Fast Statistical Homogenization Procedure (FSHP) for Particle Random Composite

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Abstract

Composites materials, used in many engineering applications or present in nature, exhibit a microstructure made of randomly distributed inclusions (particles) embedded into a dissimilar matrix. Examples of such materials are polymer, ceramic, metal matrix composites, but also granular materials, concrete, masonry made of crushed stones casually arranged in the mortar and even porous rocks.

A key aspect, recently investigated by many researchers, is the evaluation of appropriate mechanical properties to be adopted for the study of their behaviour. Homogenization procedures may be adopted for the definition of equivalent moduli able to take into account at the macroscale the material properties emerging from the internal microstructure [1].

Respect to the classic homogenization approach, in the case of materials with random microstructure it is not possible to 'a-priori' define a Representative Volume Element (RVE), this being an unknown of the problem. A possible way to solve this problem is to approach the RVE using finite--size scaling of intermediate control volume elements, named Statistical Volume Elements (SVEs), and proceed to homogenization [2]. Here homogenization, consistent with a generalized Hill-Mandel condition [3], is adopted in conjunction with a statistical procedure, by which scale-dependent bounds on classical moduli are obtained using Dirichlet and Neumann boundary conditions for solving boundary value problems (BVPs). The outlined procedure has provided significant results, also extended to non-classical continuum formulations [4], but with high computational cost which prevents the possibility to perform series of parametric analyses [4][5].

The Fast Statistical Homogenization Procedure (FSHP), here proposed automates all the steps to perform: from the simulations of each random realization of the microstructure to the solutions of the boundary value problems for the SVEs, up to the evaluation of the final size of the RVE for the homogenization of the random medium. Moreover, the adoption of an innovative computational method, such as the Virtual Element Method (VEM) [6], allow us to reduce the computational burden [6][7].

The VEM methodology has many computational advantages such as robust stiffness matrix (can be exactly computed in precision machine) and accuracy versus the number of degrees of freedom. For the numerical analysis we adopt a polygonal mesh for the matrix and a single VEM element for the inclusions.

The results obtained by adopting this integrated homogenization procedure with VEM are compared with the results previously obtained, by some of the authors, using a standard Finite Elements procedure taking into account two different types of inclusions, either stiffer or softer than the matrix. Several simulations are then performed by modifying the material contrast (ratio between the moduli of the materials components) deriving the size of the RVE for performing homogenization on various kinds of two--phases random composites.

Keywords: **Statistical Homogenization, Representative Volume Element, Composite Materials, Virtual Element Method**

Acknowledgment. This work is supported by Italian Ministry of University and Research (P.R.I.N. National Grant 2015, B86J16002300001; Sapienza Research Unit) and by Sapienza University Grant 2016, B82F16005920005

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