Fourier transform analysis of convergence properties of MultiGrid V-cycle algorithms

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Abstract

MultiGrid (MG) method has been widely used to solve the large-scale algebraic system of equations, due to its fast (close to linear) convergence property. This paper aims to provide an insight on the mechanism for the fast convergence of MG V-cycle algorithms. Our study will reveal a number of features of these MG algorithms, including a novel relationship among the iteration numbers need for converged solution, condition number of the system matrix, and step-length. We apply the Fourier transform analysis to the residual error resulted from the MG iterations to compute quantitatively the frequency components of the error spectrum. When the number of iterations in different layers is fixed, the optimal iteration mode is determined according to the attenuation of the high and low frequency error components. Various numerical experiments have been conducted by solving algebraic equations of FEM models of heat transfer problems. It is found that the iteration with numbers of 3-2-3 is the most efficient a three-layer grid. Our studies also show that the complexity of the MG method is close to O (*N*), meaning that it is a nearly-linear solver, which echo the findings given by others in the literature.

Key words: MultiGrid; finite element method; restriction operator; Fourier transform; high and low frequency components