# 2-D Numerical Simulation of Grounded Electrical-source Airborne Transient Electromagnetic Exploration based on Meshfree Method

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## Abstract

Grounded Electrical-source Airborne Transient Electromagnetic（GREATEM）is a new geophysical exploration method. When using the traditional numerical simulation methods to deal with the forward modeling, there exits some problems such as mesh generation and low accuracy. In view of the above problems, the meshfree collocation method based on the radial basis function is applied to calculate the response of GREATEM. We derive the control iterative equations based on the Maxwell's equations, the boundary condition and stability condition are also discussed. We compare the meshfree solution with the analytical and the finite difference method in a homogeneous half space model, and calculate a circular low resistivity abnormal body model. The calculation results show that the meshfree collocation method to discrete control equation is simple and has high calculation precision, it is easy to simulate the complex models. The meshfree method is expected to be widely used in the numerical simulation of grounded airborne transient electromagnetic exploration, it will provide a new idea for the geophysical exploration modeling.

**Keywords:** GREATEM;  meshfree; collocation; iterative equations.

## Introduction

Grounded Electrical-source Airborne Transient Electromagnetic（GREATEM）is a new and hot geophysical electromagnetic exploration style in recent years. In this method. the current source is transmitted on the ground, while the receiver in the flying platform receives the electromagnetic signal. GREATEM combines the advantages of ground time-domain electromagnetic system and air time-domain electromagnetic system, it has the advantages of large depth exploration, high vertical resolution, simple and easy operation. It has been widely used in the investigation of mineral resources and engineering environment (Ito et al. 2011; Allah et al. 2013; Ji et al. 2016). Numerical simulation is an effective way to study the electromagnetic response variation law of GREATEM. The commonly used electromagnetic numerical simulation methods include finite difference method (FDM), finite element method(FEM), finite volume method(FVM) etc. These methods are based on grids, the solution domain is often divided into several certain shape meshes. In the process of dealing with the irregular abnormal body or the undulating terrain, it is necessary to divide the solution domain into small part meshes, which wastes lots of time, and the simulation result is not better.

Meshfree method is a new numerical calculation method,  it does not require the predefined meshes to construct the shape functions, which can completely or partially eliminate the dependency of meshes. Meshfree method has the characteristics of simple pre-processing and easy to simulate the complex models, which has been widely concerned and used in the field of engineering calculation. In the field of geophysical electromagnetic exploration, Dai et al. (2014) carried out the 2D Ground Penetrating Radar (GPR) forward simulation with the improved Sarma boundary condition by using Eelement-Free Galerkin method. Wittke and Tezkan (2014) used MLPG method to simulate the 2-D magnetotelluric response. Ji et al. (2016) studied the magnetotelluric response under the undulating terrain and the anisotropic media. At present, the application of meshfree method in the field of geophysics is mainly concentrated in the frequency domain and the weak form, the relevant results based on strong form (collocation method) has not been published yet.

Meshfree collocation method is a pure meshfree method, which has the advantages of direct, simple and high efficiency. In view of the characteristics of GREATEM, we apply the meshfree method to the numerical simulation of the 2-D GREATEM forward modeling in this paper. Based on the Maxwell's equations, the diffusion equation of the electric field along the strike direction is derived. We use the radial basis functions to discretize the spatial domain, and the C-N difference scheme to discretize the time, the numerical simulation is realized, and some electromagnetic response characteristics are also discussed.

## Theory

### *GREATEM governing equation*

The measurement principle of GREATEM is shown in Fig. 1. It adopts a grounded long conductor laid on the ground as the transmitting source to establish the primary field , when the current turns off, the aircraft equipped with the receiving sensor receives the second induced electromagnetic field which excites from the underground anomaly. GREATEM not only has the advantages of large depth and high resolution, but also has the advantages of wide range and high speed, it is especially suitable for mountains, forest coverages, swamps and other special landscape areas to detect the resources.

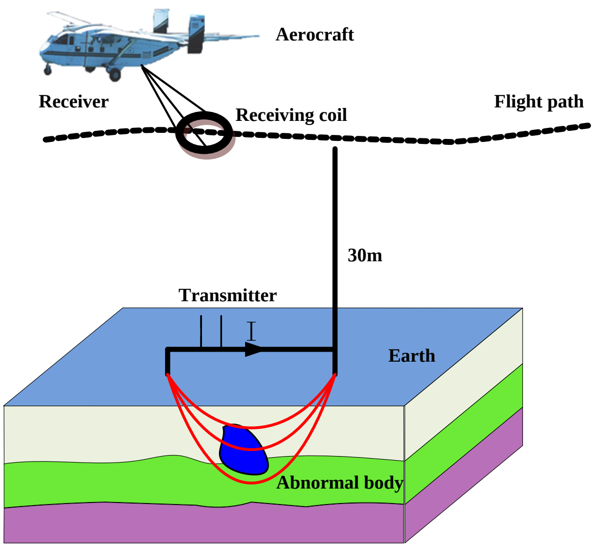


**Figure 1. The measurement principle of GREATEM**

Maxwell's equations are the basis of the electromagnetic exploration theory, when the electromagnetic waves propagate in a homogeneous, lossy, non-magnetic medium, the Maxwell's equations can be written as:

$$\begin{cases} \nabla \times E = -\dfrac{\partial B}{\partial t} \\ \nabla \times H = \varepsilon \dfrac{\partial E}{\partial t} + \sigma E \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \nabla \cdot E = 0 \\ \nabla \cdot H = 0 \end{cases} \tag{1}$$

Where $E$ is the electric field intensity, $H$ is the magnetic field intensity, $B$ is the magnetic flux density, $\sigma$ is the conductivity, $\varepsilon$ is the dielectric constant, $t$ is the time.

When the low frequency electromagnetic wave propagates in the lossy earth, the displacement current is relatively small, the conduction current is dominant, so the Maxwell's equation can be approximated under the quasi-static condition, we use the vector identity after ignoring the displacement current:

$$\nabla \times \nabla \times \mathrm{A} = \nabla(\nabla \cdot \mathrm{A}) - \nabla^2 \mathrm{A} \tag{2}$$

Then we can derive the diffusion equation of the electric field:

$$\nabla^2 \boldsymbol{E}(r,t) - \mu\sigma(r)\frac{\partial \boldsymbol{E}(r,t)}{\partial t} = 0 \tag{3}$$

When the length of the grounded conductor line is much greater than the distance from the observation point, the line source can be considered as an infinite 2-D source. We set the strike direction of the line source is along $y$ axis, then there are only three components of the electromagnetic field:

$$E(x,z,t) = E_y y \qquad H(x,z,t) = H_x x + H_z \hat{z} \tag{4}$$

Therefore, the 2-D GREATEM control equation is:

$$\nabla^2 \boldsymbol{E}_y(r,t) - \mu\sigma(r)\frac{\partial \boldsymbol{E}_y(r,t)}{\partial t} = 0 \tag{5}$$

*Meshfree collocation iterative equation*

The Radial Basis Function (RBF) is based on the spatial distance between the center point and the collocation point, which has the advantages of simple form and isotropy (Lai et al., 2008). It is very suitable for the engineering calculation. Generally, the field function value $u(\boldsymbol{r})$ in the domain $\Re$ can be approximated by the RBF of each central node $\boldsymbol{r}_i$ :

$$u(\boldsymbol{r}) \approx u^h(\boldsymbol{r}) = \sum_{i=1}^{N} \lambda_i \phi_i(\boldsymbol{r}) \qquad r \in \Re \tag{6}$$

Where $u^h(\boldsymbol{r})$ is the approximate field function, $\lambda_i$ is the undetermined coefficient, $\phi_i(\boldsymbol{r}) = \phi(|\boldsymbol{r} - \boldsymbol{r}_i|)$ is the RBF between collocation point $\boldsymbol{r}$ and the central node $\boldsymbol{r}_i$ . $N$ is the number of central nodes in the domain.

Considering the MQ function has good interpolation properties and has been widely used (Kansa 1990; Cheng et al. 2003), so we use the MQ function to solve the 2-D GREATEM modeling. The basic form of difference approximation is shown in table 1.

The forward and backward difference schemes are conditionally stable, while the C-N scheme is the best of the four difference schemes, it has the highest precision and is used in this paper.

For any time $t_0$ in equation, the field $\boldsymbol{E}_y$ can be separated into

$$\boldsymbol{E}_y\left(r,t_0\right)=T\left(t_0\right)\boldsymbol{E}_y\left(r\right) \tag{7}$$

The C-N difference scheme is used to iterate each time step. The iterative relation in the space domain is:

$$u^{n+1}=\left(\nabla^2-\frac{\Delta t}{2K}\right)^{-1}\left[\nabla^2+\frac{\Delta t}{2\mu\sigma}\right]u^n \tag{8}$$

**Table 1. Time domain difference scheme for the control equation**

| Difference scheme | Discrete form |
|---|---|
| Forward scheme | $\nabla^2(\boldsymbol{E}_y{}^n)-\mu\sigma\dfrac{\boldsymbol{E}_y{}^{n+1}-\boldsymbol{E}_y{}^n}{\Delta t}=0\Rightarrow\boldsymbol{E}_y{}^{n+1}=\boldsymbol{E}_y{}^n+\dfrac{\Delta t}{\mu\sigma}\nabla^2(\boldsymbol{E}_y{}^n)$ |
| Backward scheme | $\nabla^2(\boldsymbol{E}_y{}^n)-\mu\sigma\dfrac{\boldsymbol{E}_y{}^n-\boldsymbol{E}_y{}^{n-1}}{\Delta t}=0\Rightarrow\boldsymbol{E}_y{}^n=\left(1-\dfrac{\Delta t}{\mu\sigma}\nabla^2\right)^{-1}\boldsymbol{E}_y{}^{n-1}$ |
| Richardson scheme | $\nabla^2(\boldsymbol{E}_y{}^n)-\mu\sigma\dfrac{\boldsymbol{E}_y{}^{n+1}-\boldsymbol{E}_y{}^{n-1}}{2\Delta t}=0\Rightarrow\boldsymbol{E}_y{}^{n+1}=\boldsymbol{E}_y{}^{n-1}+\dfrac{2\Delta t}{\mu\sigma}\nabla^2(\boldsymbol{E}_y{}^n)$ |
| Crank-Nilcoson scheme | $\dfrac{1}{2}(\nabla^2(\boldsymbol{E}_y{}^{n+1})+\nabla^2(\boldsymbol{E}_y{}^n))-\mu\sigma\dfrac{\boldsymbol{E}_y{}^{n+1}-\boldsymbol{E}_y{}^n}{\Delta t}=0\Rightarrow\boldsymbol{E}_y{}^{n+1}=\left(\nabla^2-\dfrac{\Delta t}{2\mu\sigma}\right)^{-1}\left[\nabla^2+\dfrac{\Delta t}{2\mu\sigma}\right]\boldsymbol{E}_y{}^n$ |

Bring the RBF approximation into Eq.(8):

$$\sum_{i=1}^{N}\lambda_i^{n+1}\phi\left(r_i\right)=\left(\nabla^2-\frac{\Delta t}{2K}\right)^{-1}\left(\nabla^2+\frac{\Delta t}{2K}\right)\sum_{i=1}^{N}\lambda_i^n\phi\left(r_i\right) \tag{9}$$

We rewrite it as matrix form:

$$\Phi_{M_1\times N}\lambda_{N\times1}^{n+1}=\alpha\left(x\right)_{M_1\times1} \tag{10}$$

The boundary matrix can be obtained by substituting the boundary conditions:

$$\Phi_{M_{2\times N}}\lambda_{N\times1}^{n+1}=\psi\left(t\right)\Big|_{M_2\times1}^{t=(n+1)\Delta t} \tag{11}$$

In the next time step, the weight coefficients should satisfy both the regional governing equation and the boundary condition, we combine the above matrices and then obtain $\lambda^{n+1}$ :

$$\begin{bmatrix}\Phi_{M_1\times N}\\ \Phi_{M_2\times N}\end{bmatrix}\begin{bmatrix}\lambda_1^{n+1}\\ \lambda_2^{n+1}\\ \cdots\\ \lambda_N^{n+1}\end{bmatrix}=\begin{bmatrix}\alpha\left(x\right)_{M_2\times1}\\ \psi\left(t\right)\Big|_{M_2\times1}^{t=(n+1)\Delta t}\end{bmatrix} \tag{12}$$

*Initial condition*

Assuming that the earth is homogeneous, the response of the 2D current source in the homogeneous half space is taken as the initial field, its expression is (Oristaglio M L,1982):

$$E_y(x,z,t)=\frac{I}{\pi\sigma}\left\{\left[\frac{z^2-x^2}{R^2}+\frac{2z^2}{T}\right]\frac{e^{-R^2/T}}{R^2}-\frac{2ze^{-z^2/T}}{\sqrt{\pi}R^2}\left[\frac{1}{T^{1/2}}-2xF\left(xT^{-1/2}\right)\left(\frac{1}{T}+\frac{1}{R^2}\right)\right]\right\}+\frac{1}{\pi\sigma}\frac{x^2-z^2}{R^4}\left[1-erf\left(zT^{-1/2}\right)\right] \tag{13}$$

On the earth surface, it can be simplified as:

$$E(x,z=0,t)=\frac{I}{\pi\sigma}\frac{1}{x^2}(1-e^{-x^2/T}) \tag{14}$$

*Boundary condition*

The electric field in the TE mode is always continuous in the solution region. When the calculation domain is large enough and the boundary is far away from the abnormal body, we can set it be zero. On the ground-air boundary, the initial value can be calculated by the analytical formula. The electric field in the air can be realized by upward continuation theory (Wang and Hohmann,1993).

In order to save computation time, the step length is gradually increased with time, and the maximum time step can be:

$$\Delta t_{\max}=\frac{\left(\mu\sigma_{\min}t\right)^{1/2}\Delta_{\min}}{2} \tag{15}$$

Where $\Delta$ is the minimum node spacing.

## Model calculation

*Algorithm verification*

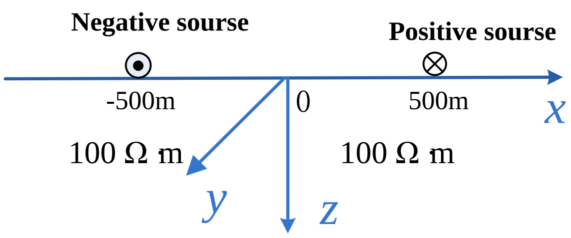


**Figure 2 Homogeneous half space model**

In order to verify the correctness of the meshfree method, a homogeneous half space model is built as shown in figure 2, the resistivity is $\rho$=100 Ω·m, there are two emission line source ,the positive source is located at *x*=500m, while the negative one is in *x*=-500m. The nodes are distributed evenly, the nodes near the source are dense while the nodes far from the source is gradually increasing. The number of the nodes and the node spacing are shown in Table 2.

**Table 2. Node number and node spacing**

| *x* direction | | *z* direction | |
|---|---|---|---|
| Node No.（i） | spacing（m） | Node No.（j） | spacing（m） |

| | | | |
|---|---|---|---|
| 1-10 | 240 | 1-53 | 10 |
| 11-15 | 120 | 54-58 | 15 |
| 16-20 | 60 | 59-63 | 30 |
| 21-25 | 30 | 64-68 | 60 |
| 26-35 | 15 | 69-73 | 120 |
| 36-165 | 10 | 74-78 | 240 |
| 166-175 | 15 | | |
| 176-180 | 30 | | |
| 181-185 | 60 | | |
| 186-190 | 120 | | |
| 191-200 | 240 | | |

We respectively calculate the homogeneous half space model by using analytical method , the Dufort-Frankel finite difference method (Oristaglio et al, 1984) and the meshfree collocation method under the same conditions. The calculation results of the meshfree collocation method and the analytical method at different time are shown in Figure 3. The error comparison curve of the meshfree method and the finite difference method is shown in figure 4.

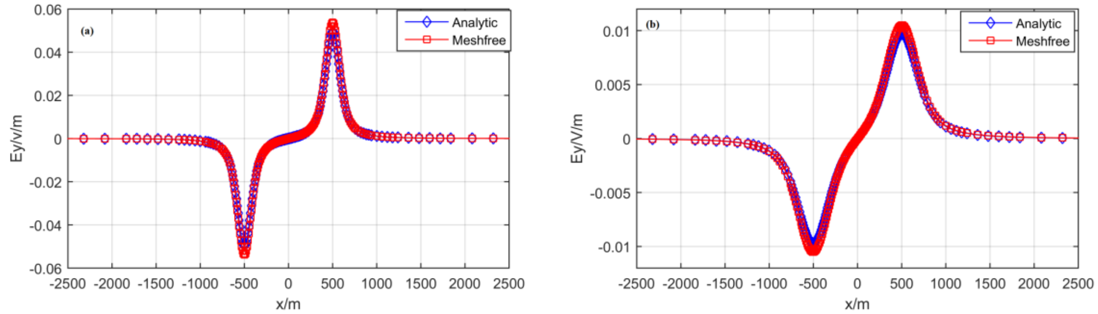


**Figure 3 The comparison between the meshfree method and the analytical method**
**（a）0.2ms （b）1ms**

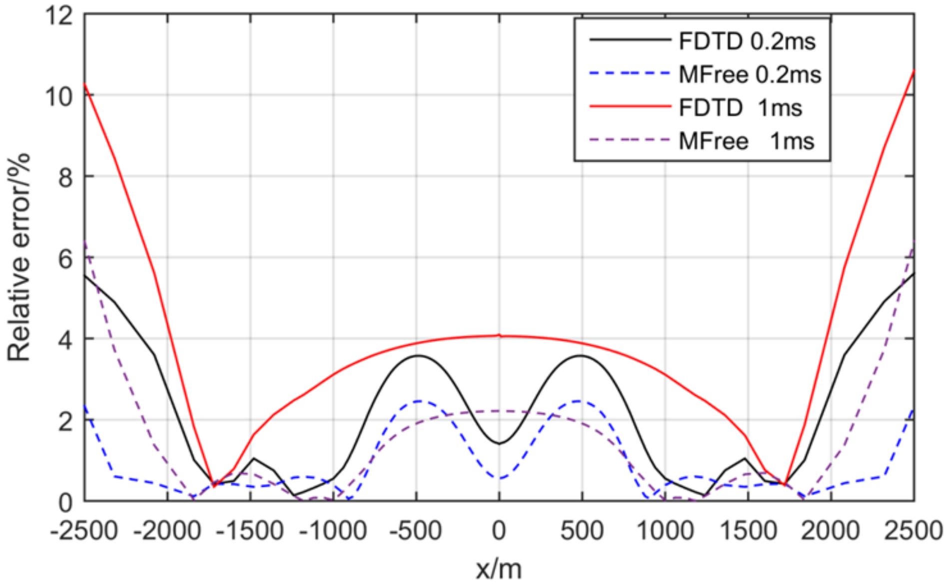


**Figure 4 The error comparison curve of the meshfree method and the finite difference method**

As ban be seen from Figure 3, the meshfree method and the analytical method fit well at different time, the maximum relative error is less than 3% in 1ms. The results show that meshfree method can effectively calculate 2D GREATEM response. Under the same time step and node distribution, the Dufort-Frankel finite difference solution is also calculated. As can be seen from Figure 4, after 1000 iterations to 0.2ms, the average relative error of FDTD is about 2.54%, while the meshfree method is about 1.21%, After 9000 iterations to 1ms, the average relative error of FDTD is about 3.84%, while the meshfree method is about 1.75%, its calculation precision is higher than the finite difference method. At the same time, the meshfree method pretreatment is simple and direct, which solves the problem of mesh generation and local approximation.

*Anomaly model*

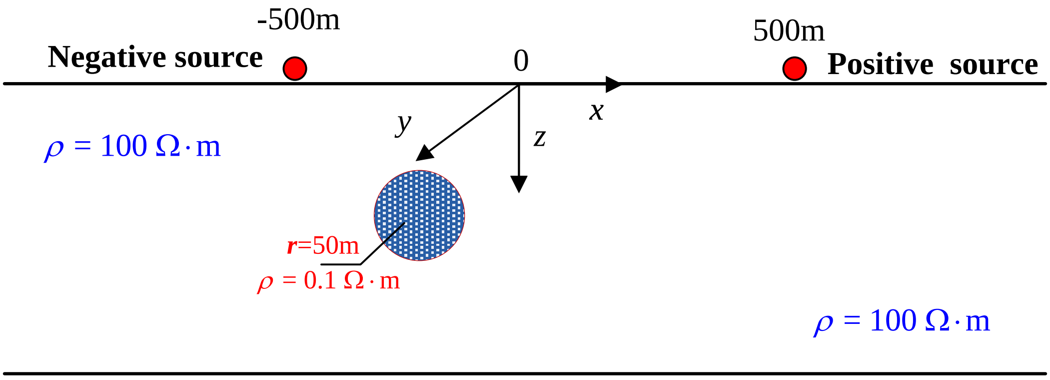


**Figure 5 Circular low resistivity anomaly model**

We establish a low resistivity anomaly model as shown in Figure 5, the resistivity of the homogeneous half space is 100 Ω·m, the resistivity of the circular anomaly bodyis 10 Ω·m. The center of the anomalous body is located in $x$=-200m, $z$=100m, its radius is 50m. The emission current is 100A, which is located at $x$=±500m. The node distribution are same as Table 2. The electric field section and profile at different time are respectively shown in Figure 6 and Figure 7.

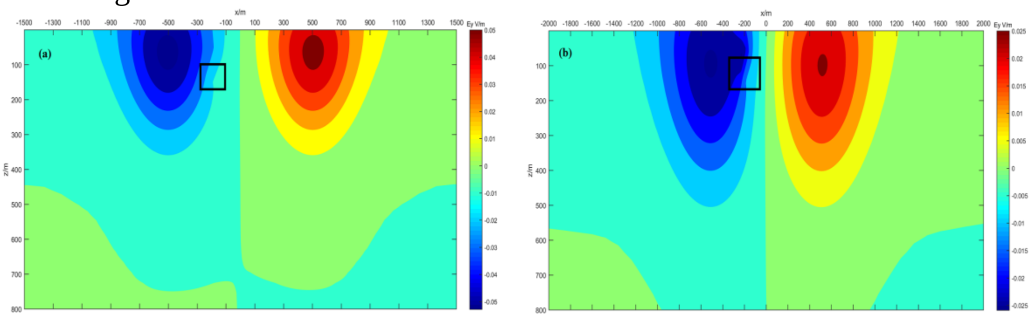


**Figure 6 The electric field section of circular low resistivity anomaly model**
**（a）0.2ms （b）0.5ms**

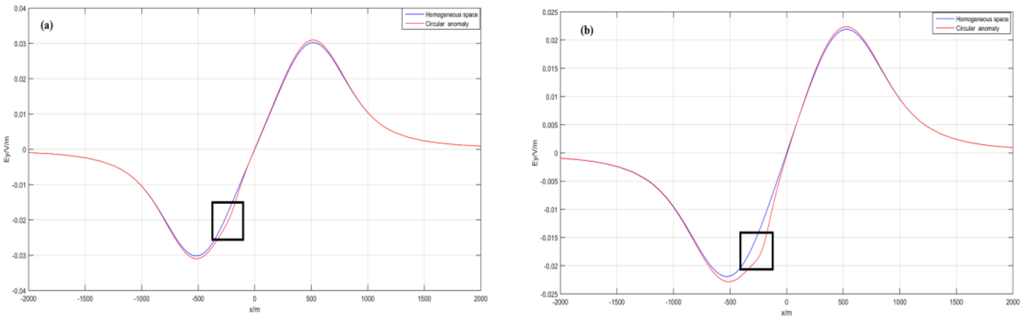


**Figure 7 The electric field profile of circular low resistivity anomaly model （a）0.2ms （b）0.5ms**

The anomaly body in Figure 5 is circular, and it is difficult to simulate with the traditional mesh-based method. Although it can be simulated by using a sufficiently small mesh or adaptive finite element method, it greatly increases the cost of computation. The meshfree method is out the restriction of the grid, and is more flexible than the grid-based method in dealing with the terrain undulation  interface, it is easy to simulate this model.

When the homogeneous earth contains low resistivity anomalous body, the diffusion of electric field is distorted due to the attraction of the low resistivity anomalous body. The electric field contour near the low resistivity abnormal body becomes denser and the gradient becomes larger. The anomalous body has an aggregation effect on the electric field, and the diffusion velocity of the induced eddy current becomes slow. Figure 6 is the electric field section under the circular low resistivity body. It can be seen that  the electric field near the low resistivity anomaly body distorts, we can judge the position of the circular anomaly body by the position of the electric field distortion. Figure 7 is the comparison of the electric field profile and the homogeneous half space , as can be seen from the figure, the electric field profile is no longer symmetrical, a large separation in the anomalous body position are engendered. So the GREATEM has a good ability to distinguish the perfect conductor, the modeling also provides a theoretical basis for the GREATEM to detect the metal ore and water geological structures.

## Conclusions

In this paper, the 2-D  numerical simulation of GREATEM is studied based on the theory of electromagnetic exploration and meshfree method, the principle and key techniques of the meshfree collocation method for the numerical simulation are presented. The main conclusions are as follows:

1. Radial basis functions have good fitting characteristics. The meshfree collocation method based on RBF to discrete control equations are simple and direct, it is easy to simulate the complex models and  make up for the deficiency of grid-based methods to some extent. It will become a new geophysical numerical simulation method;
2. The homogeneous half space and the low resistivity anomaly model are respectively calculated , the results show that the accuracy of the meshfree collocation method is higher than that of the finite difference method under the same conditions. GREATEM is more sensitive to the low resistivity anomalies, it has a good ability to distinguish the perfect conductor and provides a basis for the data processing and inversion.
3. Radial basis function is a function of distance, which has the characteristics of simple form and isotropy. It is easy to extend to the analysis of high dimensional problems. The application of the meshfree method to the numerical simulation of 3D electromagnetic exploration will be the next step in our research.

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