Limit analysis for a safety evaluation of masonry pavilion domes on octagonal drum subjected to increasing vertical loads

***†C. Anselmi[1], F. Galizia[1], and E. Saetta[1,2]**

[1]Department of Structures for Engineering and Architecture, University of Naples "Federico II", Italy
[2]Department of Structural and Geotechnical Engineering, Sapienza University of Rome, Italy

*Presenting author: anselmi@unina.it
†Corresponding author: anselmi@unina.it

Abstract

This paper aims at analyzing the different behavior of a pavilion dome according to the construction technologies adopted: in particular, if in correspondence with the ribs the bricks of the contiguous sails are well interlocked (as in the Santa Maria del Fiore dome) the dome shows a much greater load-bearing capacity compared to the case in which this interlocking between the bricks is completely missing (Santa Maria dell'Umiltà dome in Pistoia).

In several previous works [1,2,3] we had already dealt with rotation domes subject to vertical and horizontal loads, using the static theorem of the limit analysis applied to the masonry structure discretized in rigid macro-blocks of variable shape aligned along the parallels and the meridians in order to evaluate their degree of safety through the search for the load multiplier promoting the collapse. The yield domain conditions for the quadrilateral interfaces between the macro-blocks are expressed in terms of stress resultants, in observance of the following mechanical assumptions: inability to carry tension, unlimited or limited compressive strength (in the second case the domain is suitably linearized), frictional sliding with dilatancy. This last hypothesis, even if not realistic, has been assumed because in some cases the influence of sliding failure is negligible in the kinematic mechanism of the domes; thus it may be convenient to use linear programming with considerable reduction of calculation time and uniqueness of solution.

Compared to previous tackled problems, in this work there are two new aspects: a different type of dome, because it is set on octagonal and not circular drum, and the role played by the interlocking between the bricks in correspondence of the ribs on the bearing capacity of the pavilion dome.

As for vertical loads, we studied a dome segment - corresponding to one sixteenth - between two contiguous meridian symmetry planes, one passing through the center line of a sail, the other for a rib. In the horizontal section of the dome, the angle between the two symmetry planes is $\pi/8$, and the segment can be studied - at least in this first approach - as fixed at the dome' base and subject to dead loads due to their own weight and live loads corresponding to increasing overloads, as well as to the actions on the interfaces belonging to the symmetry planes, which one can consider as the redundant unknowns of the static problem.

In this context, we developed a program using Excel and its Solver to solve the optimization problem that provides the collapse multiplier. Although has always been used the same discretization in eight macro-elements (six for the dome and two for the drum), the built program is sufficiently versatile and, in addition to the mechanical characteristics, allows to define the intrados profile, the thickness variability, as well as to insert any window opening in the drum, the lantern at the top and the hoops at each level.

A fundamental aspect in the adopted modelling approach is that the yield conditions on the interfaces of the macro-blocks are always introduced with regard to the radial faces and those that belong to the symmetry meridian plane of the sail, while they can be introduced or not on the interfaces belonging to the symmetry meridian plane passing through the rib. These alternative hypotheses allow us to simulate the lack of interlocking between the bricks at the ribs (the failure is possible) or its presence (the failure is not possible).

It is also important to specify that the unknowns actions on the radial faces of the macro-blocks are six, corresponding to the resultants of stress state in three-dimensional problems, because the single analyzed segment is not symmetrical with respect to the vertical plane

bisecting the π/8 angle between the two meridian planes delimiting it, while the redundant unknowns on the faces of the aforesaid meridian planes - which are symmetry planes - are only three (the normal force and the two bending moments) and hence on these faces crisis just for rocking appear.

Once the kinematic problem has been solved, again using Excel, making use of suitable matrices of directional cosines, the kinematic mechanism is represented at the instant in which the collapse is reached.

The so far carried out applications, in the case of possible crisis at the ribs, have shown the effectiveness of the hoops, that increases as the pre-tensioning stress increases or, alternatively, according to hoops number placed; however, for an reasonable overall force of pre-tensioning, the value of the multiplier obtained is anyway inferior to that computed in the opposite case concerning the presence of interlocking between the bricks at the ribs. For the dome discretization into six blocks, the introduction of the hoops also seems to produce better effects if it's concentrated at the fifth ring starting from the top, rather than if distributed over several rings, for the same overall force. The applications have also confirmed the effect of window openings in the drum: the value of the collapse multiplier decreases as the openings width increases.

For the most part, the applications were carried out by increasing, through a multiplier, the overload acting on the dome, intended as a coating load applied on the extrados surface of the dome and also of the lantern small dome, if present. However, in order to simulate the vertical component of earthquake action, also were considered applications in which the weight of the blocks directly is amplified by a multiplier; the value that is obtained corresponds to a sort of safety coefficient related to the case of vertical component of seismic action.

In perspective, in addition to using the program in Excel to investigate other aspects, such as the problem of the double-shell domes and the search for the minimum thickness of the pavilion domes, we are implementing a program in Matlab in order to refine the solutions already obtained and to study also the pavilion domes under the action of horizontal loads.

Keywords: Masonry, dome on octagonal drum, limit analysis, no-tension material, yield surface.

References

[1] Anselmi, C., De Rosa, E. and Fino, L. (2004) Limit analysis of masonry structures, *Proceedings of the 4th International Seminar on Structural Analysis of Historical Constructions (edited by: C. Modena, P. B. Lourenço, P. Roca)*, Padova, Italy **1**, 545–550.

[2] Anselmi, C., De Rosa, E. , Galizia, F. and Maniello, D. (2006) Evaluation of the safety coefficient of axi-symmetric masonry domes with drum and lantern having variable profile and carrying their own weight, *Proceedings of the Seventh International Masonry Conference*, London, GB **5**, 1–6.

[3] Anselmi, C., De Rosa, E. , Galizia, F. and Maniello, D. (2009) Limit analysis of masonry domes subjected to horizontal loads, *Protection of Historical Buildings - Prohitec 09 (edited by F.M. Mazzolani)*, Rome, Italy **2**, 1097-1101.