Genetic Algorithms In Electoral Districting

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Abstract

In this paper, we present a mechanism that uses the concepts of genetic algorithms for electoral districting. Principles of genetic algorithms are adopted in finding the better districting solutions. The geometric information as well as the attribute data provided through the geographic information systems are used in selection, crossover, or mutation emulations. We also design various quantitative measurements to evaluation the performance (fitness) of our mechanism. We have applied this mechanism to CEC's result of Taipei City and the results show that our mechanism works.

Keywords: electoral districting, genetic algorithm, geographic information system

1. Introduction

Electoral districting is one of the traditional problems in political science for centuries. It may influence the election results directly. Electoral redistricting is normally required due to electoral regulation changes or due to new census results.

One of the crucial problems in electoral districting is to find a fair districting result efficiently. Traditionally, electoral districting was done manually and the process was time consuming and required huge amount of resources. Most important, manual districting normally introduce controversies among political parties.

In this paper, we present a mechanism that can be used to improve an existing electoral districting result. One can choose the electoral districting result announced by the Central Election Commission or developed in our early results. And then apply the principles of genetic algorithm to search for a new and better electoral districting result. In the searching processes, the concept of selection, crossover, and mutation operators are emulated for finding the better solutions. The geometric information as well as the attribute data provided through the geographic information systems are used in these operators. The fitness function or the performance evaluation function can also be defined using these data.

In order to obtain a fair electoral districting result, the election outcome is not considered in developing the entire mechanism. Only the general districting principles are included when making the decisions. However, one can evaluate the preference of different electoral districting results by the election outcomes afterward and chose a favorite one. The following three general principles are considered in our mechanism: (1) population equality: the populations in every district are close enough; (2) region contiguity: the units in the same district must be connected; and (3) shape compactness: the shape of each district shall be good.

2. Related Works

One of the most critical issues in solving the electoral districting problem is to find a fair method in order to avoid the Gerrymandering phenomenon [1] and satisfy the preset requirements.

Figure 1. Gerrymandering phenomenon

Electoral districting is one of the most important problems in political sciences. Due to a recent change of local legislative election regulations [2], many researches have been reported recently. Hsieh [9] and Pan [12] discussed the mechanism and standards. Yu [13] presented more complete discussions on the regulation changes as well as the underline legal issues. Others gave simulated studies in predicting the election outcomes [10]. Rectangle method by Harris [4] and linear programming method by Hess [6], modified by Helbig [5] are the early districting methods that we can trace. Kaiser's [11] model in evaluating population equality probably is among the earliest papers used computers to simulate electoral districting. Others may deal with finding a solution from a particular initial configuration but did not explore too many choices.

We have proposed various mechanisms in solving the electoral districting problems in the last decade [7][8]. The first method we proposed is the bricklayer method that uses only vertical and horizontal lines in the districting processes. Then we developed the two-partitioning method that uses the knowledge of computational geometry as well as the concepts adopted from dynamic programming. A weighted Voronoi diagram mechanism is also proposed in finding an initial districting result that reduced the post-processing time. However, there are two major issues have to be answered in solving the electoral districting problems. The first one is getting a solution that satisfies all the preset requirements. The second one is can we improve a given districting result without violating the preset requirements.

3. Methods

In this paper, we use the principles of evolutionary algorithms in finding better solutions for a given districting result. Evolutionary algorithms adopt the concepts of keeping the better members among the new generations. This can normally be described in the following diagram.
In general, there are three operators can be used in the evolutionary process, namely, reproduction, crossover, and mutation. Evolving through these operators may generate huge amount of members in the next generations. Only the better or the best members will be survived through the life competitions. One normally mimic these competitions using a

selection mechanism in cooperate with some evaluation functions. The members receive the higher scores will be selected and kept.

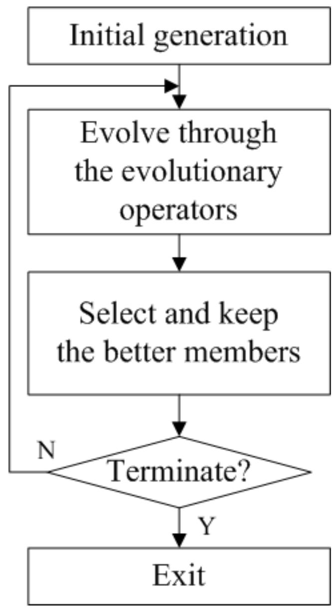

Figure 2. Concept of the evolution flow.

However, in some applications, producing huge amount of next generations and select the better members may not be practical due to the limited computation resources. One can search for the better members among the possible new generations before the actual evolving process take places. This can prevent unnecessary producing the dummy generations or members that will not be kept in any future references. We use a modified flow chart in our studies under this assumption. This is the actual evolutionary flow diagram as depicted in Figure. 2. In this diagram, the selection is implemented using heuristic searches so that the algorithm always selects the better results to be evolved. Researchers may argue that whether one shall check for better solutions or not before the actual evolving process. Nevertheless, there are two aspects must be considered in designing a system evolving through genetic algorithms, namely, how to evolve into the next generation and how to evaluate the result is better or not. The first aspect is generally accomplished through the combination of selection and actual evolving. The second aspect is generally practiced through the comparison of the fitness functions. We will discuss these issues in the following sections.

3.1 The Selection Considerations

Selecting the target for evolving may or may not include the evaluation issues before and after the evolutions. In the contrast, the evaluation process generally does not consider the selection issues in the evaluating mechanisms. If the evaluation issues are not included in the selection considerations, random search is generally used in most of the similar applications. However, random search suffers slow convergent speed and is not suitable in our studies. We shall integrate heuristic search into the evolutionary processes in our studies.

Recall that genetic algorithms normally use three operations, namely, reproduction, crossover, and mutation, for evolving into the next generation. We use the general term “selection” to represent these operations in electoral districting since they all involve selecting a target village to be evolved. In the evolving process of electoral districting, one can simply select a target village, from another district, to be merged into an existing district or can also consider find a village, in another district, to be exchanged with a village currently in an existing district. We shall refer the first strategy as “simple selection” and the second strategy as “trading”.

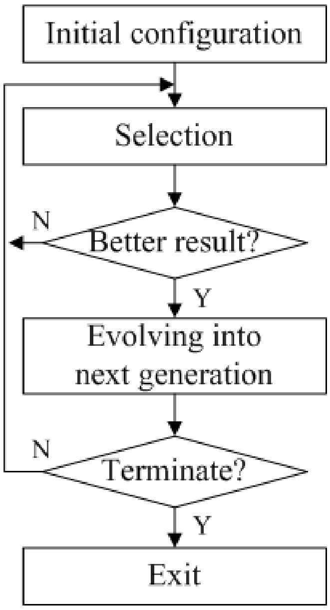

Figure 3. Evolution flow diagram.

Note that a simple selection is an operation of selecting a village v from a district D_j and merging it into another district D_i. If we use superscripts to indicate the time stamps and subscripts to indicate the districts, then we can write the districts after the selection operation, D^{t+1}, in terms of the districts before the selection operation, D^t, as follows:

$$\begin{aligned} D_i^{t+1} &= D_i^t + \{v\} \\ D_j^{t+1} &= D_j^t - \{v\} \end{aligned} \tag{1}$$

where the operator “+” and “-“ indicate “union” and “subtraction” of the set operations, respectively. Similarly, the trading operation can be written as

$$\begin{aligned} D_i^{t+1} &= D_i^t - \{v_i\} + \{v_j\} \\ D_j^{t+1} &= D_j^t + \{v_i\} - \{v_j\} \end{aligned} \tag{2}$$

where $v_i \in D_i^t$ and $v_j \in D_j^t$.

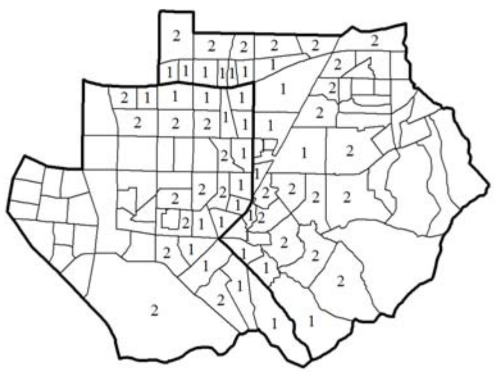

Figure 4. Districting diagram

Either simple selection or trading, the principle of population equality and region contiguity must be satisfied. In our system, we do want to select a better target for evolving into the next generation. Hence, both strategies were considered.

Refer to Figure 4, the districting diagram. There are two adjacent districts in this figure. The villages on the boarder are labelled “1” and the villages next to them are labelled “2”. (These labels indicate that the village is either 1 village away from the boarder or 2 villages away from the boarder, etc.) In our studies, crossover is considered as a trading between the “1-villages” that belong to different districts and mutation is considered as a trading when a district wants a “2-village” from another district and the associated connecting “1-village(s)” must be traded together with this “2-village”.

Algorithms for selection

A naïve algorithm can use only the distance function to select the target to be evolved. However, we are searching for a better target for evolving in the sense of “easy” producing a good next generation. Hence, in addition to the distance function, we also consider the population issue and geometry issues when searching for a better village in the selection processes. The following function, f_s, is used in selecting a target village v and add it to a particular district D:

$$f_s = \alpha\, f_d(D,v) + \beta\, f_P(D,v) \qquad (3)$$

where $f_d(D,v)$ is the distance evaluation function of a village v and a district D and $f_P(D,v)$ is the population evaluation function, after v is added to D. Note that α and β are the weighting coefficients satisfying $\alpha + \beta = 1$.

3.2 The Evaluation Principles

Three evaluation functions are considered in our studies. They reflect the issued related to distance, population, as well as geometry aspects and shall be discussed in this section.

Distance evaluation function

In the simple selection or trading operation considerations, as the distance between D and v increases, v is less likely to be selected. Thus $f_d(D,v)$ is inverse proportion to the distance, $d(D,v)$, between them.

Population evaluation function

For a given district, D_i, the population evaluation function of it, $f_P(D_i)$, reflects the population deviation of D_i to the average district population, P_a. We can write the related evaluation function as

$$f_P(D_i) = \frac{|P_a - P(D_i)|}{P_a} \quad \text{or} \quad f_P(D_i) = |1 - P(D_i)/P_a|. \qquad (4)$$

And the total population evaluation function, $f_P(D)$, of a particular district method for the given county, D, can be expressed as the sum of the individual functions. Hence, we have

$$f_P(D) = \sum_i f_P(D_i). \qquad (5)$$

4. Results

We use Taipei City as an example in illustrating our mechanism. Taipei city has 12 counties consists of 488 villages that will be districting into eight districts. The population of Taipei city is around 2.6 million, hence, the average population of each district is 0.32 million. For simplicity, we choose the initial configuration to be the following diagram, Figure 5, where nine counties are districted into eight district and 3 counties remain undissolved.

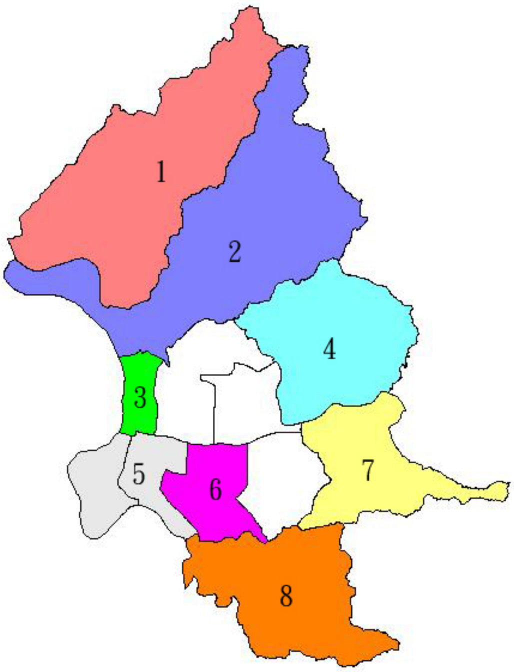

Figure 5. Taipei city, initial districting configuration

After applying our method, one can see that, for instance, some villages in district 4 have been "immigrated" to district 2, whereas some villages in district 2 have been immigrated to district 1. Figure 6 shows the districted results and the populations are listed in Table 1. As indicated in Table 1, one can see that the population difference is all within the preset requirement, 15%.

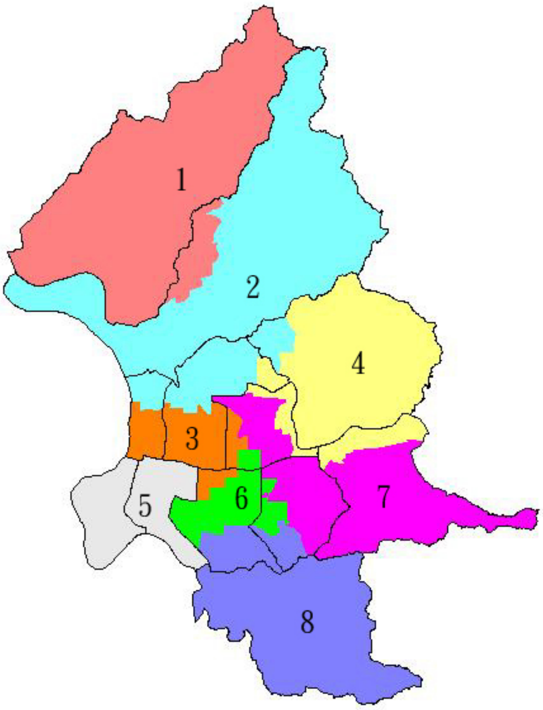

Figure 6. Taipei city, final districting configuration

Table 1. Taipei city, populations for the final districting configuration

District ID	Including counties	District population	Population error (%)
1	Baitou, Shilin	326,240	-0.7
2	Shilin, Zhongshan, Datong, Neihu	324,135	-1.3
3	Datong, Zhongshan, Daan, Songshan	326,004	-0.7
4	Neihu, Nangang, Zhongshan, Songshan	323,369	-1.5
5	Zhongchen, Wanhua	359,865	9.6
6	Daan, Xinyi, Songshan	323,538	-1.5
7	Nangang, Xinyi, Songshan	322,139	-1.9
8	Wenshan, Daan, Xinyi	321,848	-2.0

Conclusion

We have proposed a genetic algorithm that uses the concept of evolution for solving the electoral districting problem. The mechanism has been applied to district the Taipei City with successful results.

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