On Improving Evolutionary Algorithms and Acceleration Techniques Based on Estimation of Convergence Point Population for Chosen Optimization Problems of Mechanics

Janusz Orkisz, *†Maciej Glowacki

Institute for Computational Civil Engineering, Cracow University of Technology, Poland

*Presenting author: mglowac@gmail.com
†Corresponding author: mglowac@gmail.com

Abstract

Several issues regarding development of highly accelerated and efficient Evolutionary Algorithms (EA) for solving large, non-linear, constrained optimization problems are considered in this work. In particular, we briefly present here advances in development of already proposed acceleration techniques, including smoothing and balancing, adaptive step-by-step mesh refinement, as well as a'posteriori error analysis and related techniques. Our most recent research has been focused mainly on searching of efficient combination of the proposed techniques and their parameters, as well as on development of some new concepts based on estimation of the convergence point of population. The improved EA-based approach provides significant speed-up of solution process and/or possibility of solving such large problems, when the standard EA methods fail.

Keywords: Evolutionary Algorithms, acceleration techniques, large non-linear constrained optimization problems, convergence point of population

Introduction

Many important problems of computational mechanics may be formulated in terms of constrained optimization. Complexity of these problems may result mostly from their non-linearity, as well as from a large number of decision variables and constraints involved. Thus, we consider here a wide class of large, non-linear, constrained optimization problems. Due to the size and complexity of such problems, this research is focused, first of all, on the significant efficiency increase of the solution algorithms applied. Our solution approach is based on the EA, which on the contrary to most deterministic methods may be successfully applied to the both convex and non-convex problems [1]. However, general efficiency of the standard EA is rather low. Therefore, significant acceleration of the solution process is often needed. The forthcoming engineering objective of this long-term research includes residual stresses analysis [2][6] in railroad rails, and vehicle wheels, as well as a wide class of problems resulting from the Physically Based Approximation (PBA) of experimental and/or numerical data [4].

General problem formulation and solution algorithms

In the analyzed wide class of optimization problems, a function given in the discrete form, e.g. expressed in terms of its nodal values, is sought. These nodal values are defined on a mesh formed by arbitrarily distributed nodes. The optimal solution usually has to satisfy numerous equality, and inequality constraints. To obtain discrete formulation of optimization problem, any discretization method can be applied, including Finite Element, as well as Meshless Finite Difference Methods used here.

The EA are understood here as real-value coded genetic algorithms consisting of selection, crossover, and mutation operators [1]. We have proposed and tested so far several new, simple but effective EA acceleration techniques with various variants, including solution smoothing and balancing, an adaptive step-by-step mesh refinement, as well as a'posteriori solution error analysis and related techniques [3]. Appropriate constraint handling techniques were investigated as well. Our most recent research has been focused on further development of techniques based on various variants of estimation of the convergence point of a population considered. Reference [5] introduces a general idea and a few methods for estimation of the convergence point for the moving vectors of individuals between two subsequent generations. Such convergence point indicates the neighborhood of the optimum (see Fig. 1). It presents a powerful individual for the optimization process.

Considered is a population of M individuals

$$\mathbf{u}^j = [u_1^j, u_2^j, u_3^j, ..., u_n^j] \ , \quad j = 1, 2, 3, ..., M \tag{1}$$

in a n-dimensional space. Moving vectors are calculated between individuals $\mathbf{u}^{j,k}$ from k-th generation and their offspring $\mathbf{u}^{j,k+1}$ from (k+1)-th generation. $\tilde{\mathbf{u}}^k$ is a convergence point, and $\bar{\mathbf{u}}$ is the optimum point.

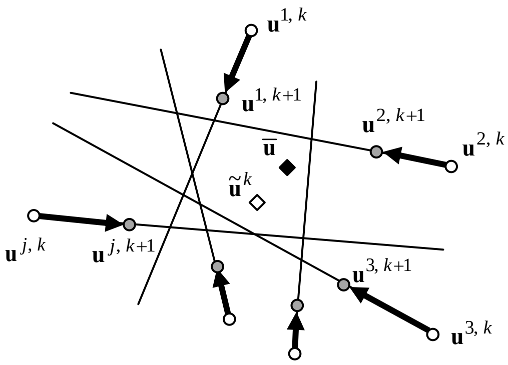


Figure 1. General idea of estimation of the convergence point of population

Various approaches, namely the exact, approximated and iterative ones are discussed in [5]. Approximated approach, which was presented as the best one, is based on the truncated Neumann series expansion. In this case, estimated convergence point is calculated in a following way:

$$\tilde{\mathbf{u}}^k \approx \tfrac{1}{M}\sum_{j=1}^{M}\mathbf{u}^{j,k} - \tfrac{1}{M}\sum_{j=1}^{M}((\mathbf{u}^{j,k})^{\mathrm{T}}\mathbf{b}^{0j,k})\,\mathbf{b}^{0j,k} \tag{2}$$

where

$$\mathbf{b}^{j,k} = \mathbf{u}^{j,k+1} - \mathbf{u}^{j,k} \ , \ \text{and} \ \ \mathbf{b}^{0j,k} = \mathbf{b}^{j,k} / \| \mathbf{b}^{j,k} \| \tag{3}$$

All general approaches presented in [5] can be applied to almost any population-based computations. We have proposed and preliminarily evaluated a specific formulation and implementation of these general approaches used for the EA acceleration. We have also proposed several new, original concepts for estimation of the convergence point of population, which can be used alternatively for these proposed in [5]. However, they still need further evaluation and development.

On benchmark problems

The efficiency of the new algorithms was examined using various demanding benchmarks involving large number of decision variables and constraints, including residual stress analysis in chosen elastic-perfectly plastic bodies, such as thick-walled cylinder, under various cyclic loadings. These benchmarks allow to choose almost any number of decision variables involved. The largest executed numerical tests involved more than 3000 decision variables. Several inverse problems were analyzed as well, including reconstruction of residual stresses. Such analysis used experimentally measured data, and the PBA approach. For example, the following sample optimization problem given in the polar coordinates for residual stresses in the thick-walled cylinder under cyclic internal pressure was investigated [6].

Find the minimum of the total complementary energy:

$$\min_{\sigma_r^r,\sigma_t^r,\sigma_z^r} \tfrac{1}{2E}\, 2\pi L \int_a^b [(\sigma_r^r - \sigma_t^r)^2 + (\sigma_t^r - \sigma_z^r)^2 + (\sigma_z^r - \sigma_r^r)^2]\, r\, dr \tag{4}$$

subject to the equilibrium equation

$$\tfrac{\partial \sigma_r^r}{\partial r} + \tfrac{\sigma_r^r - \sigma_t^r}{r} = 0 \tag{5}$$

boundary conditions

$$\sigma_{r|a}^r = 0, \qquad \sigma_{r|b}^r = 0 \tag{6}$$

the incompressibility equation

$$\sigma_z^r = \nu(\sigma_r^r + \sigma_t^r) \tag{7}$$

and the yield condition

$$\phi(\sigma_r^r, \sigma_t^r, \sigma_z^r, \sigma^E) \le \sigma_Y \tag{8}$$

where $\sigma_r^r, \sigma_t^r, \sigma_z^r$ are respectively the radial, circumferential and longitudinal stresses, $\sigma^E = \{\sigma_r^E, \sigma_t^E, \sigma_z^E\}$ is the purely elastic solution of the problem, σ_Y is the yield stress, a, b are respectively the internal and external radii, L is the cylinder length, and E is the Young modulus.

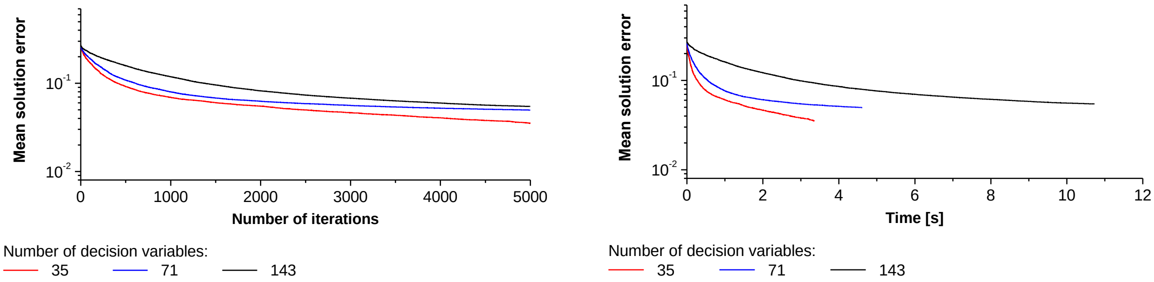
Sample of numerical results

In Fig. 2-5 one may find typical numerical results obtained for our efficiency analysis. They present convergence of mean solution error for residual stress analysis in cyclically pressurized thick-walled cylinder used as a benchmark problem. Due to stochastic nature of evolutionary computation, all results shown here were averaged over 20 independent solution processes.

Fig. 2 shows results obtained for the standard EA approach, consisting of selection, crossover, and mutation operators only, without any additional acceleration techniques. Results for three different number of decision variables are presented. Each decision variable corresponds to one nodal value of residual stresses searched.

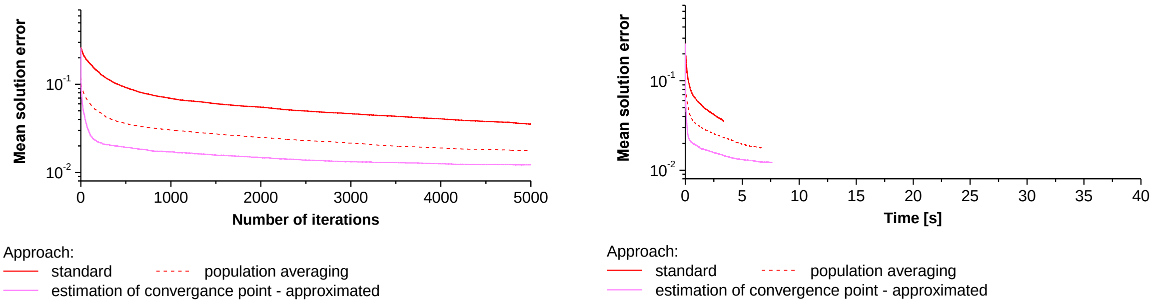
Comparison of accelerated EA is shown in Fig. 3-5. Approach based on approximated estimation of convergence point of population is compared to the standard EA, as well as to EA using simple averaging of population.

All optimization processes were calculated for the same number of generations (iterations) of EA – see figures (a). On the other hand, in figures (b) one may find time of computation needed to process these iterations.



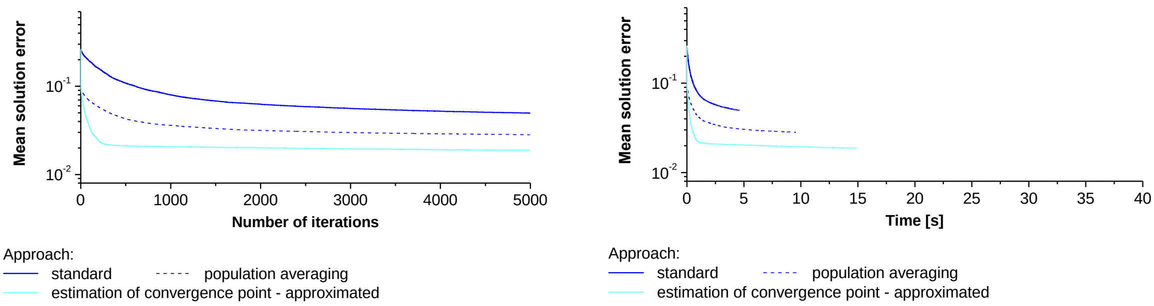
(a) (b)

Figure 2. Efficiency analysis of the standard EA for different number of decision variables



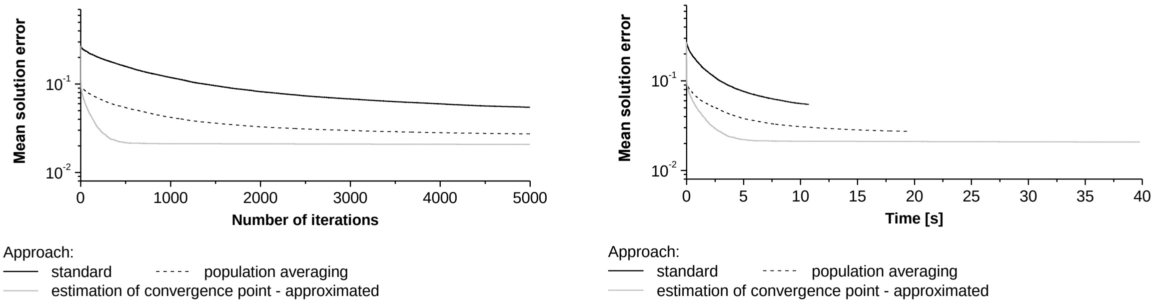
(a) (b)

Figure 3. Comparison of accelerated EA for 35 decision variables



(a) (b)

Figure 4. Comparison of accelerated EA for 71 decision variables



(a) (b)

Figure 5. Comparison of accelerated EA for 143 decision variables

Techniques based on estimation of the convergence point of population allowed to obtain acceleration up to about 40 times. It is still less than in the case of our earlier approach based on step-by-step mesh refinement combined with smoothing and a'posteriori error analysis (about 140 times), but this methods may be still improved.

In numerical results presented, approach based on approximated estimation of convergence point is better than EA using simple averaging of population. However, this method was not so efficient in all benchmark problems considered. Thus, averaging of population should also be taken into account as one of possible acceleration techniques worth applying.

Final remarks

Numerical results obtained indicate possibility of practical application of the improved EA to real complex optimization problems involving large number of decision variables and constraints. Numerical analysis also shows possibilities of further development of speed-up techniques considered, e.g. by means of combining various variants.

Future research will be mostly focused on application of the improved EA to engineering problems of mechanics.

References

[1] Engelbrecht, A.P. (2007) *Computational intelligence: an introduction*, Wiley, Chichester.

[2] Hill, R. (2004) *The Mathematical Theory of Plasticity*, Oxford University Press, New York.

[3] Glowacki, M., Orkisz, J. (2015) On increasing computational efficiency of evolutionary algorithms applied to large optimization problems, *2015 IEEE Congress on Evolutionary Computation*, 2639-2646.

[4] Karmowski, W., Orkisz, J. (1993) Physically based method of enhancement of experimental data – concepts, formulation and application to identification of residual stresses, *IUATAM Tokyo 1992 Symposium on Inverse Problems in Engineering Mechanics*, Springer-Verlag, 61-70.

[5] Murata, T., et al. (2015) Analytical estimation of the convergence point of populations, *2015 IEEE Congress on Evolutionary Computation*, 2619-2624.

[6] Orkisz, J. (1992) Prediction of actual residual stresses by constrained minimization of energy, *Residual Stress in Rails*, Vol. 2, Kluwer Acad. Publisher, 101-124.