Dynamics and vibration stability analysis of a rotating composite beam under harmonic base excitation

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Abstract

This study considers the stability of vibration of a rotating structure consisting of a rigid hub and a flexible thin-walled laminated composite beam performing the additional to-and-fro motion. The partial differential equations of motion representing a complex elastic deformation of the blade including bending, shear and twisting effects have been derived by the Hamilton's least action principle. Next, these equations have been transformed to a dimensionless ordinary differential form by adopting the Galerkin method. It is shown the final equation of motion includes time-varying coefficients that depend on the system angular velocity as well as on the base excitation frequency. Due to the doubly periodic external excitation terms this form of the governing equation is different from the typical Mathieu-Hill's equation. Two numerical examples are presented to illustrate the influence of selected model parameters on the dynamic stability of the system.

Keywords: **rotating beam, base excitation, parametric vibrations, stability analysis**

1. Model of the structure

Let us consider a uniform slender and perfectly elastic thin-walled composite beam built into a rigid hub of radius R_0 as shown in Figure 1. The structure is rotating at a constant angular velocity $\dot{\psi}(t) = \Omega$ about a vertical axis Z_0. Moreover, the centroid of the hub C is performing harmonic to-and-fro motion in the rotor plane. Thus, the temporary position of the hub is given by the translational and angular coordinates $\xi(t)$ and $\psi(t)$, respectively. The beam is clamped to the hub so that the flapwise bending plane coincides with the rotation plane.

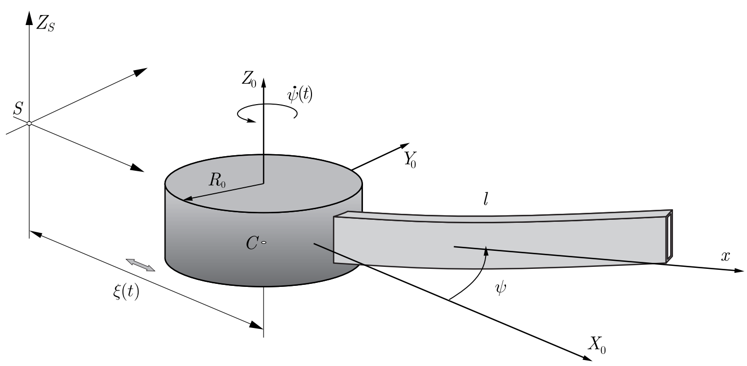

Figure 1: Schematic diagram of a rotating beam with in-plane base movement

1.1. *Governing equations*

The equations of motion of the beam are derived according to the Hamilton's principle of the least action

$$\delta J = \int_{t_1}^{t_2} \left(\delta T - \delta U\right) \mathrm{d}t = 0 \tag{1}$$

where J is the action, T is the kinetic energy, U is the potential energy and operator δ represents an infinitesimal change of the corresponding functions. Based on the previous authors research [1], and confining the analysis to the flapwise bending, shear and profile twisting, the partial differential equations of beam motion are as follows:

- $$\begin{aligned} b_1\ddot{w}_o - 2b_1\dot{u}_o\,\dot{\psi}(t) - b_1 w_o\,\dot{\psi}^2(t) - b_1(R_0+x+u_o)\,\ddot{\psi}(t) \\ + b_1\ddot{\xi}\sin\psi(t) - a_{55}\vartheta_y' - a_{55}w_o'' - (T_x w_o')' = 0 \end{aligned} \tag{2}$$

with boundary conditions $\quad w_o\big|_{x=0} = 0, \qquad \left(\vartheta_y + w_o'\right)\big|_{x=l} = 0;$

- $$B_4\ddot{\vartheta}_y - B_4\dot{\psi}^2(t)\vartheta_y + B_4\ddot{\psi}(t) + a_{55}(\vartheta_y + w_o') - a_{33}\vartheta_y'' - a_{37}\varphi'' = 0 \tag{3}$$

with boundary conditions $\quad \vartheta_y\big|_{x=0} = 0, \qquad \left(a_{33}\vartheta_y' + a_{37}\varphi'\right)\big|_{x=l} = 0;$

- $$(B_4+B_5)\ddot{\varphi} + (B_4-B_5)\dot{\psi}^2(t)\varphi - a_{37}\vartheta_y'' - a_{77}\varphi'' - (T_r\varphi')' = 0 \tag{4}$$

with boundary conditions $\quad \varphi\big|_{x=0} = 0, \qquad \left(a_{37}\vartheta_y' + a_{77}\varphi'\right)\big|_{x=l} = 0$

In the above relations B_i and b_i factors depict the inertia terms and a_{ij} ones correspond to the appropriate beam stiffnesses. The unknowns $w_0, \vartheta_y, \varphi$ are the transverse displacement, shear deformation and profile twist, respectively. The term $T_x(x)$ is defined as $b_1(L-x)\left\{\dot{\psi}^2(t)\left[R_0 + \frac{1}{2}(L+x)\right] - \ddot{\xi}\cos\psi(t)\right\}$ and it represents system stiffening/softening force resulting from both transportation motions, while $T_r(x) = {}^{(B_4+B_5)}/_{m_0\beta} \cdot T_x(x)$. Quantity m_0 is mass of the beam per its unit length and β is a perimeter of the profile cross-section.

The derived system of partial differential governing equations is transformed into ordinary differential ones taking into account the normal modes projection and the associated orthogonality condition. To this aim the Galerkin procedure for the first natural mode is applied. Next, the system is converted to the dimensionless notation, and the final form is obtained

$$\ddot{q} + \left(\alpha_1 + \alpha_3\Omega^2\right) q + \varepsilon\tilde{X}_0\nu^2\sin(\nu\tau)\cos(\Omega\tau)\alpha_p q = -\alpha_e\tilde{X}_0\nu^2\sin(\nu\tau)\sin(\Omega\tau) \tag{5}$$

where q is the generalized coordinate corresponding to the studied coupled flexural-torsional motion. Coefficients α_i ($i = 1\ldots,4$), α_e, α_p result from the Galerkin's projection. The terms $\tilde{X}_0$ and ν are the dimensionless amplitude and frequency of the hub translational motion, respectively.

When studying the final equation (5) one observes it contains doubly periodic excitation terms, namely the parametric and external one. Therefore, this form of the governing equation is different from the typical Mathieu-Hill's equation as often met in other engineering problems – e.g. column buckling under time periodic compression or pendulums systems.

2. Solution procedure and numerical results

The derived equation of motion (5) is solved to determine the instability boundaries of the system. To this aim the method of multiple scales [2] is adopted and the uniformly valid expansion is assumed as

$$q(\tau) = q_{(0)}(T_0,T_1,T_2) + \varepsilon q_{(1)}(T_0,T_1,T_2) + \varepsilon^2 q_{(2)}(T_0,T_1,T_2) + \mathscr{O}(\varepsilon^3) \tag{6}$$

Upon substituting this expansion into the governing equation (5) and equating coefficients of like powers of ε yields a set of three perturbation equations. Successive solution of these equations is followed by eliminating the troublesome secular and small divisor terms in higher order components $q_{(1)}, q_{(2)}$ that depend on the system resonant combinations of frequencies. As a final result the boundaries separating the stable and unstable regions on the frequency ν and amplitude εX_0 plane can be found.

The presented numerical examples involve two simulation tests. In the first experiment the impact of the rotating speed $\Omega = \dot{\psi}$ on the location and size of the unstable zone is examined. In the second one the influence of the fibre orientation angle in the beam laminate material is tested. The results are presented in Figures 2 and 3, respectively.

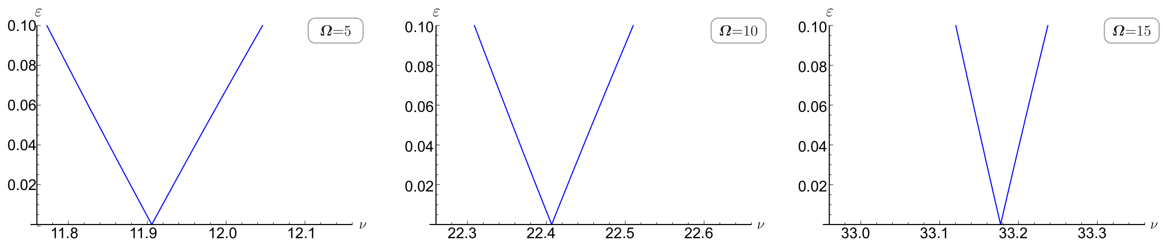

Figure 2: Primary unstable region of the tested beam – impact of the rotating speed Ω; Coefficients in (5)**:** $\alpha_1 = 3.2651, \alpha_3 = 0.352719, \alpha_p = 1.5777, \tilde{X}_0 = 0.1$

Studying Fig. 2 one observes the stabilising impact of the rotating speed Ω. This is confirmed by the reduced width of unstable zones for higher rotating speeds. At the same time these unstable regions undergo the right shift due to the centrifugal stiffening effect.

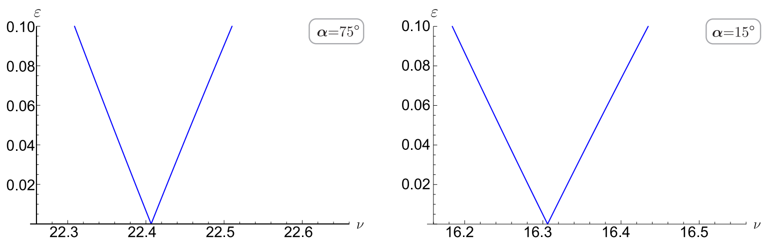

Figure 3: Primary unstable region of the tested beam – impact of the laminate reinforcing fibres orientation angle α; Case (a) $\alpha = 75^\circ$ **coefficients in** (5) $\alpha_1 = 12.3364, \alpha_3 = 0.34987, \alpha_p = 1.3217$**; Case (b)** $\alpha = 15^\circ$ **coefficients in** (5) $\alpha_1 = 3.2651, \alpha_3 = 0.352719, \alpha_p = 1.5777$**; Excitation amplitude** $X_0 = 0.1$

Results presented in Fig. 3 reveal discrepancies in the widths of unstable zones. The effect is explained by the different mutual modal bending/twisting components ratios that depend on the reinforcing fibres orientation angle α. This conclusion is further confirmed by the horizontal shift of the unstable zone as a consequence of significant difference in the specimen stiffness.

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References

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