Weight is a potential motion. The analysis of vaulted structures with a regularized Non-Smooth-Contact-Dynamic approach

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Abstract

Weight is a potential motion. With this statement Martino Bassi, in his treatise of 1572, admirably summarizes the underlying rationale of the method for the evaluation of the stability of arcs and domes that today we would call "*kinematic method of limit analysis*". The *kinematic theorem*, and the *complementary static theorem*, allow us to easily evaluate the load bearing capacity of structures susceptible to collapse due to loss of stability for the formation of a mechanism. However, the validity of the theorems is not assured in the case of structures composed of blocks of finite dimension in contact with non-associated friction, especially when collapse involves the sliding of the blocks.

To analyze the motions, both in "potentiality" and in "actuality", of structures composed of rigid blocks, we propose a model based on the non-regular dynamic formulation of the unilateral contact with friction, following the theory originally proposed by J.J. Moreau (Non-Smooth-Contact-Dynamics). According to this, velocities are functions of bounded variation, so that the impulsive nature of the forces, and the consequent accelerations, can be interpreted in the space of measures. Friction is treated as a cone complementary problem, introducing a potential *à la* De Saxcè. The variational inequalities describing the problem are implemented in a general code (Project Chrono), which interfaces with a parametric drawing program (Grasshopper @ for Rhino @ CAD software).

The model allows us to consider the importance of the masonry bond pattern, which has always represented the most creative and essential design aspect of the art of building, but which is lost when considering masonry as a simple solid not resistant to tractions (no-tension material). There are cases in which the collapse involves shear sliding between the blocks, in association with rigid movements of macroblocks depending on the texture of the comments. More generally, it is shown that the tensile strength of the individual blocks can be exploited thanks to the contact with friction between the joints, leading in the case of domed structures to an equivalent tensile strength capable of balancing, at least in part, the hoop stresses. This effect is strongly influenced by the geometry of the dome (circular, oval, octagonal), by the height of the drum and by the weight of the overhanging tiburium, when present.

Keywords: **Masonry, Rigid blocks, Frictional contact, Non-Smooth-Contact-Dynamics (NSCD).**