Analysis of hydraulic massive concrete structures using stochastic finite element method

†Jing Cheng[1], Peicong Li[1], Fanxuan Meng[1], Jinpeng Wei[1]

[1]College of Water Conservancy and Hydropower Engineering, Hohai University, Nanjing, P. R. China.
†Corresponding author: mscj042@hotmail.com

Abstract

Analysis tools available for massive concrete technology are still limited to deterministic models, which can't be used for the various uncertainty quantification of the structural performance. In this paper the spectral stochastic finite element (SSFEM) analysis theory of the mass concrete structures is developed. First the random field is discretized with the K-L expansion and the Galerkin approach; together with the polynomial chaos expansion (PCE) for output stochastic field, the SSFEM framework is then formulated for mechanic problems. Special items such as the reliability concerning displacement are also addressed. Practical gravity dam problem is provided as an illustration and verification.

Keywords: Spectral stochastic finite element method; Karhunen-Loève expansion; Polynomial chaos; Massive concrete structure; Reliability

1. Random field discretization with K-L expansion

A scalar random material property ($H(\boldsymbol{x}, \boldsymbol{\theta})$, $\boldsymbol{x} \in \Omega$, $\boldsymbol{\theta} \in S$) with mean value $\mu(\boldsymbol{x})$ and autocovariance function $C(\boldsymbol{x}_1, \boldsymbol{x}_2)$ can be approximated with the following series,

$$H(\boldsymbol{x},\theta) \approx \hat{H}(\boldsymbol{x},\theta) = \mu(\boldsymbol{x}) + \sum_{l=1}^{M} \sqrt{\lambda_l}\,\xi_l(\theta)\varphi_l(\boldsymbol{x}) = \mu(\boldsymbol{x}) + \sum_{l=1}^{M} H_l(\boldsymbol{x})\,\xi_l(\theta) \tag{1}$$

2 Stochastic field expansion with polynomial chaos

The response $U(\theta)$ can be represented as a series of polynomials in the standard normal variables $\{\xi_i(\theta), i=1\ldots M\}$. With the orthogonal PCE (polynomial chaos expansion), it can be written as,

$$U(\xi) \approx \sum_{m=0}^{P\text{-}1} U_m \psi_m(\xi) \tag{2}$$

3. Structural reliability analysis based on SSFEM

In this section the calculation of structural reliability P_r is addressed. Taking the following structural function as an example,

$$Z = U_t - U_r \tag{3}$$

where U_t , U_r are the tolerance value and simulated value for the displacement of a designed spatial location, respectively. Substituting Eq.(2) into Eq.(3) results in,

$$Z \approx Z_\xi = g_\xi(\xi) = U_t - \sum_{m=0}^{P\text{-}1} U_m \psi_m(\xi) \tag{4}$$

4. Application

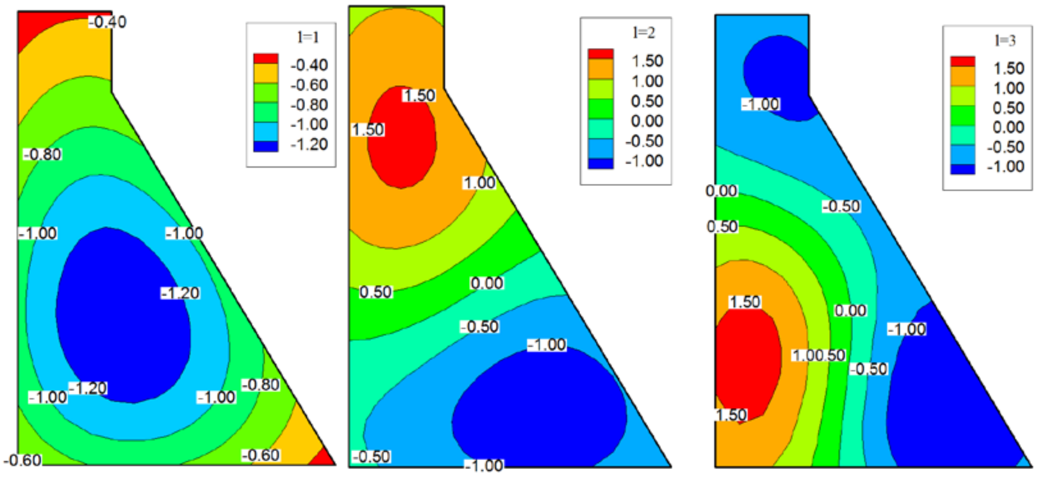

Fig. 1. Selected eigenvectors φ_l'($\mathbf{x}$) for the corresponding Gaussian random field for 2D dam (l=1, 2, 3).

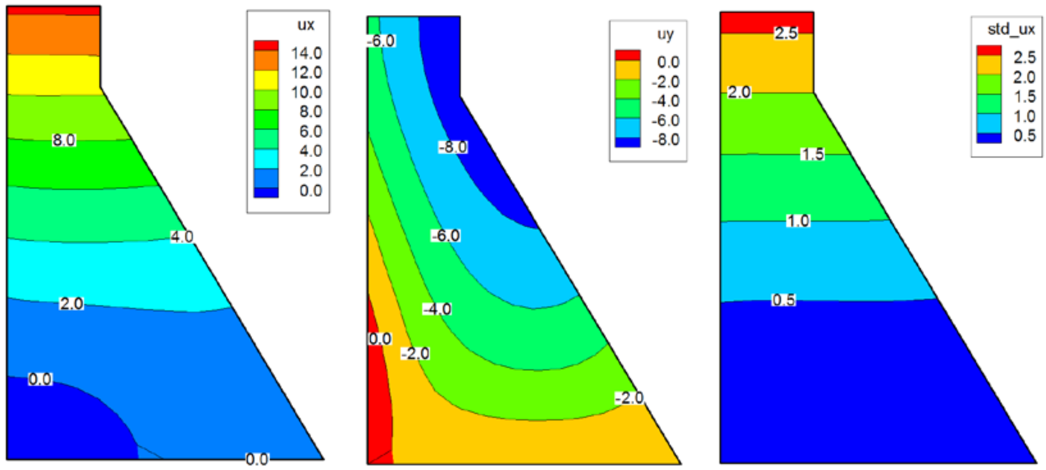

Fig. 2. Simulated mean value and standard deviation of displacements in x & y direction.(mm)

References

[1] G. Stefanou. The stochastic finite element method: Past, present and future [J]. Computer Methods in Applied Mechanics and Engineering, 2009, **198**(9-12): 1031-1051.

[2] W. Betz, I. Papaioannou and D. Straub. Numerical methods for the discretization of random fields by means of the Karhunen-Loeve expansion [J]. Computer Methods in Applied Mechanics and Engineering, 2014, **271**: 109-129.

[3] K.K. Phoon, S.P. Huang and S.T. Quek. Implementation of Karhunen-Loeve expansion for simulation using a wavelet-Galerkin scheme [J]. Probabilistic Engineering Mechanics, 2002, **17**(3): 293-303.

[4] N.D. Lagaros, G. Stefanou and M. Papadrakakis. An enhanced hybrid method for the simulation of highly skewed non-Gaussian stochastic fields [J]. Computer Methods in Applied Mechanics and Engineering, 2005, **194**(45-47): 4824-4844.

[5] R. Ghanem. The nonlinear Gaussian spectrum of log-normal stochastic processes and variables [J]. Journal of Applied Mechanics-Transactions of the ASME, 1999, **66**(4): 964-973.