Comparation with different interface capturing schemes based on

gradient smoothing method using unstructured meshes

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Abstract

This paper focuses on comparing the present advection schemes to capture the interface without reconstruction. The VOF (volume of fluid) equation is solved based on gradient smoothing method. With the help of blending function, the interface capturing schemes are devised as a blend of high-resolution and compression schemes. There are three well-known schemes to be selected, including CICSAM (Compressive Interface Capturing Scheme for Arbitrary meshes CICSAM), FBICS (Flux-Blending Interface-Capturing Scheme) and CUIBS (Cubic Upwind Interpolation based Blending Scheme). Using gradient smoothing operation, the variables of upwind points can be calculated by interpolation on gradient smoothing domains. Two benchmark tests are adopted. Numerical results show that CICSAM scheme produces more numerical error with the increase of Courant number because of numerical diffusion, while FBICS and CUIBS schemes can obtain satisfactory predictions at different Courant numbers.

Keywords: gradient smoothing method (GSM), normalized variable diagram (NVD), unstructured meshes, volume of Fluid (VOF) method.

1 Introduction

In past two decades, computational fluid dynamics (CFD) plays an important role in the safety evaluation of ship and ocean structured (e.g., sloshing, ship slamming, green water impact, etc). With the growing need in ship building industry, CFD as an engineering tool is facing new requirements. Multiple flows is a major challenge in solving naval hydrodynamics problems using CFD, especially the development of more accurate mathematical model using unstructured meshes. Thus, several interesting methods were introduced and developed to solve multiple flows problems, such as level set method [1], particles on interface [2] or smoothed particle hydrodynamics [3].

One convenient and powerful method based on Eulerian mesh is the Volume of Fluid (VOF) method, which was first developed by Nichols and Hirt [4]. In the VOF method, a volume fraction function is introduced, which represents the fraction of a local cell volume occupied by one of the fluids. And the volume fraction function is governed by a scalar convection equation through flow domain. For maintain the sharpness of the captured interface, one class of schemes is introduced with using interface reconstruction and high-resolution differencing schemes. Noh and Woodard [5] approximated the interface of each cell by vertical or horizontal lines, which is named simple line interface calculation (SLIC). For improving accuracy, the piecewise linear interface calculation (PLIC) was proposed by Youngs [6] using an oblique lines to reconstruct the interface. However, it is not difficult to see that the application is very complicated on unstructured meshes. To avoid reconstructing the interface, another class of approaches is to combine high-resolution schemes with compressive schemes. Over the past decades, many such improved schemes have been developed, among them: HRIC [7], CICSAM [8], STACS [9], FBICS [10] and CUIBS [11] schemes.

More recently, gradient smoothing methods (GSM) has been developed to solve compressible flows problems using unstructured meshes [12]. And GSM also was applied to solve the steady state and transient incompressible flow problems using the artificial compressibility method [13]. The method is effective for various types of fluid dynamics problems by combining with the major features of FVM and some meshfree techniques [14]. Because of different alternative smoothing functions and quadrature schemes for gradient approximation [15], the method has advantages on versatility and flexibility. Thus, the upwind variables can interpolated with the help of gradient smoothing operation, because the upwind points are need for constructing high-resolution schemes. For solving free surface problems using GSM, VOF is introduced in this paper. Thus, different advection schemes is performed and discussed.

Accordingly, several high-resolution, compressive advection schemes are compared in the context of GSM on unstructured meshes. In this article, a brief principle of the GSM is presented. Then the general methodology in interface-capturing schemes is clarified and concisely described, especially, three classical advection schemes are used. Finally, the results related to two advection cases obtained using several schemes at different Courant number values are presented and discussed.

2 Gradient smoothing method

Liu and Zhang developed generalized gradient smoothing technique [16]. Based G space and weakened weak formulation, smoothed point interpolation is presented and used for solving solid mechanics problems. Further, Liu and Xu introduced the method to solve strong–form governing equations for fluid dynamic problems [12]. Variable information is stored on the nodes and their derivatives at various locations are approximated with gradient smoothing operation over relevant gradient smoothing domains.

2.1 Gradient smoothing operation

The gradients of a field variable U at an arbitrary point at xi in domain Ω_i can be approximated in the form of

$$\nabla U_i \equiv \nabla U(X_i) \approx \int_{\Omega_i} \nabla U(X) \,\widehat{\omega}(X - X_i) dV \tag{1}$$

By integrating Eq. (1) by part and using divergence theorem, it becomes

$$\nabla U(X_i) \approx \int_{\Gamma_i} U(X) \,\widehat{\omega}(X - X_i) \boldsymbol{n} \mathrm{d}\Gamma_i - \int_{\Omega_i} U(X) \,\widehat{\omega}(X - X_i) \mathrm{d}\Omega_i \tag{2}$$

where \Box is the gradient operator; $\hat{\omega}$ is the smoothing function; Γ_i denotes the boundary of the gradient smoothing domain Ω_i ; and *n* represents the outward-pointing unit normal vector on Γ_i , as shown in Fig. 1.





Based on some essential conditions, e.g. the unity and compact conditions [17], the smoothing function is chosen properly to satisfy requirement of numerical solution. Accordingly, the smoothing functions in our study can be designed to be piecewise constant as follow

$$\widehat{\omega}(X - X_i) = \begin{cases} 1/A_i, X \in \Omega_i \\ 0, X \notin \Omega_i \end{cases}$$
(3)

where A_i stands for the area of the gradient smoothing domain Ω_i ; Thus, the second term on right-hand-side in Eq. (2) will vanishes, which reduces to

$$\nabla U(X_i) \approx \frac{1}{A_i} \int_{\partial \Omega_i} U(X) \, \boldsymbol{n} \mathrm{d}S \tag{4}$$

2.2 Construction of smoothing domains

The smoothing domains are constructed based on a set of primitive cells which are connected by nodes in computational domain. And the values of field functions are stored at those nodes. There are three types of gradient smoothing domains in GSM, respectively, the node-based gradient smoothing domain (nGSD), midpoint-based gradient smoothing domain (mGSD) and centroid-based gradient smoothing domain (cGSD). The nGSD is formed by connecting the centroids of relevant triangles with midpoints of influenced cell-edges, as shown in Fig. 2 (a). The mGSD is the connection of two end-nodes of the edge with the centroids on the both sides of the cell-edge, as shown in Fig. 2 (b). And the cGSD is formed by a primitive cell, as shown in Fig. 2 (c).



2.3 Approximations of spatial derivatives

2.3.1 First-order derivatives at nodes

One-point quadrature scheme (chosen as the midpoint) for each edge is used, and it is assumed that

$$U_{ck} = U_{ck-1} = U_{mk} \tag{5}$$

Using gradient smoothing operation of Equation (4), first-order derivatives at nodes can be approximated as

$$\begin{cases} \frac{\partial U_i}{\partial x} \approx \frac{1}{A_i^{nGSD}} \sum_{k=1}^{n_i} (\Delta \boldsymbol{S}_x)_{ij_k} U_{mk} \\ \frac{\partial U_i}{\partial y} \approx \frac{1}{A_i^{nGSD}} \sum_{k=1}^{n_i} (\Delta \boldsymbol{S}_y)_{ij_k} U_{mk} \end{cases}$$
(6)

where

$$(\Delta S_x)_{ij_k} = (\Delta S_x)_{ij_k}^{(L)} + (\Delta S_x)_{ij_k}^{(R)}, \qquad (\Delta S_y)_{ij_k} = (\Delta S_y)_{ij_k}^{(L)} + (\Delta S_y)_{ij_k}^{(R)}$$
(7)

In above equations, A_i^{nGSD} is the area of nGSD; n_i denotes the number of

supporting nodes around node i; $(\Delta S_x)_{ij_k}$ and $(\Delta S_y)_{ij_k}$ represent the sum of normal vectors of domain edges $m_k c_k$ and superscripts (L) and (R) are pointers to the two domain-edge associated with cell edge ij_k ; $m_k c_{k-1}$ associated with cell edge ij_k over nGSD shown in Fig. 2 (a),

$$\begin{cases} (\Delta S_x)_{ij_k}^{(L)} = r_{m_k c_k} (\boldsymbol{n}_x)_{m_k c_k}, & (\Delta S_y)_{ij_k}^{(L)} = r_{m_k c_k} (\boldsymbol{n}_y)_{m_k c_k} \\ (\Delta S_x)_{ij_k}^{(R)} = r_{m_k c_{k-1}} (\boldsymbol{n}_x)_{m_k c_{k-1}}, & (\Delta S_x)_{ij_k}^{(R)} = r_{m_k c_{k-1}} (\boldsymbol{n}_x)_{m_k c_{k-1}} \end{cases}$$
(8)

where r is the length of domain face and n_x and n_y represent the two components of a domain edge vectors.

The values of field variables U at midpoint are evaluated by simple liner interpolation:

$$U_{mk} \approx \frac{U_i + U_{j_k}}{2} \tag{9}$$

2.3.2 First-order derivatives at midpoints

The gradient at midpoint can be approximated with Eq. (4) over mGSM shown in Fig. 2 (b). They are approximated as follows:

$$\frac{\partial U_{mk}}{\partial x} \approx \left[\frac{1}{2} (\Delta \mathbf{S}_{m}^{x})_{ic_{k-1}} (U_{i} + U_{c_{k-1}}) + \frac{1}{2} (\Delta \mathbf{S}_{m}^{x})_{c_{k-1}j_{k}} (U_{c_{k-1}} + U_{j_{k}}) + \frac{1}{2} (\Delta \mathbf{S}_{m}^{x})_{j_{k}c_{k}} (U_{j_{k}} + U_{c_{k}}) + \frac{1}{2} (\Delta \mathbf{S}_{m}^{x})_{c_{k}i} (U_{c_{k}} + U_{i})\right] \frac{1}{A_{i}^{mGSM}}$$
(10)

$$\frac{\partial U_{mk}}{\partial y} \approx \left[\frac{1}{2} (\Delta \mathbf{S}_{m}^{y})_{ic_{k-1}} (U_{i} + U_{c_{k-1}}) + \frac{1}{2} (\Delta \mathbf{S}_{m}^{y})_{c_{k-1}j_{k}} (U_{c_{k-1}} + U_{j_{k}}) + \frac{1}{2} (\Delta \mathbf{S}_{m}^{y})_{j_{k}c_{k}} (U_{j_{k}} + U_{c_{k}}) + \frac{1}{2} (\Delta \mathbf{S}_{m}^{y})_{c_{k}i} (U_{c_{k}} + U_{i})\right] \frac{1}{A_{i}^{mGSM}}$$
(11)

where A_i^{mGSD} is the area of mGSD; ΔS_m^x and ΔS_m^y represent the components of a respective face vector of mGSD; The face vectors is computed in the similar way as the face vectors for nGSD. And the values of field variables U at centroid are calculated by simple liner interpolation:

$$U_{ck} \approx \frac{U_i + U_{j_k} + U_{j_{k+1}}}{3}$$
 (12)

2.3.3 First-order derivatives at centroids

Analogous to the discretization at nodes and midpoints described above, the gradient at centroids can be approximated over cGSM shown in Fig. 2 (c).

$$\frac{\partial U_{ck}}{\partial x} \approx \left[\frac{1}{2} (\Delta \mathbf{S}_{c}^{x})_{ij_{k}} (U_{i} + U_{j_{k}}) + \frac{1}{2} (\Delta \mathbf{S}_{c}^{x})_{j_{k}j_{k+1}} (U_{j_{k}} + U_{j_{k+1}}) + \frac{1}{2} (\Delta \mathbf{S}_{c}^{x})_{j_{k+1}i} (U_{j_{k+1}} + U_{i})\right] \frac{1}{A_{i}^{cGSD}}$$
(13)

$$\frac{\partial U_{ck}}{\partial y} \approx \left[\frac{1}{2} \left(\Delta S_{c}^{y}\right)_{ij_{k}} \left(U_{i} + U_{j_{k}}\right) + \frac{1}{2} \left(\Delta S_{c}^{y}\right)_{j_{k}j_{k+1}} \left(U_{j_{k}} + U_{j_{k+1}}\right) + \frac{1}{2} \left(\Delta S_{c}^{y}\right)_{j_{k+1}i} \left(U_{j_{k+1}} + U_{i}\right)\right] \frac{1}{A_{i}^{CGSD}}$$
(14)

where ΔS_c^x and ΔS_c^y denote the two components of a respective face vector for cGSM and A_i^{cGSD} is the area of the cGSM.

In this paper, GSM is adopted to solve VOF equation. Because only first order derivative need to calculated, GSM can be treat as vertex-centered FVM. However, the gradient operation is applied for the reconstruction of upwind point. This will be introduced in Section 5.

3. The VOF model

The various fluids are assumed to be incompressible and solutions are obtained by solving the following the conservation of mass and momentum equations [10]:

$$\frac{\partial(\rho \vec{v}_j)}{\partial x_j} = 0 \tag{15}$$

$$\frac{\partial \rho \vec{V}_j}{\partial t} + \frac{\partial}{\partial x_j} \left(\rho \vec{V}_i \vec{V}_j \right) = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} + \rho g_i \tag{16}$$

where \vec{V}_j is velocity vector, p is the pressure, τ_{ij} is the viscous stress tensor and g_i is gravitational acceleration.

And the volume fraction φ is governed by a simple advection equation:

$$\frac{\partial\varphi}{\partial t} + \frac{\partial(\vec{v}_j\varphi)}{\partial x_j} = 0 \tag{17}$$

The density is calculated by $\rho = \varphi \rho_1 + (1 - \varphi)\rho_2$ and viscosity by $\mu = \varphi \mu_1 + (1 - \varphi)\mu_2$. The subscripts 1 and 2 respectively denote the two fluids.

In this work, VOF equation is solved without interface reconstruction explicitly. The key is the spatial discretization of the advection equation on unstructured meshes. Thus, the convection term in Eq. (17) over a cell can be approximate as

$$\int_{\Omega} \frac{\partial(\vec{v}_j \varphi)}{\partial x_j} d\,\Omega = \int_{\Gamma} \vec{n} \cdot (\vec{V}_j \varphi) \, d\Gamma \approx \sum_f (\vec{n}_f \cdot \vec{V}_f) \varphi_f S_f \tag{18}$$

where Γ denotes the boundary of the control volume Ω ; S_f is the area of each face and f is the number of faces of the control volume Ω . For the temporal discretization, the Crank-Nicholson scheme is employed.

4. The present interface capturing schemes

From previous study, it is obvious that the key of the VOF method without interface

reconstruction focus on the interface capturing schemes used in advection equation. The schemes can ensure sharp resolution of the discontinuity, meanwhile avoid an over compressed interface. The design of interface scheme possess the two following basic fundamentals:(a) The interface scheme is a combination of Compressive (BD) and High-Resolution (HR) schemes;(b) Based on the angle between the interface direction and the grid orientation, a blending function should be obtained, preferably in a continuous fashion. Generally, the normalized value of φ at the control volume face can be obtained by blending the two schemes involved BD scheme and HR scheme:

$$\tilde{\varphi}_f = \tilde{\varphi}_f^{BD} f(\theta) + \tilde{\varphi}_f^{HR} [1 - f(\theta)]$$
(19)

where $f(\theta)$ is blending function which varies from 0 to 1 and θ is the angle between the normal unit vector of the interface and the vector pointing from centre point C to downstream point D. And $\tilde{\varphi}$ is the normal value which is introduced by Gaskell and Lau [18] and Leonard [19]:

$$\tilde{\varphi} = \frac{\varphi - \varphi_U}{\varphi_D - \varphi_U} \tag{20}$$

where the index U, C and D denote upwind point, centre point and downwind point in GSM, respectively, as depicted in Fig. 3. It is clear that when fluid flows from the upwind cell to the interface, if the interface is parallel to the cell face, the compressive scheme should be employed; and if perpendicular to the cell face, only high-resolution is used.



Figure 3. Illustration of the upwind, centre and downwind points on unstructured meshes

4.1 Compressive Interface Capturing Scheme for Arbitrary meshes, CICSAM

The CICSAM scheme was developed by Ubbink and Issa for interface capturing [20]. The scheme switches between the HYPER-C scheme and ULTIMATE QUICKEST (UQ) scheme. Both two schemes need to satisfy the Convection Boundedness Criterion (CBC). The HYPER-C scheme combined the CFL condition and CBC, and is expressed as:

$$\tilde{\varphi}_{fHYPER-C} = \begin{cases} \min\left(1, \frac{\tilde{\varphi}_C}{c_f}\right) & 0 \le \tilde{\varphi}_C \le 1\\ \tilde{\varphi}_C & otherwise \end{cases}$$
(21)

And the UQ scheme adopt a blend of upwind and QUICK schemes with a Courant number, the normal face value is defined as:

$$\tilde{\varphi}_{fUQ} = \begin{cases} \min\left(\frac{8C_f \tilde{\varphi}_C + (1 - C_f)(6\tilde{\varphi}_C + 3)}{8}, \tilde{\varphi}_{fHYPER-C}\right) & 0 \le \tilde{\varphi}_C \le 1\\ \tilde{\varphi}_C & otherwise \end{cases}$$
(22)

where C_f is the value of local Courant number and defined by $C_f = \frac{\sum_f \max(V_f \cdot S_f \Delta t, 0)}{V}$.

Furthermore, using a blending function, CICSAM switches the both schemes and can be written as:

$$\tilde{\varphi}_f = \tilde{\varphi}_{fHYPER-C} f(\theta_f) + \tilde{\varphi}_{fUQ} [1 - f(\theta_f)]$$
(23)

where $f(\theta_f)$ is blending function of angle θ_f between the gradient of the volume fraction at the interface and the normal to the cell face. The blending function and the angle are calculated by

$$f(\theta_f) = \min[\frac{\cos(2\theta_f) + 1}{2}, 1]$$
(24)

and

$$\theta_f = \arccos \left| \frac{\nabla \varphi_f \cdot \boldsymbol{d}_f}{\| \nabla \varphi_f \| \| \boldsymbol{d}_f \|} \right|$$
(25)

The NVD of CICSM is drawn in Fig. 4.



Figure 4. NVD of the CICSAM scheme

4.2Flux-Blending Interface-Capturing Scheme, FBICS

Tsui and co-workers have developed two interface-capturing scheme based on flux blending, FBICS-A and FBICS-B [10]. And in this paper, the FBICS-A is referred to simply as FBICS. Compared with other present schemes, FBICS scheme was the most accurate in capturing interface at different Courant number. FBICS uses Fromm's scheme as the basic scheme in HR and is built to satisfy CBS, is expressed:

$$\tilde{\varphi}_{fHR} = \begin{cases} 3\tilde{\varphi}_C & 0 < \tilde{\varphi}_C \leq \frac{1}{8} \\ \tilde{\varphi}_C + \frac{1}{4} & \frac{1}{8} < \tilde{\varphi}_C \leq \frac{3}{4} \\ 1 & \frac{3}{4} < \tilde{\varphi}_C \leq 1 \\ \tilde{\varphi}_C & otherwise \end{cases}$$

$$\tilde{\varphi}_{fBD} = \begin{cases} 3\tilde{\varphi}_C & 0 < \tilde{\varphi}_C \leq \frac{1}{3} \\ 1 & \frac{1}{3} < \tilde{\varphi}_C \leq 1 \\ \tilde{\varphi}_C & otherwise \end{cases}$$

$$(26)$$

The scheme is depicted on normalized variables diagram in Fig. 5. Different from CICSAM, FBICS is not dependent on the Courant number.



Figure 5. NVD of the FBICS scheme

4.3Cubic Upwind Interpolation based Blending Scheme, CUIBS

A new scheme is proposed for interface capturing, which is inspired by the study of Waterson and Deconinck based on the $GPL - \kappa$ class of schemes [21]. The CUIBS scheme is design to solve VOF model using unstructured meshes and shows a performance that is independent of Courant number [11]. In CUIBS scheme, limited CUI scheme is used as HR scheme and the BD scheme for the compressive is employed which is same as that used for FBICS. The normalized variable diagram of the CUIBS scheme is shown in Fig. 6. The HR and BD scheme is expressed as

$$\widetilde{\varphi}_{fHR} = \begin{cases}
3\widetilde{\varphi}_C & 0 < \widetilde{\varphi}_C \leq \frac{2}{13} \\
\frac{5}{6}\widetilde{\varphi}_C + \frac{1}{4} & \frac{2}{13} < \widetilde{\varphi}_C \leq \frac{4}{5} \\
1 & \frac{4}{5} < \widetilde{\varphi}_C \leq 1 \\
\widetilde{\varphi}_C & otherwise
\end{cases}$$

$$\widetilde{\varphi}_{fBD} = \begin{cases}
3\widetilde{\varphi}_C & 0 < \widetilde{\varphi}_C \leq \frac{1}{3} \\
1 & \frac{1}{3} < \widetilde{\varphi}_C \leq 1 \\
\widetilde{\varphi}_C & otherwise
\end{cases}$$
(28)
$$(28)$$



Figure 6. NVD of the CUIBS scheme

5. The upwind point reconstruction on unstructured meshes

Because of the more intricate geometrical computational field, it is difficult to implement TVD scheme on unstructured meshes. The value of point *C* and *D* can be easily obtained from known variables on unstructured meshes, but the location and variable value of upwind point are unknown. Three interpolation scheme (node gradient smoothing method, nGSM, midpoint gradient smoothing method, mGSM and centre gradient smoothing method, cGSM) based on gradient smoothing method are proposed in our previous study, as shown in Fig. 7, it is demonstrated that cGSM lead to a better performance in terms of accuracy and monotonicity. The information at upwind point can be calculated with interpolation on cGSD, is expressed as

$$\varphi_U = \varphi_U + \dot{d}_{UC} \cdot (\nabla \phi)_C \tag{30}$$

where \vec{d}_{UC} is the vector from point U to centroidC and $(\nabla \phi)_C$ is the gradient of centroid C. $(\nabla \phi)_C$ is calculated based on the gradient smoothing domain of centroidC.



Figure 7. Upwind point reconstruction using three schemes based on GSM

6. Numerical test

In this section, the three interface capturing schemes including CICSAM, FBICS and CUIBC schemes are compared to evaluate the relative performance. Two cases are selected for testing: (a) advection of hollow square in an oblique flow; (b) advection of a circle in shear flow. The tests are performed with low and high Courant number. Three Courant numbers are performed and denoted by low, medium and high in

present tests, they approximates 0.1, 0.5 and 0.7, respectively.

The relative error in numerical solutions is defined as

$$E = \frac{\sum_{i=1}^{N} |\phi_i^n - \phi_i^a|}{\sum_{i=1}^{N} |\phi_i^a|}$$
(31)

where N is the total number of nodes in the domain. ϕ_i^n and ϕ_i^a respectively denote the numerical solution and analytical solution.

6.1 Advection of hollow square in an oblique flow

To confirm the performance of interface capturing schemes, a hollow square which the outer width is 0.8 and the inner width is 0.4 and initially centred at (0.8, 0.8) ,transports in an oblique velocity field (V = (u, v) = (2, 1)). The domain is set to be 4×4 square. There are 11419 nodes and 22436 cells in unstructured triangular meshes.

After 1 unit of time, the contours of the volume fraction on unstructured meshes including CICSAM, FBICS and CUIBS schemes are depicted in Fig. 8- Fig. 10, and are over the range from 0.05 to 0.95 in interval of 0.1.



Figure 10. Contour plots for advection of hollow square in oblique flow using CUIBS scheme

The result shows that CICSAM scheme deteriorate at high Courant number because of numerical diffusive, as shown in Fig. 8. And the other two schemes can capture a sharp interface, as depicted in Fig. 9 and Fig. 10. To verify the influence of Courant number, the relative error for the hollow square is calculated using Eq. (31) and summarized in Table 1. With the increasing of Courant number, the error of CICSAM increases, while FBICS and CUIBS scheme are just opposite. Thus, it is also demonstrated that the Courant number has effect on CICSAM scheme and the error of FBICS and CUIBS scheme change very little at different Courant number. This indicates FBICS and CUIBS perform satisfactorily, regard less of the Courant number.

 Table 1. Relative error of hollow square in an oblique flow with different

 Courant numbers

Scheme	Low	Medium	High	
CICSAM	0. 13793	0.14222	0.18928	
FBICS	0.14328	0.13411	0.13049	
CUIBS	0. 14249	0.13180	0.12897	

6.2Advection of a circle in a shear flow

To further compare the ability of the three schemes for capturing interface, a circle in shear flow as a benchmark was tested. The problem reflects the interface twisted by a shear flow field, which is subjected to flow straining and deforms continuously. The computational field was set as a square with the size of $\pi \times \pi$. There are a circle of

radius
$$0.2\pi$$
 centred at $\left(\frac{\pi}{2}, \frac{(1+\pi)}{5}\right)$. The velocity field is assumed

$$\begin{cases} u = \sin(x)\cos(y) \end{cases}$$
(32)

 $\begin{cases} u = \sin(x)\cos(y) \\ v = -\cos(x)\sin(y) \end{cases}$ (32)

Simulations are performed using unstructured triangular meshes included 26142 nodes and 51682 cells. The circle is strained for N time units in forward step, then the velocity is reverses and the circle returned to its original configuration by the backward of N time units. Similar with advection of hollow square in oblique flow, the contours of the volume fraction for N=8 are depicted in Fig. 11-Fig. 13with different Courant numbers, which are over the range from 0.05 to 0.95 in interval of 0.1.



Figure 11. Contour plots for advection of a circle in a shear flow of the forward and backward using CICSAM scheme



and backward using CUIBS scheme

By comparing the contours obtained over unstructured meshes, the results show that CICSAM scheme has evidently dependence on C_f and the predicted interface become evidently diffusive with increasing Courant number. Table 2 presents the relative error variation of the three schemes with Courant number. It should be noted that FBICS and CUIBS scheme lead to more accurate numerical predictions at medium and high C_f , while has slight more numerical diffusion at low C_f .

Table 2. Relative error of a circle in a shear flow with different Courant numbers

Scheme	Low	Medium	High
CICSAM	0.02613	0.04609	0. 43241
FBICS	0. 03009	0.02533	0.02727
CUIBS	0.03067	0.02549	0.02889

6. Conclusions

Three present interface capturing schemes are implemented and compared in this study. The VOF model is solved by gradient smoothing method without explicitly interface reconstructing. On unstructured meshes, the variables on upwind points are calculated by cGSM model for improving the numerical accuracy. For comparing the ability of the three advection schemes for interface capturing, two benchmark tests are used at different Courant numbers. The results indicate that accuracy of CICSAM scheme is depended on Courant number and has serious numerical diffusion at high Courant number. While FBICS and CUIBS schemes can produce accurate numerical predictions even at high Courant number. Thus the two schemes will be alternative in application to free surface problems using GSM in future.

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