A finite element based procedure for accurate determination of mode I SIF of orthotropic materials based on two parameter strain series

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Abstract

In strain gage based determination of stress intensity factor (SIF), the degree of accuracy of strain readings is affected by the radial position of the gage ahead of the crack tip. In this paper, a theoretical frame work based on two parameter strain series has been developed for accurate determination of mode I SIF (K_I). Based on the two parameter strain series a finite element analysis (FEA) based methodology is presented by which the limit of the radial location ($r_{\max}$) up to which the two parameter strain series is valid could be determined. Therefore knowing the $r_{\max}$ for a specimen and placing the strain gages within $r_{\max}$ will ensure accurate determination of K_I. Even though two parameter strain series for determination of K_I in orthotropic laminates was used by earlier researchers, radial location of the strain gage was decided by trial and error and hence accuracy could not be ensured. Using the proposed methodology, optimal radial location for the strain gage has been determined for $[0_2/90]_{2s}$ glass-epoxy edge-cracked laminates and numerical simulations were performed for determination of K_I. Results from numerical simulations show that the present technique using two parameter strain series not only allows the strain gage to be placed at a radial distance sufficiently away from the crack tip compared to that reported by earlier researchers, but also ensures accurate determination of K_I in orthotropic composites.

Keywords: **Stress intensity factor, Orthotropic composites, Strain gage, Finite element**

Introduction

The use of strain gages for the determination of SIF was first proposed by Irwin many years back [1]. However, complications related to a strain gage based application like high strain gradients, 3D effects etc. restricted the usage of strain gages to its full potential. Dally and Sanford [2] then proposed a single strain gage technique for the determination of K_I in isotropic materials. They employed a truncated three parameter strain series representation for the strain field around the crack tip and after careful mathematical simplification they devised that a single strain gage placed sufficiently away from the crack tip with certain orientations and location (decided by θ and ϕ) (Fig. 1) can be used to measure K_I for isotropic materials. In the case of orthotropic composites, the development of similar technique was little more involved due direction dependent properties. Shukla and co-workers [3] were the first to make a similar attempt for a single strain gage technique for the

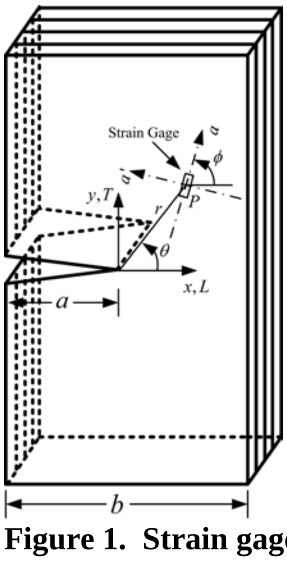


Figure 1. Strain gage location ahead of the crack tip

determination of K_I of orthotropic materials. However, instead of a three parameter series as originally proposed by Dally and Sanford [2], they used a two parameter series representation of the strain field around the crack tip to make the analysis simpler. They selected the radial distance for mounting the strain gage as 5 mm and 9 mm based on previous experience and compared the results and did not present any further explanation for the selection of gage locations. It is important that for accurate determination of SIF, the gage needs to be placed at an optimum location which is neither very near to (where strain gradients and 3D effects are prevalent) nor far away from the crack tip (where other terms in addition to the selected two terms of the infinite series become significant).The lower limit of the gage location was established experimentally [3-4]. However, the upper limit of the strain gage though so important was purely based on intuition and was not addressed. Only recently Sarangi et al. [5-6] came up with an approach to determine the upper limit of the radial distance ($r_{\max}$) of placing the strain gage in the Dally and Sanford technique for determination of K_I for isotropic materials. Even though, recently, Chakraborty et al [7-8] extended Dally and Sanford technique to orthotropic composites, no such attempts have been made to determine $r_{\max}$ corresponding to the relatively simpler single strain gage method proposed by Shukla et al. [3] for determination of K_I for orthotropic materials. The present work attempts at establishing a theoretical basis for developing a procedure for determination of $r_{\max}$ using FEA corresponding to the two parameter method. Numerical simulations are also presented supporting the present theoretical formulation.

Theoretical Development and Numerical Simulation

The two parameter strain field representation of the normal strain component ε_{aa} at an angle ϕ with the crack axis (CCW direction with crack axis is positive) at a point P located by r and θ (Fig. 1) obtained using strain transformation laws as [3]

$$\varepsilon_{aa} = A_0 \left\{ \begin{aligned} &\left[\frac{1}{\sqrt{r_1}} \left(\left(\cos\frac{\theta_1}{2} \frac{\alpha-\beta}{2\alpha} \begin{pmatrix} \cos^2\phi\left(-a_{11}(\alpha+\beta)^2 + a_{12}\right) \\ +\sin^2\phi\left(-a_{12}(\alpha+\beta)^2 + a_{22}\right) \end{pmatrix} \right) - \left(\sin\frac{\theta_1}{2} a_{66} \sin\phi\cos\phi \left(\frac{\alpha^2-\beta^2}{2\alpha} \right) \right) \right) \right] \\ &+ \left[\frac{1}{\sqrt{r_2}} \left(\left(\cos\frac{\theta_2}{2} \frac{\alpha+\beta}{2\alpha} \begin{pmatrix} \cos^2\phi\left(-a_{11}(\alpha-\beta)^2 + a_{12}\right) \\ +\sin^2\phi\left(-a_{12}(\alpha-\beta)^2 + a_{22}\right) \end{pmatrix} \right) + \left(\sin\frac{\theta_2}{2} a_{66} \sin\phi\cos\phi \left(\frac{\alpha^2-\beta^2}{2\alpha} \right) \right) \right) \right] \end{aligned} \right\}$$

$$+ B_0 \left\{ \frac{\beta}{2\alpha} \left[(\alpha+\beta)^2 - (\beta-\alpha)^2 \right] \left(a_{11}\cos^2\phi + a_{12}\sin^2\phi \right) \right\} \tag{1}$$

Inspection of Eq. (1) suggests that the coefficient of B_0 term can be eliminated by selecting an angle ϕ such that

$$\tan^2\phi = -a_{11}/a_{12} = 1/\nu_{12} \tag{2}$$

Thus, with ϕ determined from Eq. (2), the strain ε_{aa} can be written in terms of r, θ and unknown term A_0 as

$$\varepsilon_{aa} = \frac{1}{\sqrt{r}} \times A_0 \left\{ \begin{array}{l} \frac{1}{E_2}\left(\frac{1-\nu_{LT}\nu_{TL}}{1+\nu_{LT}}\right)\frac{1}{2\alpha}\left[\frac{\cos\left(\frac{1}{2}\tan^{-1}\left(\left(\beta+\alpha\right)\tan\theta\right)\right)}{\sqrt[4]{\left(\cos^2\theta+\left(\beta+\alpha\right)\sin^2\theta\right)}}\left(\alpha-\beta\right)+\frac{\cos\left(\frac{1}{2}\tan^{-1}\left(\left(\beta-\alpha\right)\tan\theta\right)\right)}{\sqrt[4]{\left(\cos^2\theta+\left(\beta-\alpha\right)\sin^2\theta\right)}}\left(\alpha+\beta\right)\right] \\ +\frac{1}{G_{12}}\left[\frac{\nu_{LT}}{\left(1+\nu_{TL}\right)\sqrt{\nu_{TL}}}\right]\frac{1}{2\alpha}\left[\frac{\sin\left(\frac{1}{2}\tan^{-1}\left(\left(\beta+\alpha\right)\tan\theta\right)\right)}{\sqrt[4]{\left(\cos^2\theta+\left(\beta+\alpha\right)\sin^2\theta\right)}}-\frac{\sin\left(\frac{1}{2}\tan^{-1}\left(\left(\beta-\alpha\right)\tan\theta\right)\right)}{\sqrt[4]{\left(\cos^2\theta+\left(\beta-\alpha\right)\sin^2\theta\right)}}\right] \end{array} \right\} \quad (3)$$

Therefore, by placing a single strain gage as shown in Fig. 1 at a radial distance r from the crack tip along the line at angle θ and oriented at an angle ϕ, the measured strain ε_{aa} can be equated to Eq. (3) to obtain the value of unknown coefficient A_0. The mode I SIF can then be determined using $K_I = \sqrt{2\pi}A_0$. For a given cracked configuration, applied load and material properties and after a careful selection of the angle θ, Eq. (3) can be written as

$$\varepsilon_{aa} = \frac{C}{\sqrt{r}} \quad (4)$$

where C is a constant. Taking logarithm on both sides of Eq. (4) results as

$$\ln\left(\varepsilon_{aa}\right) = -0.5\ln\left(r\right) + \ln(C) \quad (5)$$

A plot of Eq. (5) on log-log axes depicts a straight line of slope equals to – 0.5, with an intercept of $\ln\left(C\right)$. If $r_{\max}$ is the extent of valid two parameter zone theoretically, the straight line property will break beyond $r > r_{\max}$ as more than two parameters would be needed in Eq. (3) to estimate the ε_{aa}. Thus, Eq. (5) is valid along the gage line determined by the angle, θ for $r \le r_{\max}$.

Numerical Simulation, Results and DIscussions

In the present study, a $[0_2/90]_{2s}$ glass-epoxy edge-cracked configuration with $a/b = 0.4$ as used by Shukla et al. [3] for their experimentations has been considered. Table 1 lists the material properties, loading and geometric parameters of the cracked orthotropic panel considered in this section. The parameters α = 0.9684, β = 1.4496 and orientation ϕ = 68.01° are determined from the material properties. The exact model used by Shukla and co-workers [3] has been duplicated so that the results can be easily compared and the value of θ as suggested after taking care of various factors like strain gradients, averaging error due to finite gage size and the like is found to be $\theta = 38^{\circ}$.

Table 1. Geometry and material parameters for edge-cracked $[0_2/90]_{2s}$ glass-epoxy

b (mm)	h/b	ν_{LT}	E_L (GPa)	E_T (GPa)	G_{LT} (GPa)	σ (MPa)
50	3	0.163	33.3	24.6	5.2	100

Finite element analysis for the present example has been carried out in ANSYS employing eight noded isoparametric quadrilateral elements. A typical finite element mesh used in the present analysis considered after proper convergence study is shown in Fig. 2(a). Following Eq. (5), the plots of $\ln(\varepsilon_{aa})$ versus $\ln(r)$ obtained from the FEA is shown in Fig. 2(b). It may be observed from Fig. 2(b) that plot consists of well demarcated zones defining the linear and non-linear portions as predicted by the theory. The extent of the linear portion which gives the value of $r_{\max}$ for that configuration is found to be 16.74 mm. The numerically determined SIF values computed at different locations are compared with the analytical results. The analytical expression for mode I SIF of this configuration is given by [3]

$$K_I = Y_I\left(a/b\right)\sigma\sqrt{a} \tag{6}$$

where σ is the applied stress, a is the crack length and Y_I is the specimen geometric factor given by [3]

$$Y_I = 1.99 - 0.41\left(a/b\right) + 18.7\left(a/b\right)^2 - 38.48\left(a/b\right)^3 + 53.85\left(a/b\right)^4 \tag{7}$$

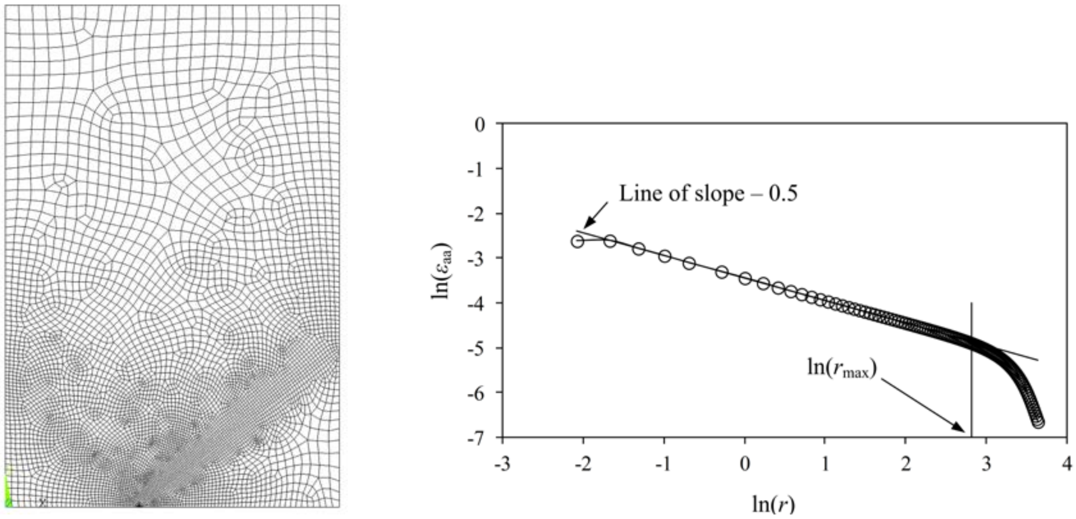
The computed strains at all the nodes on the gage line are considered as the measured strains using a single strain gage oriented at angle of $\phi = 68.01^0$ with the crack axis at the corresponding radial distances. Following the procedure explained, the $r_{\max}$ value of this configuration is found to be 16.74 mm and the thickness of the plate is set to 1 mm. Therefore, according to the present approach, any radial distance of the strain gage from the crack tip that satisfies

$$1\,\text{mm} \le r \le 16.74\,\text{mm} \tag{8}$$

is an optimal or valid gage location for accurate determination of mode I SIF for the problem considered. The gage locations for which $r \ge r_{\max}$ are invalid or non-optimal locations. Accordingly, the strain ε_{aa} is sampled at two optimal gage locations (for which, $r < 16.74\,\text{mm}$) and two non-optimal gage locations (for which, $r \ge 16.74\,\text{mm}$) as shown in Table 2. For this configuration at $\sigma = 100\,\text{MPa}$ the reference value of mode I SIF determined using Eq. (7) is $K_I = 52.80\ \text{MPa}\sqrt{\text{m}}$ .The measured mode I SIF using a single strain gage located at those optimal and non-optimal radii are determined using the simulated finite element strain values ε_{aa} at those radii (Table 2). The percent relative error in measured is computed as

$$\%\,\text{Rel. error} = \frac{K_{\text{Analytical}} - K_{\text{measured or simulated}}}{K_{\text{Analytical}}} \times 100 \tag{9}$$

Table 2 shows the comparison of numerically simulated SIFs with the analytical value and the corresponding percent error at different radial locations. It may be observed form the results of numerical simulations that for the strain readings at radial locations within $r_{\max}$, the values of SIF are highly accurate and those estimated at non-optimal radial locations (beyond $r_{\max}$) are erroneous. This also shows that the radial location (r = 9 mm) used by Shukla et al. [3] was very well within the estimated $r_{\max}$ of 16.74 mm and hence resulted in accurate K_I. However, using the proposed method, it was observed that highly accurate value of K_I could actually be obtained even by placing the strain gage well beyond 9 mm thereby substantiating the importance and usefulness of the method in fracture mechanics in accurate determination of SIFs.



(a) (b)

Figure 2. (a) FE mesh (b) Variation of ln(ε_{aa}) vs. ln(r)

Table 2. Simulated K_I at the optimal and non-optimal locations for edge cracked $[0_2/90]_{2s}$ glass-epoxy laminate

r (mm)	ε_{aa}	Analytical K_I $\left(\mathrm{MPa}\sqrt{\mathrm{m}}\right)$	Measured K_I $\left(\mathrm{MPa}\sqrt{\mathrm{m}}\right)$	% Relative Error
10.98	9.57E-03		51.96	1.59
14.29	8.26E-03	52.80	51.16	3.11
18.56	6.92E-03		48.85	8.1
22.01	5.92E-03		45.5	13.83

Conclusions

In the present work, a theoretical frame work has been developed for determination of maximum radial location (r_{max}) up to which the two parameter strain series is valid ahead of the crack tip in an orthotropic laminate. Based on the theoretical formulation, a finite element based methodology has been proposed by which the r_{max} for a particular configuration of edge cracked orthotropic composites could be determined. Numerical simulations have been performed and the results show that it is possible to accurately determine mode I SIF in orthotropic composites using a single strain gage considering two parameter strain series only when the gage is placed within r_{max}. In the absence of prior knowledge of r_{max}, and placing gages arbitrarily might lead to erroneous values of SIF. Therefore the development of this FE based procedure will be immensely useful for experimentalists using the procedure put forward by Shukla et al [3] for determination of K_I in edge cracked orthotropic composites.

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