Mode shapes complexity for damage identification of structures experiencing plasticization

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Abstract

Classically, for structures prone to earthquakes, the damage identification techniques rely on changes of modal parameters between different structural states. Recently, the effects of energy dissipation have been considered as possible alternatives to modal parameters for damage identification. In particular, it is assumed that energy dissipation causes an increase of damping non-proportionality and, consequently, an increase of complexity in the mode shapes. A number of indices have been proposed to measure the damping non-proportionality or complexity in the mode shapes. To be successful in damage detection such indices should possess at least two characteristics: monotony and sensitivity. The work aims to investigate the effectiveness of the indices for structural damage identification. To this end, numerical simulations concerning a plane frame structural model are carried out. The damage is such to produce plastic hinges at selected joints of the model where the energy dissipation is concentrated. Seismic type base motion of progressive increasing intensity is considered to show the relation between damage severity, energy dissipation, damping non-proportionality and mode shapes complexity. Signal processing time domain techniques, EMD and CPR, are applied to the structural dynamic response in order to identify the indices based on the complex mode shapes. The indices are then applied to detect the damage.

Keywords: **Damage identification; Non-proportional damping; Mode shapes complexity; Damage indices.**

Introduction

The recent and numerous seismic events that worldwide hit the existing buildings have made very timely and important the development of new techniques for the structural damage identification. In the seismic field, the analysis of the dynamic behavior of a structure can be used to identify the damage occurred. When the damaged structure preserves a quasi-linear behavior, it can be retained that its modal parameters (frequencies, mode shapes and damping) are function of the physical properties (mass, stiffness and energy dissipation). Changes in these latter properties, caused by damage, are reflected, therefore, in modal parameters changes. The comparison of the values attained by the frequencies, mode shapes and damping, or their appropriate function, between different structural states is an effective means to estimate the damage. There exist a number of techniques based on changes in modal parameters and targeted to damage assessment [1]. In general, the frequencies are sensitive to the damage but hardly allow its localization [2]. On the contrary, the modal shapes are identifiable with greater difficulty but are extremely effective to localize the damage [3]. The damping presents the same characteristics of the frequencies but it is more difficult to identify exactly [4][5][6].

Recently, the damping has been used in conjunction with the effects induced on the mode shapes in energy dissipating structures [7][8]. The basic hypothesis is to associate the damage to the energy dissipated during the structural vibrations and to measure the effects generated on the mode shapes in terms of modal complexity. In particular, it is assumed that: the greater the damage is, the greater the energy dissipation is and consequently, the more damped the dynamic response is, the higher the loss of damping proportionality is and so the more complex the mode shapes are. The measure of this non-proportionality (or complexity) is used to estimate the structural damage. A number of indices have been proposed and analyzed to provide an effective measure of non-proportionality [9]. A group of indices requires the knowledge of the damping matrix of the structure; whereas another group of indices requires the knowledge of the mode shapes. In consideration of the difficulty of the experimental identification of the damping matrix, the first group has only a theoretical value. The practical applicability is therefore restricted to the second group of indices. However, to get successful indices, two properties should be fulfilled: monotony and sensitivity both related to uniqueness aspects. In this contingency, it is possible to perform a reliable identification based on the comparison of the values attained by the indices between two different states of a structure.

The work aims to examine the effectiveness of the indices based on the mode shapes for damage identification purposes. To this end, numerical simulations are carried out. The pseudo-experimental data (i.e. the dynamic responses) are generated using a plane frame model endowed with plastic hinges localized in selected joints of the structural model where the energy dissipation, i.e. the damage, is concentrated. The damage is caused by seismic type base motion. The intensity of the base motion is progressively raised in order to increase the damage severity. Each intensity level produce a different state of the structural model. The structural response at each state is processed to identify the mode shapes according to the assumption of quasi-linear behavior of the structure. Then the indices (of the second group) are computed and compared between the different states. In this way it is possible to show the relation between damage severity, hysteretic energy dissipation, damping non-proportionality and mode shapes complexity.

Plasticity and modal complexity

The discrete form of the motion of a structure with linear behavior endowed with viscous damping has equation:

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)=\mathbf{P}(t) \tag{1}$$

in which t is the time variable; $\mathbf{x}$, $\dot{\mathbf{x}}$ and $\ddot{\mathbf{x}}$ are, respectively, the displacement, velocity ad acceleration vectors; in turn **M**, **C** and **K** are the mass, damping and stiffness matrices; **P** is the load vector.

Even though the matrices **M** and **K** are diagonalizable, **C** can be diagonalizable or not depending whether the damping is proportional or non-proportional (i.e. **C** is a combination or not of **M** and **K**). In the first case, the mode shapes are real and have components with equal phase; in the second case, the mode shapes are complex and have the components with different phase [10]. Further, the more **C** is non-proportional, the more the imaginary part of the mode shapes is high, that is to say the complexity of the mode shapes increases along with the increase of the **C** non-proportionality [9][11].

In framed structures, the damage is often confined in the beam-column joints where local plasticity occurs accompanied by hysteretic type energy dissipation. In these conditions, the discrete form of the equation of motion Eq. (1) takes the form:

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{H}[\mathbf{x}(t)]=\mathbf{P}(t) \tag{2}$$

where $\mathbf{H}(\mathbf{x})=\mathrm{d}\mathbf{F}(\mathbf{x})/\mathrm{d}(\mathbf{x})$ is the matrix of the instantaneous non-linear stiffness that depends on the reaction force $\mathbf{F}(\mathbf{x})$. The increase of plasticity (i.e. of damage) implies the stiffness reduction and the energy dissipation increase. These two effects are taken simultaneously into account in Eq. (2) through $\mathbf{H}(\mathbf{x})$ that represents their combined effect. Both effects contribute individually to make non-proportional the damping and, hence, to make complex the mode shapes. Actually, the dominant contribution is that due to the energy dissipation [7], therefore the contribution due to the stiffness is not considered in this work.

Modal complexity indices for structural damage identification

Among the indices based on the knowledge of the mode shapes, collected in [9], and for coherence with those studied in [7][8][11], in the present work five indices are considered: modal imaginary ratio (I_1), modal collinearity (I_2), modal dispersity (I_3), modal phase difference (I_4) and modal polygon area (I_5). Their formulation is given below:

$$I_1=\frac{\left\|\mathrm{Im}(\boldsymbol{\Psi}_k)\right\|}{\left\|\boldsymbol{\Psi}_k\right\|};\quad I_2=1-\frac{\left|\mathrm{Re}(\boldsymbol{\Psi}_k)^T\,\mathrm{Im}(\boldsymbol{\Psi}_k)\right|}{\sqrt{\left(\mathrm{Re}(\boldsymbol{\Psi}_k)^T\,\mathrm{Re}(\boldsymbol{\Psi}_k)\right)\left(\mathrm{Im}(\boldsymbol{\Psi}_k)^T\,\mathrm{Im}(\boldsymbol{\Psi}_k)\right)}};\quad I_3=\frac{\sum_{j=1}^{N}\left|\mathrm{Im}(\Psi_{kj})\right|}{N};$$

$$I_4=\frac{\left|\phi_{k,max}\right|-\left|\phi_{k,min}\right|}{\pi};\quad I_5=\frac{A_k}{A_{k,max}}; \tag{3}$$

in which N is the number of the degrees of freedom (dofs) of the structures, $\boldsymbol{\Psi}_k$ is the k-th mode shape, Ψ_{kj} is the j-th component of $\boldsymbol{\Psi}_k$; Re(·) and Im(·) stand for the real and imaginary part of the quantity in (·); $\phi_{k,max}$ and $\phi_{k,min}$ are, respectively, the maximum and minimum phase angle of $\boldsymbol{\Psi}_k$; A_k and $A_{k,max}$ are respectively the modal polygon area of $\boldsymbol{\Psi}_k$ and its maximum value; || · || is the Euclidean 2-norm operator and | · | the componentwise absolute value. The meaning of the indices is as follows: I_1 weighs the importance of the imaginary part with respect to the overall length of the complex mode shape. I_2 measures the degree of interdependence of the real and imaginary parts of a complex mode shape and is directly affected by the damping proportionality: the higher the damping proportionality is, the more correlated the real and imaginary parts of the mode shapes are; if the imaginary part of a complex mode shape is completely dependent on the real parts, the damping is proportional. I_3 measures the degree of the scatter of the complex mode shape that is directly related to the amplitude of the imaginary part. The idea behind the I_4 and I_5 relies on a geometric interpretation. If the components of the mode shape are plotted in the complex plane, the effects of the non-proportional damping become apparent. I_4 considers the phase differences between the dofs of a mode shape as a consequence of damping non-proportionality. In fact, each component of a mode shape of a system endowed with proportional damping lies on a straight line, whereas those of a system endowed with non-proportional damping do not; in effect, these latter exhibit an angular dispersion equivalent to the phase differences. If the individual components of a mode shape are connected by straight lines, an N-side polygon is formed. If the damping is proportional, the components of a mode shape lie on a straight line and the polygon area is zero. As the non-proportionality of the damping increases, the area of this polygon also increases. I_5 measures the area of this polygon. In practical terms, once the mode shapes are identified they are first normalized using the procedure proposed in [12] that ensures the minimization of the errors in the identification of the imaginary part of the mode shape [8]. Subsequently the normalized mode shapes are used to compute the indices of Eq. (3). Finally, the indices, scaled in the interval [0; 1] and expressed in percentage, are analyzed to infer the damage presence.

Case study and methodology

The effectiveness of the five modal complexity indices of Eq. (3) to identify the damage in structures experiencing plasticization is analyzed using as pseudo-experimental data generator a basic model of framed structure with the scheme of "strong beam – weak column".

The reference structure is a plane frame with a single span and three levels. The inter-storey height is 3 m and a mass of 10 t is condensed at each level. The columns have a constant cross-section 0,30 by 0,30 m and a Young modulus equal to $3 \cdot 10^7$ kN/m^2, so the inter-storey stiffness is $1{,}8 \cdot 10^4$ kN/m. The damage is simulated by means of plastic hinges that dissipate energy for hysteresis and are localized at the columns base (Fig. 1).

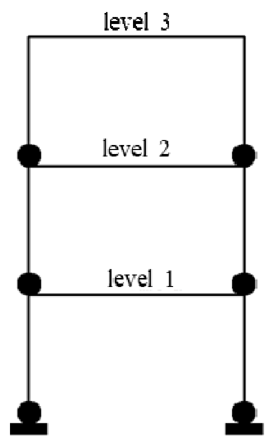


Figure 1. Reference framed structure with simulated damage using plastic hinges localized at the columns base

The excitation of the structure is a seismic type base motion. Seismic input in resonance conditions (ideal) and not (actual) are considered. The "resonance conditions" case corresponds to mono-harmonic input with the excitation frequency equal to any of the structural frequencies. This case allows to obtain output signals typical of the experimental modal analysis in real structures. The "actual conditions" case corresponds to a real earthquake input. The energy dissipation entity of the plastic hinges (i.e. of the damage) is controlled by a damage parameter function of the excitation amplitude: the higher the excitation amplitude is, the higher the value of the damage parameter is.

In order to obtain mono-harmonic output signals of the structural dynamic response, the excitation is endowed with the same natural frequency of the structure to excite, according to the mode shape to identify. The damage parameter ranges from 0 (no damage) to 1 (maximum damage). The pseudo-experimental mode shapes can be derived in principle by any identification technique. In the present context, the Complex Plane Representation (CPR) method [13] is used. Briefly speaking, the CPR method is an output-only time domain technique in which the original signal is mapped in the complex plane by computing its imaginary counterpart via the Hilbert transform [14]. This new representation makes it very simple to identify the phase shift of the motion between the different measurement points and, therefore, it is particularly effective for the identification of complex mode shapes of general viscously damped systems.

Multi-harmonic output signals of the structural dynamic response are obtained by exciting the framed structure with the Northridge earthquake, Arleta and Nordhoff Fire Station, 1994 (Fig. 2). The seismic excitation is gradually increased to increase the damage. In particular, the input intensity is progressively increased by scaling the earthquake profile to the following PGA levels: 0,1g; 0,3g, 0,5g; 0,7g; 0,9g. The elastic response spectrum of pseudo-acceleration (5% damping) is shown in Fig. 3, where the portion of interest of the spectrum is highlighted. In particular, it is important to note that this portion allows to verify the effectiveness of the modal complexity indices. In fact, as damage progresses, the first mode shape of the structure increases its natural period from 0,3 s to 0,5 s and simultaneously reduces, about 25%, the maximum pseudo-acceleration acting on the structure. As an example, the time-histories of the dynamic response in acceleration at the three levels of the structure in Fig. 1 are given below in Fig. 4 and in Fig. 5, respectively, for the minimum and maximum PGA values (0,1g and 0,9g).

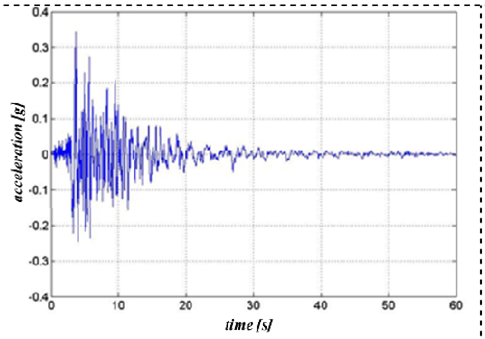


Figure 2. Northridge earthquake, comp. 90°, 1994.

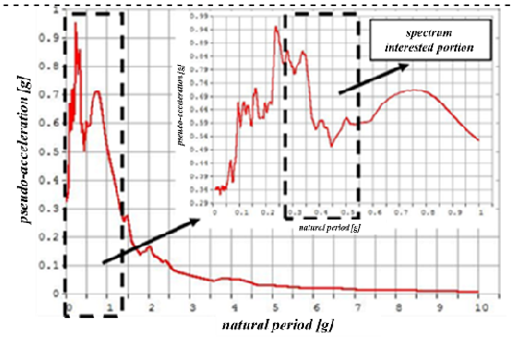


Figure 3. Elastic response spectrum - Northridge earthquake, comp. 90°, 1994.

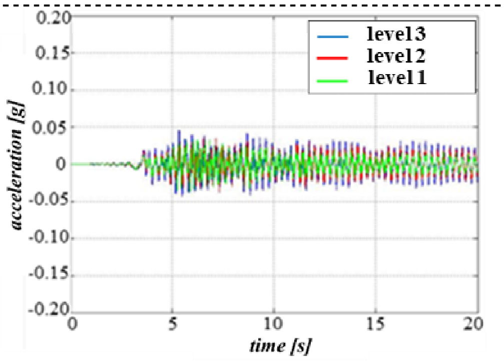


Figure 4. Dynamic response. PGA = 0,1g

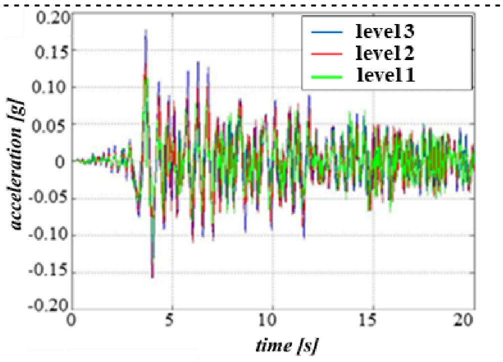


Figure 5. Dynamic response. PGA = 0,9g

As shown in Fig. 4 and Fig. 5, the structural response is composed by multi-harmonic signals. Therefore, in order to use the CPR method, to identify the pseudo-experimental mode shapes, it is necessary to decompose the signals in their individual harmonic components (each referred to a particular mode shape). Subsequently, the individual harmonics related to the same mode shape are considered to form a structural response composed by quasi-mono-harmonic signals in each dof of the structure, in such a way to recover the "resonance conditions" and hence the applicability of the CPR method. In this work, the decomposition procedure for obtaining the individual harmonic components of the response signals is carried out using the time domain Empirical Mode Decomposition (EMD) [15]. The EMD method is applied in the time interval [5,5 s; 7,5 s] where the response signals are closer to the stationary (i.e. resonance) conditions necessary to apply the CPR method, see Fig. 4 and Fig. 5.

The results obtained after to apply the EMD method are given from Fig. 6 to Fig. 11 and referred to the minimum (PGA = 0,1g) and maximum (PGA = 0,9g) earthquake intensity. More in detail, the Fig. 6-7, Fig. 8-9 and Fig. 10-11 show, in the order, the results relative to extraction of the first, second and third quasi-harmonic component (i.e. with lower, middle and higher frequency associated to the first, second and third mode shape, respectively) of the multi-harmonic response signals of each frame level.

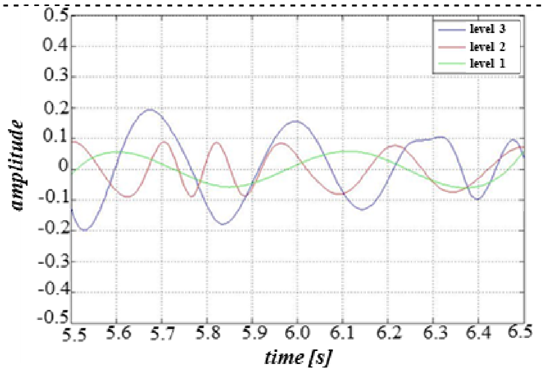


Figure 6. Quasi-harmonic component with lower frequency (first component) of the dynamic response. PGA = 0,1g

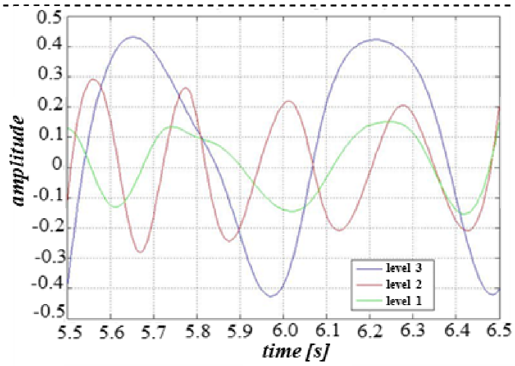


Figure 7. Quasi-harmonic component with lower frequency (first component) of the dynamic response. PGA = 0,9g

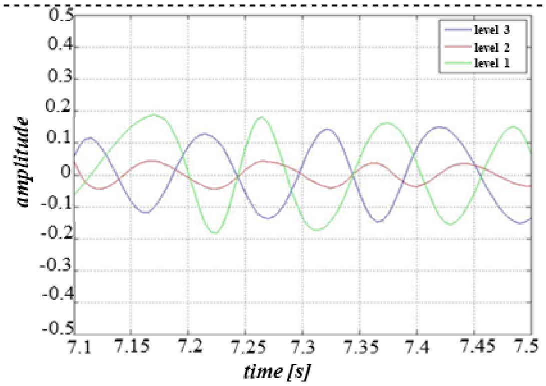


Figure 8. Quasi-harmonic component with middle frequency (second component) of the dynamic response. PGA = 0,1g

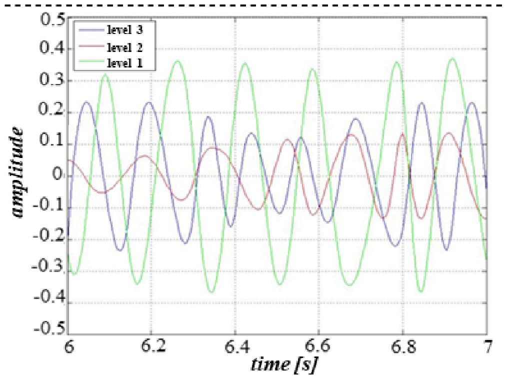
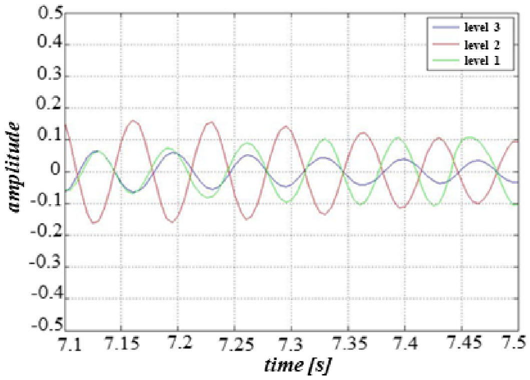


Figure 9. Quasi-harmonic component with middle frequency (second component) of the dynamic response. PGA = 0,9g



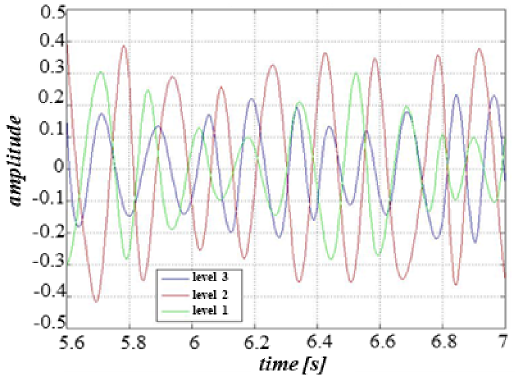
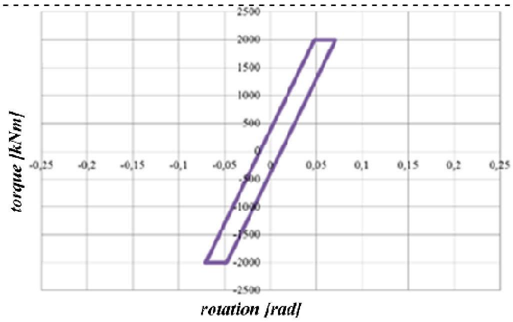


Figure 10. Quasi-harmonic component with higher frequency (third component) of the dynamic response. PGA = 0,1g

Figure 11. Quasi-harmonic component with higher frequency (third component) of the dynamic response. PGA = 0,9g

Results

As a matter of example, the results obtained in "resonance conditions" (ideal seismic excitation) are shown in the figures below for the two cases of low and high damage severity. In each case, the moment-rotation hysteretic cycle of the plastic hinge localized at the column base, Fig. 12 and Fig. 14, and the displacement time-histories at each floor level, Fig. 13 and Fig. 15, of the structure in Fig. 1, are reported.



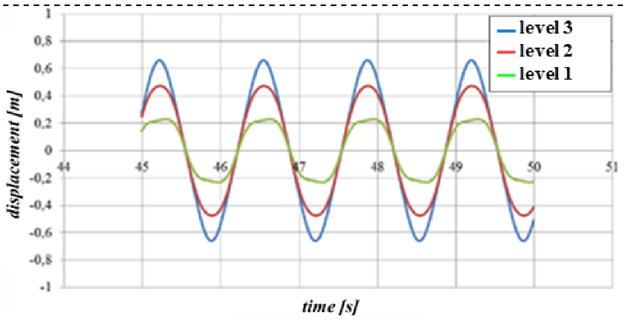
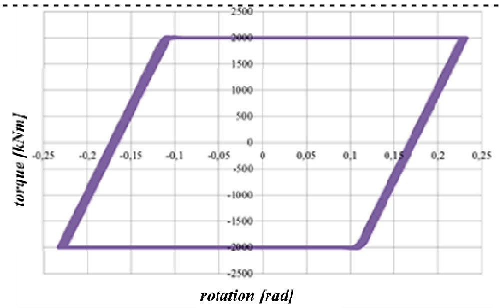


Figure 12. Moment-rotation diagram of the plastic hinge at the column base (low damage)

Figure 13. Dynamic response: displacement time history (low damage)



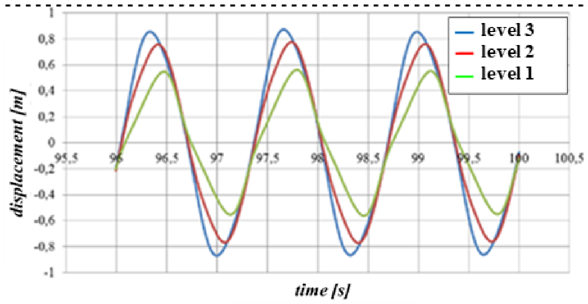


Figure 14. Moment-rotation diagram of the plastic hinge at the column base (high damage)

Figure 15. Dynamic response: displacement time history (high damage)

The analysis of the results highlights that as damage increases, both the area of the hysteresis cycle and the phase difference between the response signals of the three levels of the structure increase as well. Now, since the energy dissipation is a function of the area of the hysteresis cycle and the modal complexity is a function of the phase difference, both the energy dissipation and the modal complexity increase in turn along with the damage severity.

In the following figures, the variations of the five modal complexity indices versus the damage severity (damage parameter or PGA) are reported for the first mode shape. The cases of ideal, Fig. 16, and actual, Fig. 17, seismic input are compared.

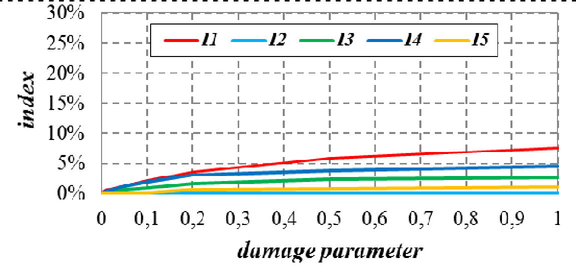


Figure 16. Indices vs. damage parameter. First mode shape

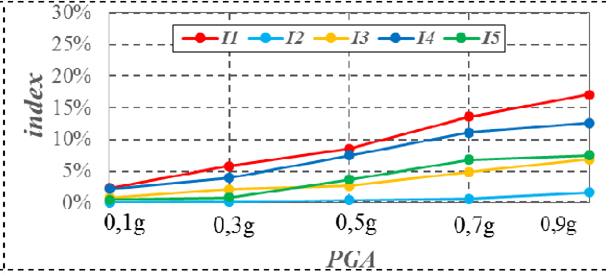


Figure 17. Indices vs. PGA. First mode shape

The analysis of Fig. 16 and Fig. 17 leads to conclude that the indices show always an increasing monotonous behavior with the damage severity. This property is fundamental when the damage identification is performed through the comparison between different states. As concerns the sensitivity, it is apparent that the indices show a significant difference among them; the more and less sensitive ones are respectively I_1 and I_2.

Similar results are obtained for the higher order modes as it is shown in Fig. 18 and Fig. 19 for the case of the Northridge seismic input. In particular, it is observed that the indices tend to increase more rapidly along with the order of the mode shape. This is a consequence of the major articulation presented by the higher order mode shapes. In fact, the higher gradient of inter-story displacement demands higher rotations of the plastic hinges and ultimately an increase in energy dissipation.

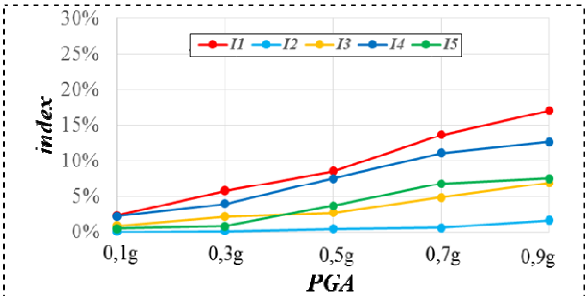


Figure 18. Indices vs. PGA. Second mode shape

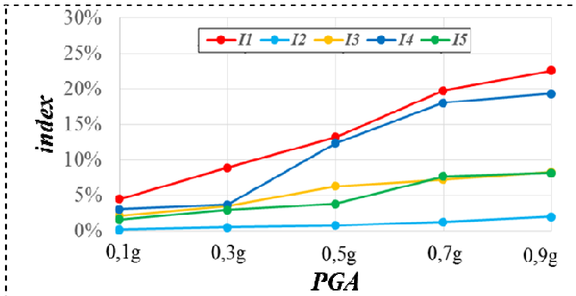


Figure 19. Indices vs. PGA. Third mode shape

However, it is observed from Fig. 17 to Fig. 19 that the indices estimated using the first mode shape are characterized by a sensitivity comparable with those of higher order mode shapes. Therefore, at least in the present case, the indices can be applied to identify the structural damage indifferently from the order of the mode shape selected. Finally, it is worth considering that the monotonic increase of the indices is preserved regardless the spectral form of the input. In fact, in the present case, a drop in the spectral ordinates is observed passing from 0,3 s to 0,5 s (period elongation due to damage), nonetheless the indices appear not to be affected by this reduction of the seismic intensity.

Conclusions

The work investigates the effectiveness of some indices based on the mode shape complexity to identify the damage in structures experiencing plasticization. The basic idea is the relation between the damage and the complexity of the mode shapes generated by the damping non-proportionality that occurs when a structure subjected to vibration plasticizes dissipating energy. The indices derive from the technical literature and transform the identified complex mode shapes into scalar quantities readily usable in one to one relationships with any measure of the input severity. Presently, the identification of mode shapes is carried out by the joint and sequential use of the CPR and EMD methods. Nonetheless, any available signal processing technique is in principle applicable.

A simple plane frame structure endowed with plastic hinges is used as a case study. The behavior of the indices with respect to the damage is studied using seismic type base motion of progressive increasing intensity. The severity of the input is measured by the PGA of the selected earthquake.

The results confirm the monotony and sensitivity of the indices respect to the structural damage. As consequence, it is possible to identify the damage presence in a very simple form by the comparison of the indices between the current state and a previous (reference) state. More in detail, the results show a higher sensitivity of the indices measured using the higher order modes. This different sensitivity is however modest and it can be concluded that any mode shape can be effectively used to identify the damage in structures. In conclusion, it can be stated that the complexity of the mode shapes is an effective tool for a reliable detection of the structural damage. However, the use of modal complexity indices for the assessment of the damage allows only its identification but not its localization. The effectiveness of these indices rely therefore on their capability of providing synthetic and reliable indications on the structural damage presence.

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