Nomograph to calculate amount of reinforcing bar against bending moment in Circular Void slabs

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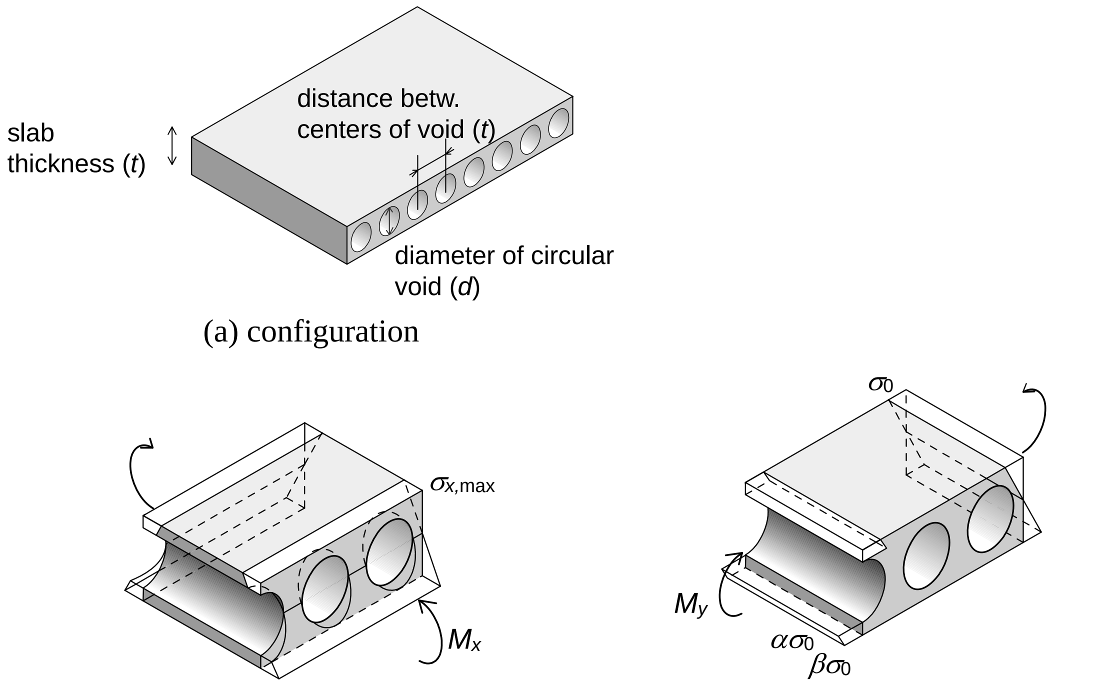
1. Introduction

Void slab has an advantage of reducing weight and increasing stiffness. In 1970s, "structural design of void slab"[1] was published by prof. Gengo Matsui, also well known as a famous structural engineer. In the published book, void slab that has long circular steel pipes etc. is treated as one way structure and is recommended to design like typical beams. For that reason, circular voids were arranged in orthogonal direction against slab boundaries and no stresses are thought to occur at supported beams whose directions are parallel to the void. So the author indicated that the stiffness of slab in orthogonal direction of voids [2] and clarified elastic characteristics occurred in such structures [3]. In these papers, such structures which have cylindrical steel pipes or other materials in one direction can be estimated as an orthotropic plate, and the bending stiffness and the stress concentration, which is clarified to be related to the ratio of the diameter to the thickness of the slab, are presented.



(a) configuration

(b) bending in void direction (c) bending in the orthogonal direction

Figure 1. Applicable void slab and stress distribution in each direction

But the distribution of stress in the paper was calculated on the assumption of the linearly elastic isotropic material. As shown in fig. 1 (b), distribution of normal stress in void direction can be formulated based on Navier's hypothesis, but that in the orthogonal direction is not straight as shown in fig. 1 (c). Figure 2 shows the distribution of normal stress in the orthogonal direction on upper and lower section of circular void calculated by BEM in previous paper. And the author had shown the scaling factor α and β to find the stresses on the surface of slab and the upper edge of the void.

As these results are obtained under the assumption that the slab has no steel bar and concrete has tensile strength, there exists stress distribution in tensile side of the concrete. Consequently, when finding amount of reinforcing bar from these results, resultant stress in the tensile side would be used.

On the other hand, amount of reinforcing bar in typical RC beam is formulated on the assumption that concrete has no tensile strength and Navier's hypothesis [4]. But the hypothesis cannot be established in void slab as shown in fig. 3, so the formulation to find the amount of reinforcing bar in the same method is difficult.

Accordingly, in this paper, amount of reinforcing bar for void slab is numerically calculated by FE analysis.

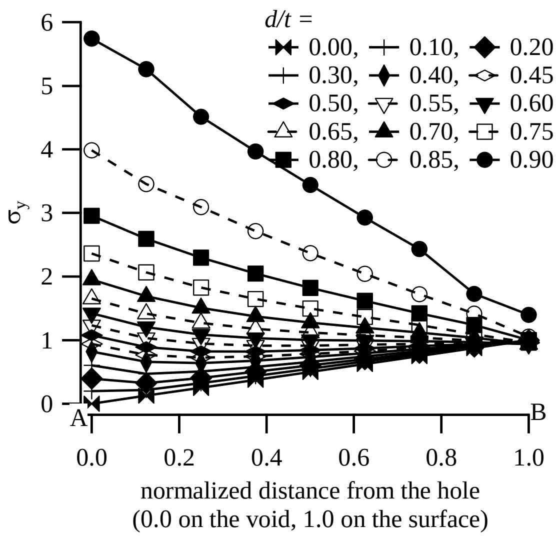


Figure 2. Normal stress distribution on upper and lower section of circular void

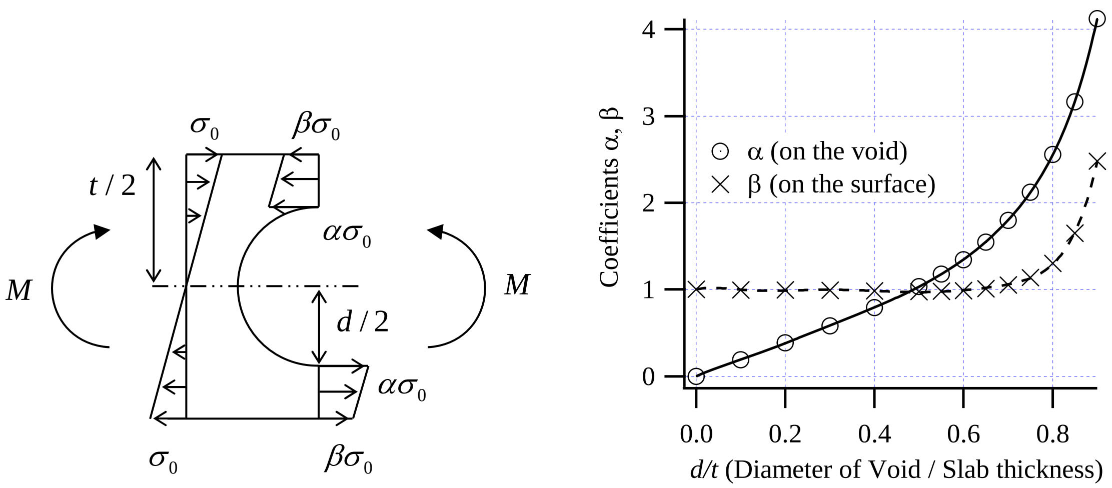
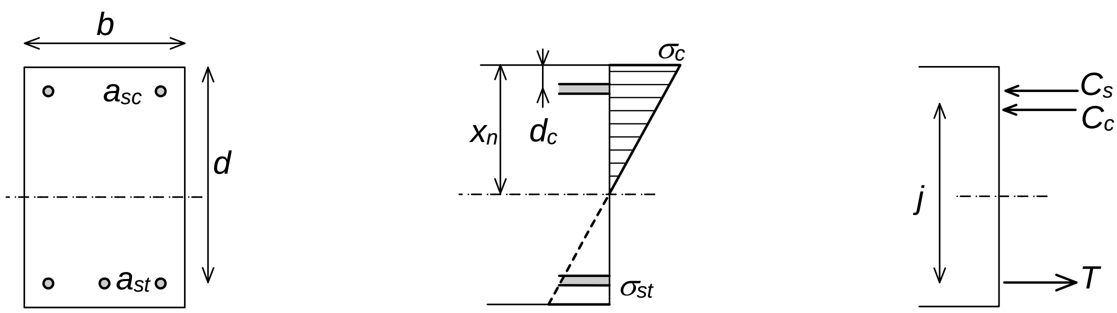


Figure 3. Scaling factor a and b to find the stresses on edge of void and on surface of slab

2. Void Slab configuration and Distribution of normal stress

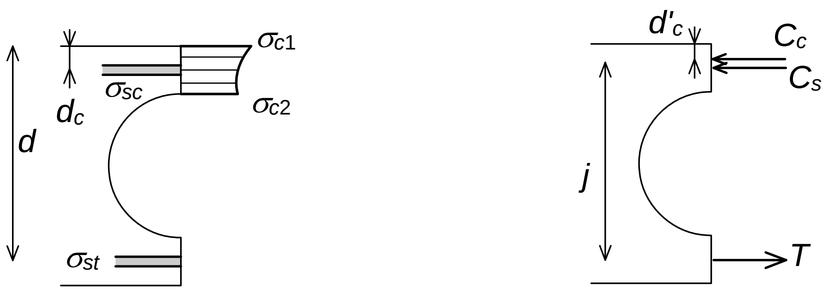
Circular void slab treated in this paper is shown in fig. 1 (a). Cylindrical steel pipes or other materials are embedded in the center part of cross section in one direction. The distance between steel pipes is same as the thickness of the slab and the diameter of the pipes is equal to the thickness minus 120 mm, which comes from cover depth of reinforcing bars and the pipes.

Figure 4 shows the distribution of normal stress in concrete and tension force in reinforcing bars in typical RC beam. a_{sc} and a_{st} are area of compression reinforcement and that of tension reinforcement, and d_c is distance between extreme compression fiber and center of compression reinforcement. By using these variables in this figure, allowable bending moment for RC beams is formulated in AIJ standard for Structural Calculation of Reinforced Concrete Structures in Japan. Assumption of the neutral axis position x_n gives the maximum stresses σ_c of compressive concrete and σ_{st} of tensile steel bar. And the equations of allowable bending moment are obtained by the maximum stress reaching the allowable strength.



Section Stress Distribution Action point of resultant force

Figure 4. RC beam [4]



Stress Distribution Action point of resultant force

Figure 5. Void slab

By considering in the same manner, the stress distribution and resultant forces around the circular hole would be illustrated as in fig. 5. The maximum compressive stress in the concrete occurs at the surface of the slab or at the top of circular void (σ_{c1} or σ_{c2}).

Similarly in formulation for RC beam, finding resultant force C_c and its acting position from distribution of compressive concrete stress, and the distance j between acting position of composite stress of compressive concrete stress C_c and compressive steel stress C_s and the center of tensile reinforcing bar gives the following two bending moment equations.

$$M_c = C_s(d - d_c) + C_c(d - d'_c)$$
$$M_t = T \cdot j$$

Considering cover depth of concrete, we assume that $d = t - 30$ [mm] and $d_c = 30$[mm]. Bending moments in which stress reaches its allowable value can be written in

$$M_{ci} = M_c \times \frac{f_c}{\sigma_{ci}} \quad (i = 1, 2), \quad M_{c3} = M_t \times \frac{f_t}{\sigma_{st}}$$

where f_c and f_t are allowable compressive stress of concrete and tensile stress of steel bar. Dividing them by bd^2 gives the coefficients C_1~ C_3 which can be similarly seen in AIJ standard for RC beam.

$$C_1 = \frac{M_{c1}}{bd^2}, \quad C_2 = \frac{M_{c2}}{bd^2}, \quad C_3 = \frac{M_{c3}}{bd^2}$$

As the distribution of stress is dependent on the ratio of the diameter to the thickness of the slab, we need to calculate amount of reinforcing bar for each thickness individually. The thickness of slab is to be set from 200 to 500 mm at 50 mm intervals. Tension reinforcement ratio *pt* (= section area of tension reinforcement / that of slab) is to be set from 0.2 to 2.0 % at 0.2 % intervals and double reinforcement ratio γ (= section area of compressive reinforcements / that of tension reinforcements) is to be set from 0.0 to 1.0 at 0.2 intervals.

3. FE models

Figure 6 shows analytical model to calculate distribution of normal stress of concrete and tension and compression stresses in reinforcing bars. Simply supported beam which has seven circular holes is loaded with constant load. Firstly, to confirm that the same distributions of normal stress indicated in fig. 2 in a section can be obtained, finely divided 2D model consists only of concrete is used. Numerical models are built by using plane elements. After that, by appending beam elements for reinforcing bars, RC models are formed.

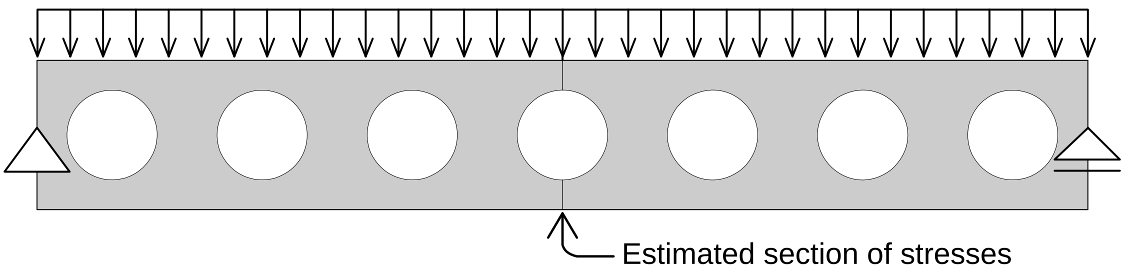


Figure 6. Analytical model and section of estimating stress

In the beginning, analytical models as shown in fig. 7 whose concrete elements around the estimated section are deleted were calculated to confirm the stress distribution. But the distributions are totally different by changing the nodal positions that connect the tension reinforcing elements and the concrete elements, and the models turn to be insufficient. Therefore, analytical models as shown in fig. 8 whose all concrete elements in lower part of beam are deleted and stiff elements are appended to let tension reinforcing bars be able to be tensioned in adequate distance.

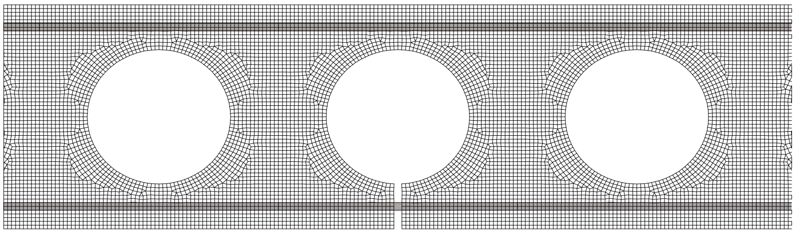


Figure 7. 1st FE model

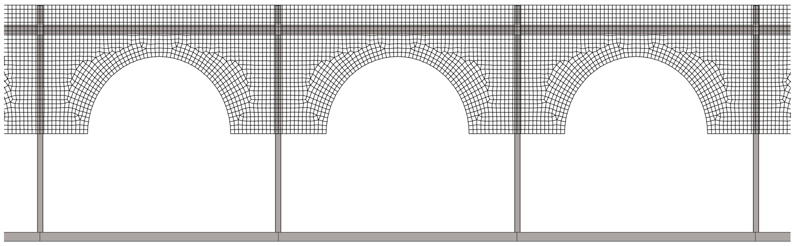
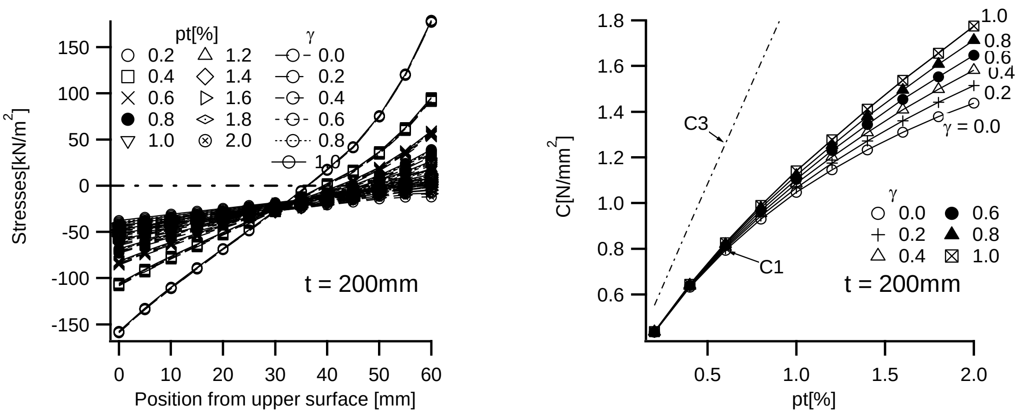


Figure 8. Analytical FE model

4. Results and conclusions

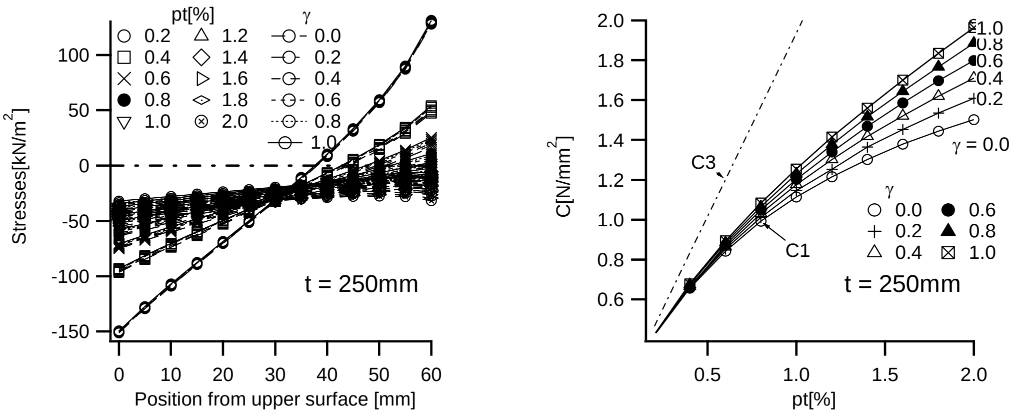
Normal stress distribution and nomograph to find allowable bending moment are illustrated in figures 9 ~ 11 in each thickness of slab. Left side of figs. is normal stress distribution and right one is the nomograph. Plus value of normal stresses means tension stresses and tension stresses are occur near the void in the lower tension reinforcement ratio *pt*. Allowable bending moment can be calculated as multiplication of minimum value of C1~C3 and bd^2. C1s illustrated in solid line show the values at which the stress σ_{c1} in fig. 3 reaches the allowable strength and C2s in dashed line show those at which the stress σ_{c2} reaches the allowable strength. And C3 in chain line shows the value at which the stress σ_{st} of reinforcing bar reaches it. As the thickness of slab becomes thicker, stress σ_{c2} at top of circular hole becomes bigger than σ_{c1} at on the surface of slab. And this phenomenon notably appears in higher *pt* value.

The problem is that tension stresses occur in concrete in some cases. To avoid this fact, we need to use FEM model in which material nonlinearity can be applied, or to use some technics to delete concrete elements that show tension stress.

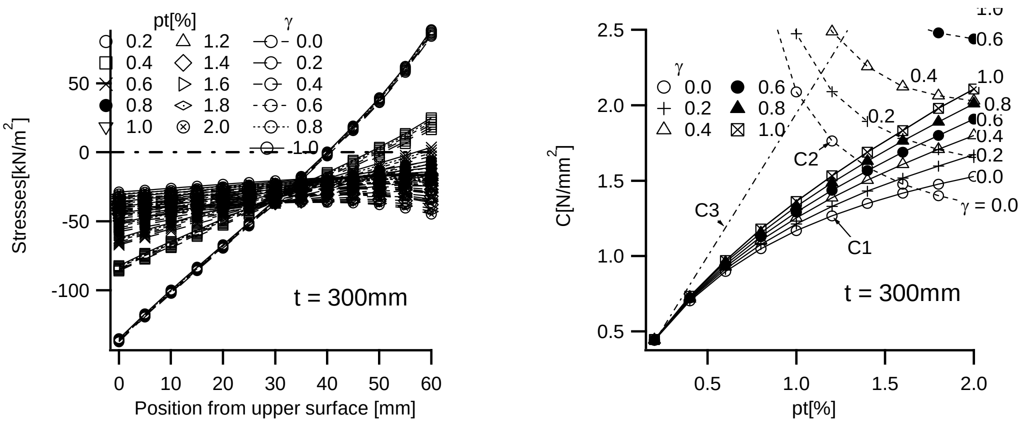


t = 200

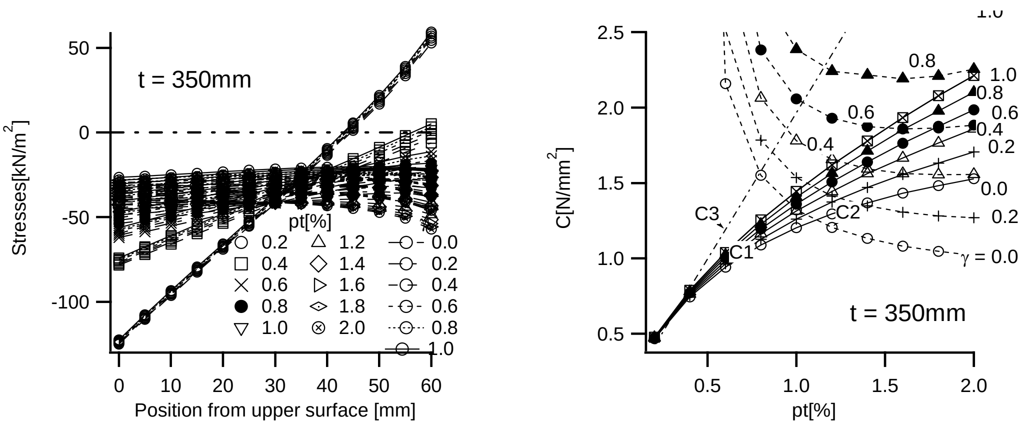
Figure 9. Allowable bending moment for long period load(Fc=24[N/mm^2], n=9.04)



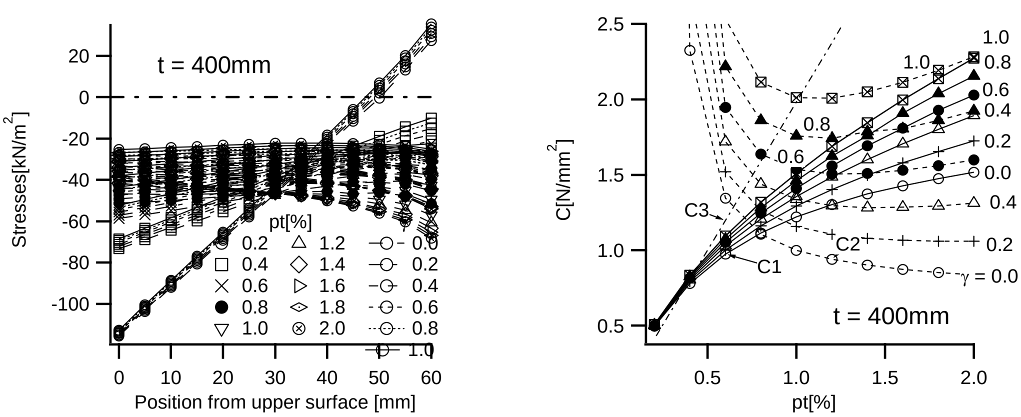
t = 250



t = 300

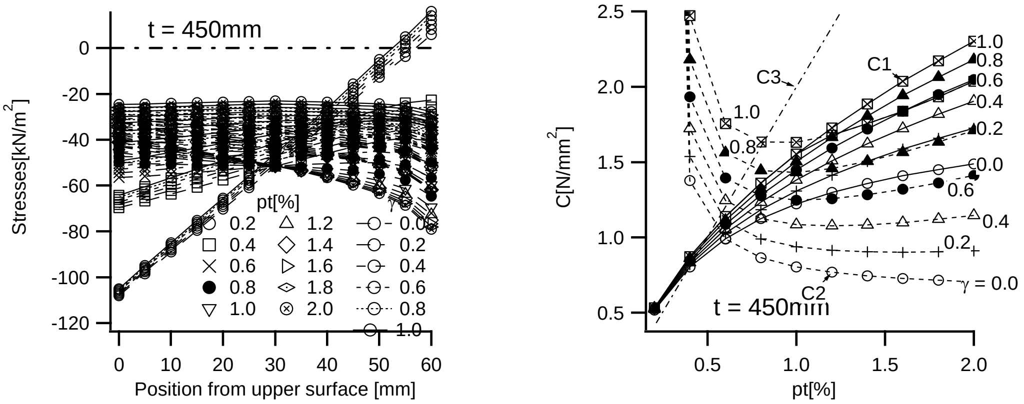


t = 350

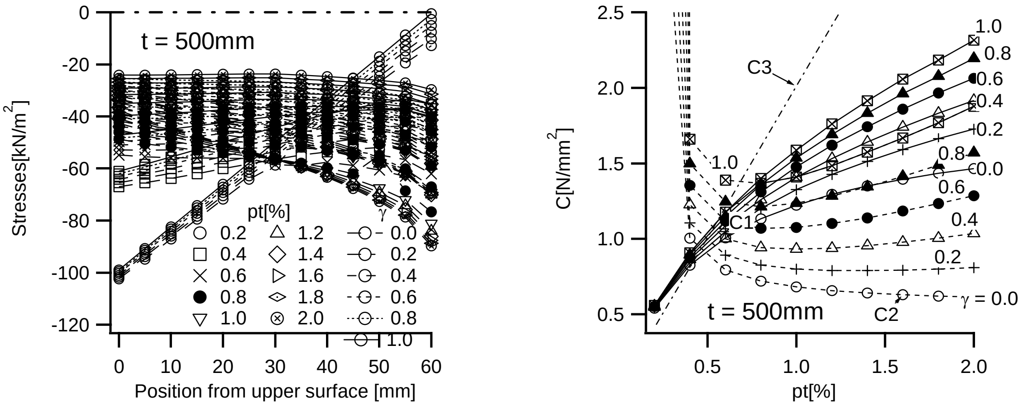


t = 400

Figure 10. Allowable bending moment for long period load(Fc=24[N/mm^2], n=9.04)



t = 450



t = 500

Figure 11. Allowable bending moment for long period load(*Fc*＝24[N/mm^2], *n*=9.04)

References

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[3] Shigehiro MOROOKA: CHARACTERISTICS OF ELASTICITY OF CIRCULAR HOLE VOID SLAB, 2003 Volume 68 Issue 565 Pages 79-85 , Journal of Structural and Construction Engineering (Transactions of AIJ) (in Japanese)

[4]AIJ Standard for Structural Calculation of Reinforced Concrete Structures -Based on Allowable Stress Concept -, revised 1999, Architectural Institute of Japan, 1999 (in Japanese)