}_b$ and global smoothed shear stiffness $\bar{\mathbf{K}}_s$ based on the edges can be assembled just as the same procedure as in the standard FEM. For overcoming the shear locking problem, the discrete shear gap (DSG) technique is applied in the calculation of global smoothed shear stiffness [11]. Then the global smoothed stiffness can be evaluated as:

$$\bar{\mathbf{K}}=\bar{\mathbf{K}}_b+\bar{\mathbf{K}}_s \tag{7}$$

Finally, the ES-FEM formulation for structural domain then can be written as:

$$(\bar{\mathbf{K}}-\omega^2\mathbf{M})\mathbf{u}=0 \tag{8}$$

3.Numerical analysis and Experimental validation of ES-FEM

In order to validate the ES-FEM, a simple and reproducible experiment is carefully designed and conducted, in which the mode values and shape of a rectangular steel plate is tested by the

LMS equipment. The rectangle flexible plate is made of steel (ρ=7800 kg/m^3, ν=0.3 and E=210Gpa) and has a dimension of 998×200mm with the thickness of 9.5mm. As shown in the Figure 2, there are 4 holes with diameter of 8mm in the corner of the rectangle plate. So that, the rectangle plate can be hanged by elastic ropes. Eight test points are evenly distributed around the plate.

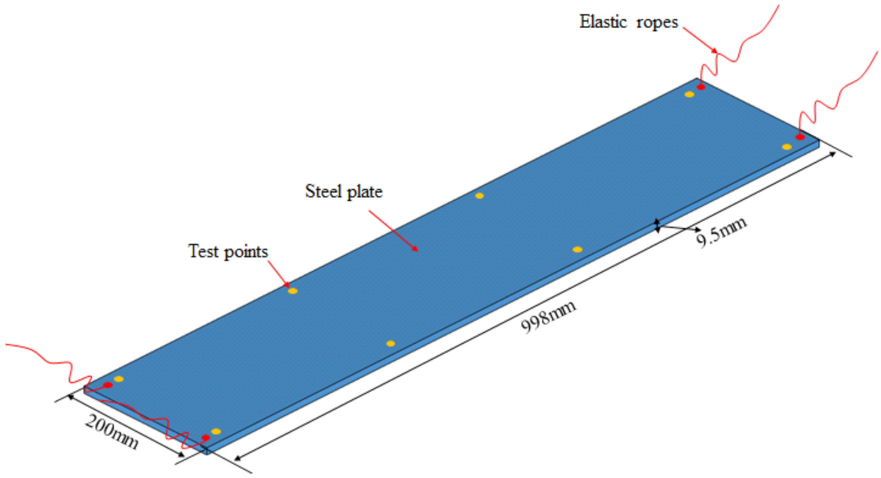


Figure 2. The test modal and geometric parameter of the steel rectangle plate.

By moving the hammer method, the transfer functions between different test points are obtained by LMS equipment and plotted as follows:

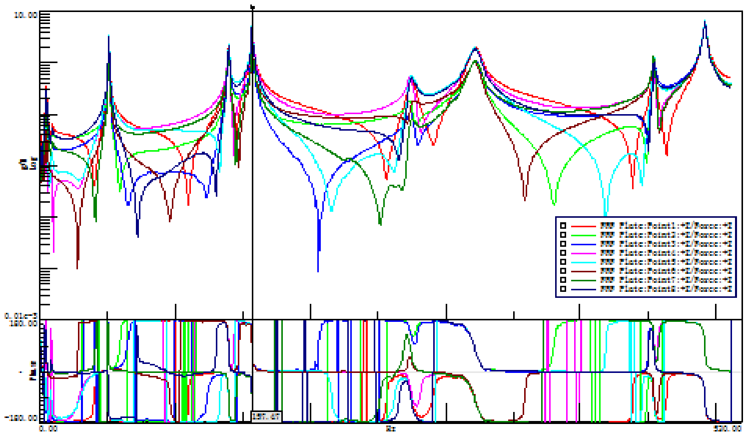


Figure 3. Transfer functions between different test points.

Meanwhile the four lowest bending modes of rectangle plate obtained by the ES-FEM、FEM and Hypermesh are also investigated in **Table 1**. The plate is discretized by 164 uniform meshes and 107 nodes. It is noted that all the results obtained from different numerical method are calculated based on the same mesh.

Table 1. Natural frequencies obtained by the calculation ES-FEM, FEM and Hypermesh and the test.

Bending Mode order	Mode shape	Test mode (Hz)	ES-FEM (Hz)	FEM (Hz)	Hypermesh (Hz)
1	First bending	50.86	50.4	50.5	50.34
2	Second bending	139.47	139.08	140.19	138.64

3	Third bending	274.97	273.01	277.38	271.38
4	Fourth bending	454.22	451.79	462.05	447.2

For a better comparison, the absolute errors relative to test value are plotted in the Figure 4.

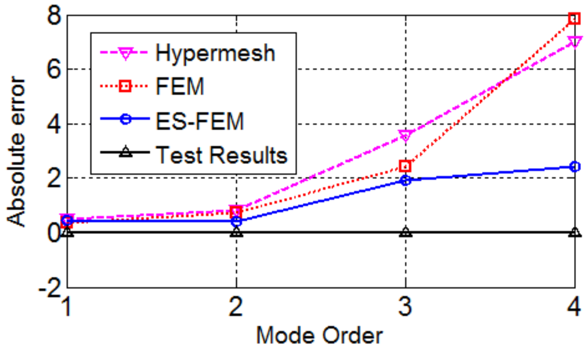
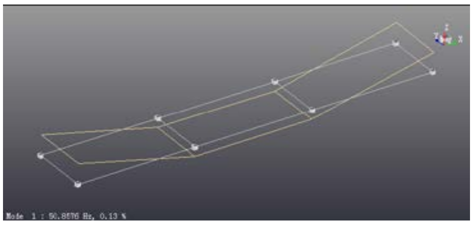
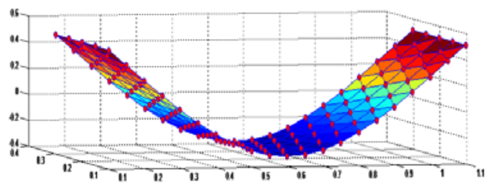
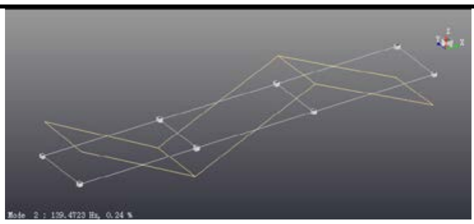
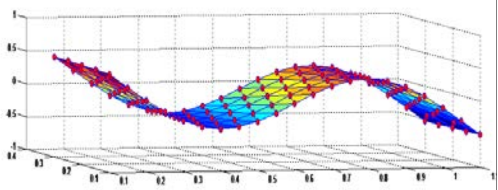


Figure 4. Comparison on the error of nature frequency obtained from different methods

As shown in the **Figure 4**, in the low frequency domain, results calculated from FEM and ES-FEM show good agreements with the Test result, suggesting that both FEM and ES-FEM can offer high accuracy results in low frequency domain. As the frequency increases, the nature-frequencies obtained from FEM become far much larger than the test results, demonstrating that the inherent drawback of "over-stiffness" in FEM leads lower accuracy in higher frequency domain. While the ES-FEM always offers much more accurate results in higher frequency range, compared to the FEM model using the same mesh.

Meanwhile, the first four lowest bending mode shapes of the rectangle plate obtained from ES-FEM and test are illustrated in **Figure 5**.

Mode order (bending)	Mode shapes (Test)	Mode shapes (ES-FEM)
1		
2		

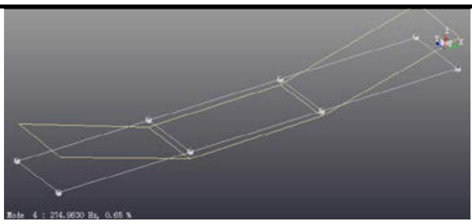
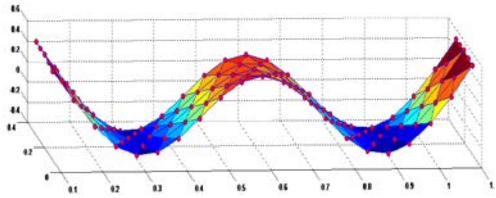
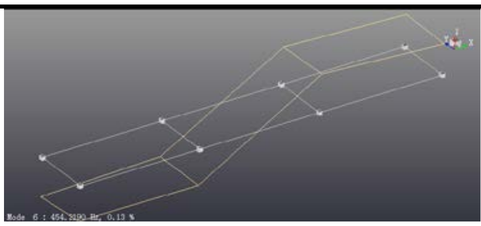
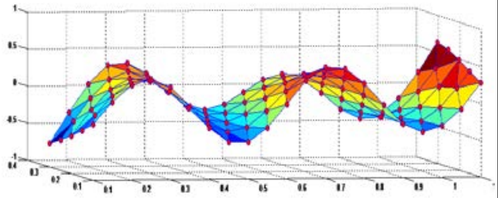
3		
4		

Figure 5. Comparison on the mode shapes obtained from different methods

It is found that the result of ES-FEM is stable for solving dynamical problems, where the physical mode shape can be clearly obtained.

4.Conclusions

In this work， we developed a test modal for investigating the property of the previous proposed ES-FEM. The ES-FEM was validated through standard benchmarking problem. Numerical examples and test results have demonstrated the following features of the ES-FEM:

(1) Compared with conventional FEM, the edged-based smoothing techniques help ES-FEM soften the stiffness of the system, thus eliminating the numerical error in standard FEM.

(2) Compared with the test results, ES-FEM is stable for solving dynamical problems, all the physical mode can be effectively obtained. What is more, ES-FEM constantly offers much more accurate nature frequencies results in higher frequency range, compared with the FEM model using the same mesh. It is a good competitor to the test methods.

5.References

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Biography

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