Hierarchical multiscale modeling of large deformation problems in granular media

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Abstract

Granular media may experience large deformation in a wide range of engineering problems. Considering large deformation behavior is key for a numerical model to provide accurate predictions of the material responses in these applications. Finite Element Method (FEM) may suffer mesh distortions in the finite strain regime in simulating granular media, which may demand for complicated remeshing and mapping schemes to be developed for the remedy. The Material Point Method (MPM) [1] has shown great advantages in tackling large deformation problems owing to its mesh-less nature, and has become rather popular across several engineering and science disciplines for material modeling. A major pitfall associated with continuum-based numerical methods, including MPM, is the necessity of assuming proper constitutive relations to describe the material behavior for each material (or integration) point. The responses of granular media are widely known as highly nonlinear, path, history and state dependent and can be intricately coupled with the hydraulic response of pore fluids. As such, the constitutive laws developed for granular media are commonly exceedingly complex and contain many phenomenological model parameters that bear no clear physical meanings and are difficult to calibrate. Meanwhile, a variety of important phenomena observed in granular media, such as strain localization and failure, have their physical origins and mechanisms from the grain scale. To relate these macro phenomena to the underlying microstructures and their changes, cross-scale characterization and simulation have become recent trends in granular mechanics modeling.

In this study, we propose a computational multiscale scheme based on hierarchical coupling between MPM and DEM (Discrete Element Method). The hierarchical framework follows a similar concept of the FEM-DEM coupling scheme we have developed earlier [2-12]. In the framework, we employ MPM to treat a typical boundary value problem of granular media that may experience large deformations, and use DEM to derive the nonlinear material response for each of the MPM material points to feed the MPM computations. The proposed coupling framework helps avoid phenomenological constitutive assumptions in a typical MPM, while inherits its advantageous features in tackling large deformation problems over FEM. It also expedites a direct micro- macro linking for us to understand complicated behavioral changes of granular media over all deformation levels, from the initial elastic stage en route to large deformation regime before failure. Demonstrative examples are shown to highlight the advantages of the new MPM-DEM framework and discuss further exploratory directions relevant to its development.

Keywords: **Hierarchical multiscale modeling, MPM-DEM coupling, large deformation, granular media**

Coupling of MPM and DEM: Methodology & Predictions

In a Material Point Method a collection of material points are typically used to discretize the domain of a boundary value problem. A regular background mesh is employed to interpolate the field variables for each computational step. The deformation mesh is restored to its initial position to resolve the mesh distortion issue commonly encountered by other mesh-based methods such as FEM [1]. MPM has been proven efficient in capturing the complicated behavior of soils undergoing large deformation and has been applied in a wide or geotechnical problems, but needs phenomenological constitutive models for material response predictions. This paper presents a novel multiscale modeling approach based on hierarchical coupling of MPM and DEM to resolve the above issues in large deformation simulation of geotechnical problems. A similar concept based on coupled FEM-DEM scheme is followed [2-12]. We employ MPM to treat a typical boundary value problem in geomechanics that may experience large deformations, and use the DEM to derive the nonlinear material response required by MPM for each of its material points. A schematic of the hierarchical multiscale coupling scheme between MPM and DEM is depicted in Figure 1. Based on such a hierarchical framework, we are able to avoid phenomenological constitutive assumptions commonly needed by a continuum-based MPM, while retaining its advantageous features in tackling large deformation problems over FEM (e.g., no need for re-meshing to avoid highly distorted mesh in FEM). The MPM-DEM coupling scheme offers a straightforward way on micro-macro bridging for us to understand complicated behavioral changes of granular media over all deformation levels, from initial elastic stage en route to large deformation regime before failure.

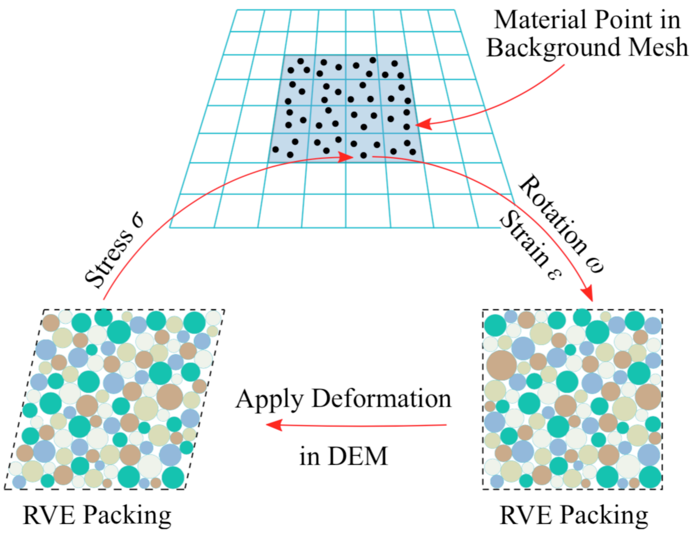

Figure 1 Schematic of hierarchical multiscale coupling of MPM×DEM for modeling large deformation in granular media [13]

As a demonstrative example, we present in Figure 2 a coupled MPM-DEM simulation of the problem of rigid footing foundation on dense sand which is widely considered in geotechnical engineering. In Figure 2a, a regular mesh is used to represent the initial domain of (12 m x 8 m). In each mesh one material point is placed. Attached to each of the material point is a dense DEM packing of 400 sphere particles. A rigid footing (H: 2 m, B:1 m) is placed on top-left corner before being pushed downwards to cause foundation soil failure. The rest of the ground surface is constrained by a surcharge pressure of 20 kPa. Figure 2b shows a final general failure mode our model captured at excessive settlement of the foundation. The maximum shear strain reaches over 100% within the shear bands and adjacent to the side-edge of the foundation. The footing soil interface can be model wit ease by the coupled

MPM-DEM scheme, while the situation is known to be notoriously troublesome for FEM simulations whereby extremely distorted meshes have been observed.

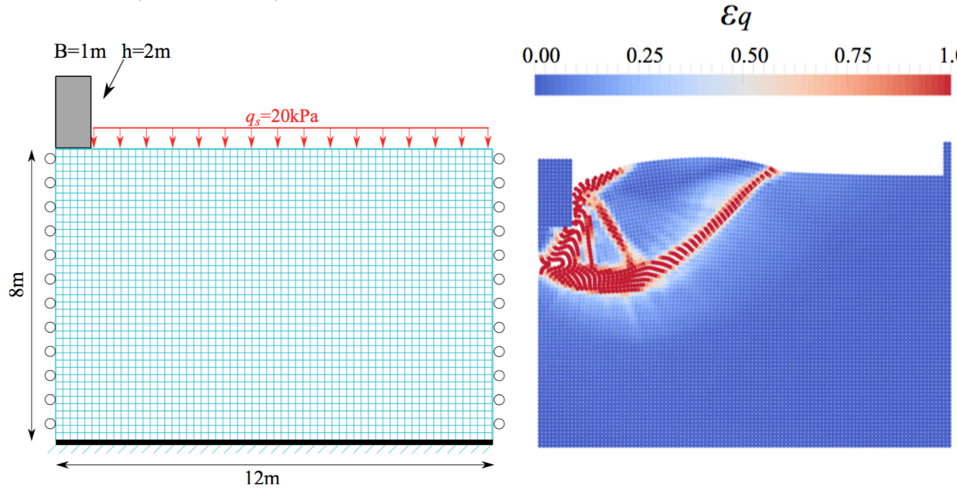

Figure 2 MPM×DEM multiscale simulation of a footing foundation experiencing general failure at large deformation [4]. (a) MPM background mesh; (b) Modeling simulation of general failure mode of footing foundation at large deformation in term of shear strain.

Acknowledgments

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References

[1] Sulsky, D., Chen, Z. and Schreyer, H. L. (1994) Particle method for history-dependent materials, *Computer Methods in Applied Mechanics and Engineering* **118**(1-2), 179–196.

[2] Guo, N. and Zhao, J.D. (2013). A hierarchical model for cross-scale simulation of granular media, *Powders and Grains 2013: AIP Conference Proceedings* **1542**, 1222-1225.

[3] Guo N. and Zhao J. D. (2014) A coupled FEM/DEM approach for hierarchical multiscale modelling of granular media, *Int. J. Numer. Methods Eng.* **99**, 789–818.

[4] Zhao J.D. and Guo N. (2015) Bridging micro and macro in granular media: A computational multiscale paradigm. In K. Soga, K. Kumar, G. Biscontin and M. Kuo (Eds.) *Geomechanics from Micro to Macro*, CRC Press, Taylor & Francis, London. Vol. **1**, 747-752.

[5] Guo N. and Zhao J.D. (2015) A multiscale investigation of strain localization in cohesionless sand. In: Chau K.T. & Zhao J.D. (eds.) *Bifurcation and Degradation of Geomaterials in the New Millennium*. Springer Series in Geomechanics and Geoengineering. Springer Heidelberg, Berlin, Germany. pp 121-126.

[6] Zhao, J.D. and Guo, N. (2015) The interplay between anisotropy and strain localisation in granular soils: a multiscale insight, *Géotechnique* **65**(8), 642-656.

[7] Guo N. and Zhao J. D. (2016) Parallel hierarchical multiscale modelling of hydro-mechanical problems for saturated granular soils, *Computer Methods in Applied Mechanics and Engineering* **305**, 37-61 (2016).

[8] Guo, N. and Zhao, J.D. (2016) 3D multiscale modeling of strain localization in granular media, *Computers and Geotechnics* **80**, 360-372.

[9] Guo, N. and Zhao, J.D. (2016) Multiscale insights into classical geomechanics problems, *International Journal for Numerical and Analytical Methods in Geomechanics*, **40**(3), 367-390.

[10] Guo, N., Zhao, J.D. and Sun, W.C. (2016) Multiscale analysis of shear failure of thick-walled hollow cylinder in dry sand, *Géotechnique Letters* **6**(1), 77-82.

[11] Zhao J.D. (2017) Hierarchical multiscale modeling of strain localization in granular materials: A condensed overview and perspective. In: E. Papamichos et al. (eds.) *Bifurcation and Degradation of Geomaterials with Engineering Applications*, Springer Series in Geomechanics and Geoengineering. Springer Heidelberg, Berlin, Germany. pp 349-359.

[12] Wu, H.R., Guo, N. and Zhao, J.D. (2017) Multiscale modeling and analysis of compaction bands in high-porosity sandstones, *Acta Geotechnica*. In press. doi: 10.1007/s11440-017-0560-2

[13] Liang, W., Zhao J. D. (2018) Multiscale modeling of large deformation in geomechanics, *Int. J. Numer. Analy. Methods Geomech.* Under review.