High-dimension basins of attraction computation on clusters (distributed memory systems) with simple cell mapping

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Abstract

High Performance Computing (HPC) platforms offer the possibility to investigate high dimensional nonlinear dynamical systems having a complex behaviour. In this work in progress we illustrate how it is possible to develop algorithms exploiting the computational power of HPC to determine attractors and basins in systems having dimensions equal or greater than six. With these results, it will be then possible to investigate the global integrity and structural stability of considered systems, which is needed for a safe design of real structures.

Keywords: **Bains of attraction, cluster computation, computational nonlinear dynamics**

Introduction

Numerical computation of attractors and basins (global analysis) of nonlinear dynamical systems, especially in higher dimensions, requires significant resources, both in computing power and storage capacity. Actually, numerical methods are constantly developed as a consequence of the lack of analytical methods to determine basins. In addition, an element of difficulty is due to the fact that frequently basins of attraction have fractal boundaries, which involves several computational issues.

Current technologies commonly used in scientific computing are cluster computers and computers with general purpose Graphical Processing Unit (GPU). Although, both offer massive parallelization capabilities, computing problems that can be effectively parallelized fundamentally differ through platforms. GPU is shared memory system which is suitable for data intensive and tightly coupled problems. Clusters, implementation of distributed memory systems, are suitable for operation intensive, weakly coupled computing tasks.

Cell mapping methods [1] aim to reduce computation time by approximating trajectories with mapping function created after short time integration (one period of system, when possible). Attractors and basins are determined by examining evolution of this map. Cell mappings developed for sequential computing are recently combined and parallelized [2]. Another numerical method, grid-of-starts [3], integrate each point of discretized state space up to the steady state behavior, to determine toward which attractor trajectories converge.

Current aim of our work is to investigate basins of attraction in higher dimensions. To have efficiently parallelizable integration process we decided to consider the Simple Cell Mapping (SCM) method. Creation of the map (integrations stage) by SCM is the computing task that can be effectively parallelized on both clusters and GPUs. The post-processing stage is more suited for shared memory systems (like GPU), but limited amount of operational memory is not enough to hold mapping required to form basins of high-dimensional systems. By using external storage memory GPU losses substantial amount of performance due to low data transfer bandwidth compared to computing speed of its processors. Based upon those facts, initial stage of our work consists of developing software tool that implements SCM on clusters with Message Passing Interface (MPI) standard.

Computing of basins

Integration process consists of large number of independent operations that can be effectively parallelized. Result of integration is mapping function that approximate the steady state of system. Certain clusters do not have all nodes of equal resources so we introduced load balancing algorithm where active nodes share portion of work with idle ones.

The map post-processing (sorting) stage that discovers attractors and basins is tightly coupled problem. Sequential unraveling algorithm [1] propagates in forward direction through map and marks periodic and transient cells. Basis for parallel approach is described in [2], where algorithm searches for covering set are illustrated. In our work we upgraded parallel algorithm to determine basins by finding and labeling periodic motions and transient cells. For consistency of attractors, marking of periodic motions is done sequentially.

Basin computation example

To illustrate computation of attractors and basins, we examined coupled pair of van Der Pol oscillators with addition of periodic excitation (in [1] the unforced case is considered):

$$\begin{aligned}\dot{y}_1 &= y_2,\\ \dot{y}_2 &= \mu(1-y_1^2)y_2-(1+\nu)y_1+\nu y_3+F\cos(\omega t),\\ \dot{y}_3 &= y_4,\\ \dot{y}_4 &= \mu(1-y_3^2)y_4-(1+\nu+\eta)y_3+\nu y_1.\end{aligned} \tag{1}$$

Phase space subset y_i=[-4,4] is divided into 6'765'202 cells (51 regular cells per system dimension and one sink cell). Figure 1(a) and (d) show $y_{1,2}$ and $y_{1,3}$ phase space projections of two periodic attractors, in this case limit cycles (SCM approximates continuous curves as repeating sequence of cells). Basin cross-section on $y_{1,2}$ plane is reported on Figure 1(b) for $y_{3,4}$=(25, 15) and in Figure 1(c) for $y_{3,4}$=(25, 35). Basin cross-section on $y_{1,3}$ plane is reported on Figure 1(e) for $y_{2,4}$=(25, 25) and on Figure 1(f) for $y_{2,4}$=(45, 25). Each dot represents center of cell and color of the basin matches color of the attractors reported on Figure 1(a) and (d). Parameter values used for integration are μ=0.1, η=0.04, ν=0.1, F=1.0 and ω=0.25.

Computing performance on clusters

Performance examination showed that sequential forward sorting is substantially faster than parallel search for covering set (implemented on distributed memory systems). Including more nodes into computation, number of communication operations drastically increase in comparison to sequential implementation. As a result, the post-processing performance becomes highly dependent on node inter-connection speed and number of communication requests.

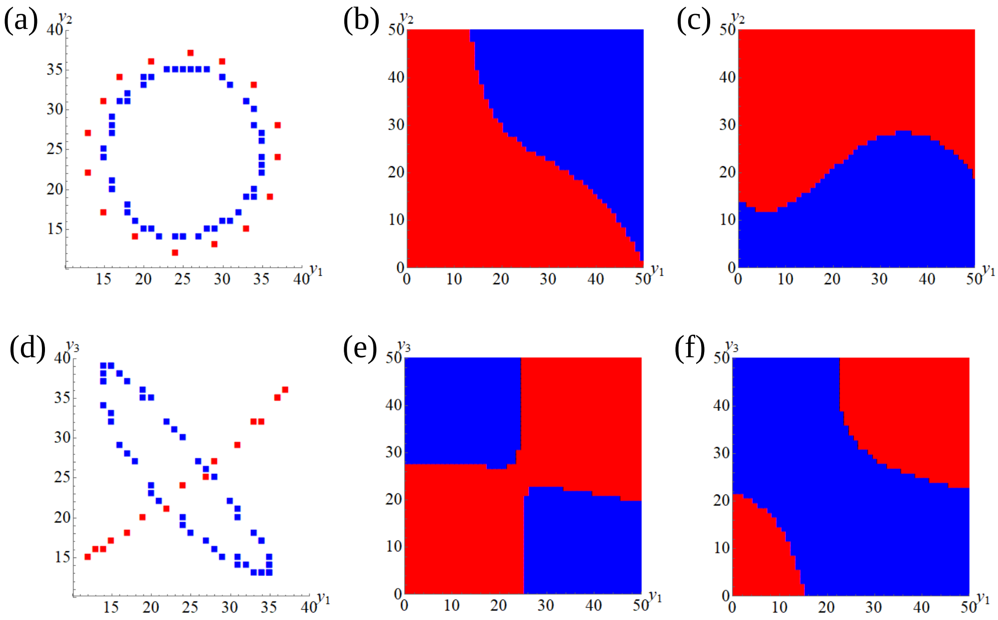

Figure 1. Computed attractors and basins of (1)

Current and future work

With main goal to analyze global dynamics through high-dimensional basins of attraction, our current work is on setting-up convenient six-dimensional system and cluster to examine it. Possibilities for future work might include analysis of eight-dimensional dynamical system, developing new parallel post-processing algorithm or reduction of communication operations during parallel search of covering set.

References

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