KBL: A knowledge based learning method for extracting formulas of aerodynamic heating*

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Abstract

Aerodynamic heating of hypersonic flights originates from the action of strong shock and viscous boundary layer. Accurate prediction of the heating flux is a classic problem in aerodynamics. Great efforts including theoretical analysis, numerical simulation and wind-tunnel/flight test, have been devoted to it since the middle of last century, and many empirical formulas for some typical shapes have been obtained, which have supported the development of current aerospace engineering. However, formulas of quantitative relationships with more universal meaning are still lacking, and conventional analytical method is difficult to solve this problem. In this work, an intelligent optimization method, knowledge based learning (KBL), is proposed to detect the underlying laws of aerodynamic heating that could unify the data distribution of different models from different wind tunnels. In KBL, the wind-tunnel data to be learned are preconditioned and enriched with expert knowledge and numerical simulation of high fidelity, and then a special genetic programming algorithm is developed to detect the underlying universal laws of aerodynamic heating. The proposed method has been applied to two groups of aerodynamic heating data collected from different wind tunnels. One is of sphere heat transfer, and the other is of a double ellipsoid configuration. The study indicates that the KBL can discovered concise formulas and is promising for detecting more universal laws of aerodynamic heating.

Keywords: **Artificial intelligence, Data correlation, Aerodynamic heating, Wind tunnel, Computational fluid dynamics**

Introduction

Aerodynamic heating of hypersonic flights originates from the action of strong shock and viscous boundary layer. It is a critical issue to consider when developing a new hypersonic vehicle [1]. Accurate prediction of the heating flux is a classic problem in aerodynamics. Great efforts including theoretical analysis, numerical simulation and wind-tunnel/flight test [5,6,9-11,18-22], have been devoted to it since the middle of last century, and many empirical formulas for some typical shapes have been obtained.

Early research was mainly based on boundary layer analysis and experimental data fitting. Empirical formulas are available for many typical configurations. The classical Fay-Riddle formula can be used in the stagnation area. The surface friction formula of Blasius and the modified Reynolds analogy can be used in the large-area of laminar flow, and the reference enthalpy method can be used to consider the effect of high-speed laminar compressibility.

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For the turbulent area, the surface friction formula of Schultz-Grunow, reference enthalpy methods and Reynolds analogy, or the surface friction formula of Spalding-chi and Reynolds analogy could be applied. To distinguish the laminar and turbulent flow, Van Driest criterion, Batt criterion, Dirling criterion, and Bishop criterion could be used to estimate the position of transition.

With the development of computer and CFD, numerical simulation plays an increasingly important role in the prediction of aerodynamic heating. However, direct CFD method is Computationally intensive. Therefore, researchers began to combine theoretical empirical formulas with simplified CFD (e.g., inviscid CFD) simulations to sketch surface streamlines and predict the aerodynamic heating. The hybrid method needs less calculation time, and does work for more complex shapes. For example, the NASA Langley Research Center developed AEROHEAT into the AA3DBL program based on the theory of three-dimensional axisymmetric boundary layer. This program can not only predict the heat flow distribution of the centerline, but also predict the lateral heat flow distribution off the centerline, which can be used for calculations such as space shuttles, etc. It can be used to calculate the heat flux distribution of three-dimensional aircrafts like the Space Shuttle. Later, a generalized body-fitted coordinate system was used to allow boundary layer calculations to be used in conjunction with inviscid flow field solution methods. Thus, it can be used to solve the heat flux distribution of any shape of aircrafts.

It is worth noting that people’s pursuit of universal laws of aerodynamic heating will never stop even if the CFD and computational capabilities become so advanced that heat flux distribution can be accurately predicted. In fact, CFD is a gray box process. The inputs are flow parameters (i.e., P, T, V, etc.), mesh grids, turbulence models, boundary conditions, etc. The output is the heat flux at each point of the grid-nodes or grid-cells, and it cannot directly depict the intrinsic quantification relationship of heat flux and flow parameters (law of aerodynamic heating).

Comparing classical mechanics with aerodynamic heating, universal laws that can quantitatively describe the relationship of different parameters such as Newton's second law (F=ma) and the law of universal gravitation (F=GMm/r^2) are still lacking.

Of course, it is not easy to obtain such universal laws. For example, Kepler's three laws of planetary motion (the Law of Ellipses, the Law of Equal Areas, the Law of Harmonies) have cost Kepler more than 8 years of hard work, based on the set of astronomical observation data collected by Danish astronomer Tycho Brahe. The discovery of Kepler's three laws has undergone two phases, knowledge accumulation (relevant theories, observation/experiment data) and formula extracting.

Comparing the present and the past, the phase of knowledge accumulation has matured. In fact, many countries have been made great efforts to develop new types of hypersonic vehicles, and have conducted a large number of wind tunnel experiments and flight tests, and accumulated a large number of aerothermal data with high-precision [16]. So it is time for the stage of formula extracting.

Note that extracting universal formulas of aerodynamic heating would be much more difficult. In fact, the aerodynamic heat flux is a gradient quantity influenced by global parameters, and it has is strong nonlinearity. Compared to Kepler's law, the formula that can depict the intrinsic quantification relationship of heat flux and flow parameters must have higher complexity. For example: the heat flux formula of Fay-Riddell [6], at the stagnation point is relatively complex:

$$q = 0.763(Pr)^{-0.6}\left(\frac{\rho_w\mu_w}{\rho_s\mu_s}\right)^{0.1}\sqrt{\rho_s\mu_s\left(\frac{du_e}{dx}\right)_s}\left[1+(Le^{\alpha}-1)\frac{h_D}{h_s}\right](h_s-h_w)$$

where $\left(\frac{du_e}{dx}\right)_s$ is the velocity gradient at the edge of boundary layer, and equals to

$$\frac{1}{2R_0}\sqrt{\frac{2(P_s-P_\infty)}{\rho_s}} \quad , \quad \text{if } Ma_\infty > 1.12$$

$$\frac{V_\infty}{R_0}(2.0-0.872Ma_\infty^2-0.328Ma_\infty^4) \quad , \quad \text{if } Ma_\infty \le 1.12$$

As above shown, conventional analytical method of manual deduction is difficult to extracting laws of aerodynamic heating. Fortunately, artificial intelligence (AI), including genetic programming [8], big data algorithm, deep learning, has made considerable progress in recent years, and has been widely concerned and applied [2,3,4,7,13,14,19]. In near future, AI as an extension of human brain will be an inevitable trend. However, general AI are not suitable for learning the wind tunnel / flight test data. In fact, compared with the big data obtained through the Internet (large amount, timeliness and weak correlation), the data of wind tunnel / flight test can be regarded as a small data with strong correlation characteristics [17]. On the one hand, it is difficult to carry out a large number of experiments to obtain massive data for each model. On the other hand, the effects of the parameters on the heat flow are almost deterministic. Furthermore, even if a general AI method (e.g., deep learning) could give an accurate model, the neural network is still a gray box, which involves a large number of coefficients. It cannot give a concise, intuitive, easy to understand law of aerodynamic heating.

In this work, a special AI method, knowledge based learning (KBL), is proposed. In KBL, the wind-tunnel data to be learned are preconditioned and enriched with expert knowledge and numerical simulation of high fidelity, and then a special genetic programming algorithm is developed to detect the underlying universal laws of aerodynamic heating. The proposed method has been applied to two sets of aerodynamic heating data collected from different wind tunnels. One is of sphere heat transfer, and the other is of a double ellipsoid configuration. Study shows that the KBL can discovered concise formulas and is promising for detecting more universal laws of aerodynamic heating.

Knowledge based learning

In order to detect universal laws of aerodynamic heating using the wind tunnel/flight test data, both aspects, regarding to data and algorithm, respectively, must be customized. On one hand, the wind tunnel/flight test data are parameterized, localized, and standardized by CFD, and enriched and transformed using expert knowledge. On the other hand, general AI algorithm is specialized to enhance the ability of nonlinear function evolution, dimension analysis, and interval analysis. Then, the specialized AI algorithm will be used to detect universal laws of aerodynamic heating using the wind tunnel/flight test data as its training, test, and validation sets. The sketch could be illustrated as Figure 1.

Data preconditioning

Usually, the information of the heat flux data from wind tunnel tests at different measurement points is very brief. The test condition parameters involve only the total temperature, the total pressure and the Mach number. In this way, not only the heat flux data of different models cannot be compared, but also the data of the same model under different wind tunnels cannot be directly compared and correlated. For example, it is usual that although the Mach number of test flows in two wind tunnels is the same, the unit Reynolds number may be quite different. In this work, CFD simulation is applied to identify the local parameter near the measurement points (i.e., the parameters on the edge of boundary layer). And then the raw data is extended with the knowledge of high temperature gas dynamics. For example, the original data (q_w, P_0, T_0, M_∞, α, ...) could be extend to (qw, Cq, St, St_e; P, T, v, Cp, Entropy, gamma, h, k, Levis, mu, nu, pr, rou, Schmidt, c, thermalDiff, M, Re, Re_x).

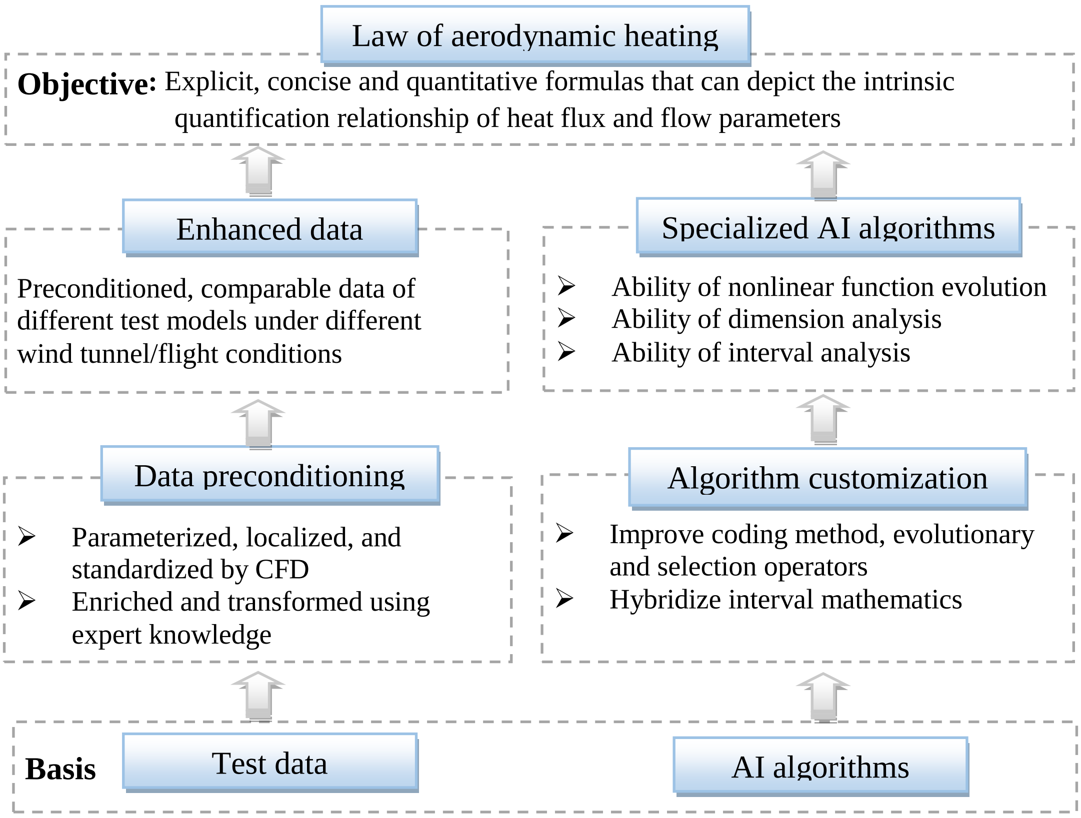


Figure 1. Sketch of knowledge based learning (KBL)

Note that the aim of CFD simulation here is to obtain the macroscopic structure of flow field and the local parameters of boundary layer outer edge, where the CFD model could be laminar, or turbulent, the real gas effect might be considered or ignored, according to different cases. Small errors are supposed to be "commonplace", since the AI algorithm to be applied is expect to be capable of handling noise data within some tolerance error. The purpose is different from both direct CFD simulation of aerodynamic heating and the CFD simulation in engineering methods. In fact, the CFD simulation in engineering methods should be laminar to save the computational cost, and the direct CFD simulation of aerodynamic heating is not only computationally intensive but also has high requirements for Y+ and orthogonality on mesh grid. Determining the macroscopic structure of flow field and the local parameters of boundary layer outer edge needs much less CPU time and requires much less on mesh grid. In fact, during the CFD iteration process, after the macroscopic structure of flow field become steady, the flow field inside the boundary layer still needs a lot of computation time to get the right heat flux.

Algorithm customization

According to the data characteristics of aerodynamic heating, it is necessary to improve the existing artificial intelligence (AI) algorithms in order to carry out intelligent learning. In this work, general AI algorithm is specialized to enhance the ability of nonlinear function evolution, dimension analysis, and interval analysis. In fact, the aerodynamic heat flux is a gradient quality and has strong nonlinearity, which requires the AI algorithm used to have a good nonlinear function evolution ability. Meanwhile, the aerodynamic heating data are strongly correlated. The expression of the quantitative relation that reflects the aerodynamic heating law should be dimensional compatible, so the algorithm needs a certain ability of dimensional analysis. Finally, the interval analysis ability of the algorithm is also obligatory. This can not only help the stability of the algorithm, but also give the confidence interval of the result. The intelligent learning algorithm, which has the capability of nonlinear function evolution, dimension analysis and interval analysis, is called specialized AI algorithm.

Preliminary applications

Two sets of aerodynamic heating data collected from different wind tunnels are selected to demonstrate the usage of KBL method. One is of sphere heat transfer, and the other is of a double ellipsoid configuration.

Formulas learned for sphere heat transfer

The data set consists 530 measurement points distributed on the windward side of a sphere of radius 0.1 meters. The data are collected under the test conditions of two wind tunnels of different freestream flows, JF-10 for real-gas flow and JF-12 for ideal-gas flow. The original data set $\left\{(\mathrm{q_w}^{(i)};\mathrm{T_0}^{(i)},\mathrm{P_0}^{(i)},\mathrm{M}^{(i)})\right\}_{i=1}^{530}$ is parameterized, localized, and standardized using inviscid CFD simulations, and extended using expert knowledge of high temperature gas dynamics. The enhanced data set is as follows.

$$\left\{\begin{matrix}(qw,\ Cq,\ St,\ St_e;\ P,\ T,\ v,\ Cp,\ Entropy,\ gamma,\ h,\ k,\ Levis,\ mu,\ nu,\ pr,\ rou,\ Schmidt,\\ c,\ thermalDiff,\ M,\ Re,\ Re_x)^{(i)}\end{matrix}\right\}_{i=1}^{530}$$

The specialized AI algorithm is then applied to automatically search the best formula to fit the data, and balance the fitting error, formula complexity and stability. Mean Absolute Error (MAE), Pearson's correlation coefficient r and the coefficient $1-R^2$ are applied to measure the goodness of a fitting formula, where $1-R^2=\frac{SSE}{SST}$. SSE is the Error Sum of Squares of observed and predicted values ($\sum_i(\mathrm{q}_i^o-q_i^p)^2$), and SST is the Total Sum of Squares of observed and average values ($\sum_i(q_i^o-\overline{q})^2$).

Table 1. Aerodynamic Heating Formulas of sphere with boundary layer parameters

Complexity	formula	MAE	r	1-R^2
3	$q_w=k\cdot P_e$	1.4E5	0.99	0.04
9	$q_w=k\cdot T_e\cdot\sqrt{P_e}$	5.3E4	0.997	0.006
17	$q_w=k\cdot(T_e)^{\frac{5}{4}}\cdot\sqrt{P_e}$	3.9E4	0.997	0.006

From Table 1, we can see that the pressure at the edge of boundary layer P_e is a key parameters for aerodynamic heating, and the heat flux is almost linear to the square root of the pressure, i.e., $q_w:\ \sqrt{P_e}$. The results indicate that the thickness of boundary layer is also linear to the square root of the pressure, i.e., $\delta:\ \sqrt{P_e}$. This result might be helpful to study the development of boundary layer.

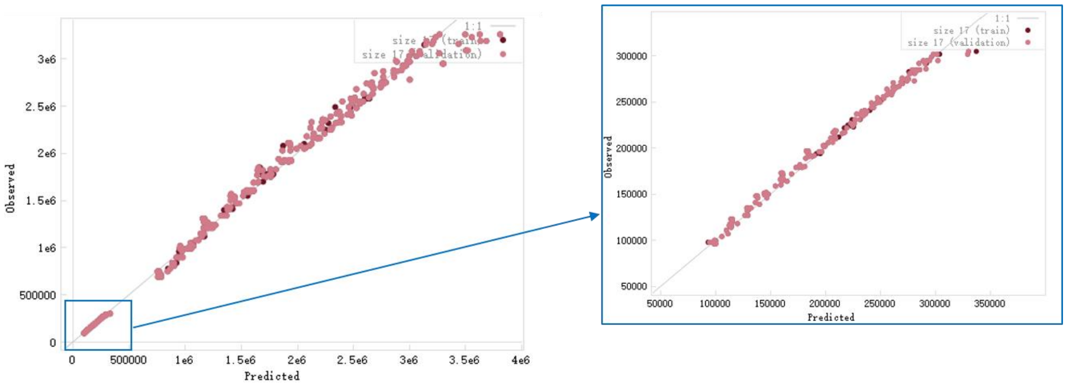
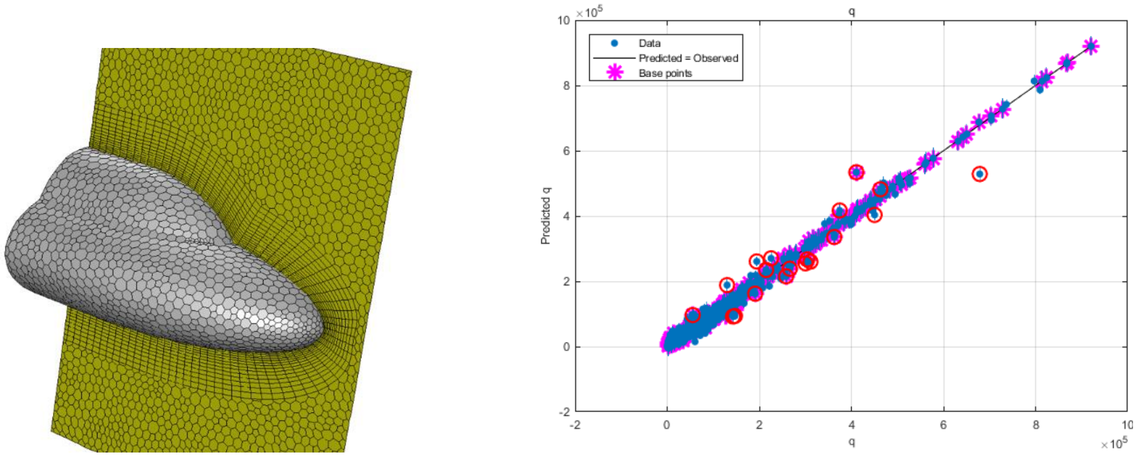


Figure 2. **Comparison of observed and predicted heat flux of the sphere** under the conditions of two wind tunnels (with $q_w = k\cdot(T_e)^{\frac{5}{4}}\cdot\sqrt{P_e}$), JF-10 and JF-12. The left figure shows the overall results, and the right figure is the local zoom of the left, results of JF-12 with smaller heat flux

Preliminary results of a double ellipsoid configuration

The data set consists 1044 measurement points distributed on both windward and leeward side of a double ellipsoid model of length 0.215 meters (see Fig. 3(a)). The data are collected from two wind tunnels, FD-14A and FD-20, of several tests [12]. For model and experimental parameter details, please refer to reference [12]. Similarly, the original data set is parameterized, localized, and standardized using CFD simulations (Fig. 3(a)), and extended using expert knowledge of high temperature gas dynamics. An enhanced data set is obtained. Then the specialized AI algorithm is applied to search the best formula automatically to fit the data, and balance the fitting error, formula complexity and stability. The study shows that the heat flux could be quantitatively determined by the air's temperature, pressure, speed at the outer edge of boundary layers, i.e., $q_w = f(\mathrm{T_e}, \mathrm{P_e}, v_\mathrm{e})$. The learning results are shown in Fig. 3(b). The Pearson's correlation coefficient r is 0.98 and the coefficient $1-R^2$=0.03. The explicit expression is omitted here since it is very complicated at this stage due to the limitations of current AI algorithm. The improvement of AI algorithm and the simplification of the expression are left for future research.



(a) Mesh grid of the test model (b) Comparison of observed and predicted heat flux

Figure 3. KBL of a double ellipsoid configuration

Conclusions

An intelligent optimization method, knowledge based learning (KBL), has been presented to extract formulas of aerodynamic heating. In KBL, the wind-tunnel data to be learned are regarded as a small data with strong correlation characteristics. They are preconditioned and enriched with expert knowledge and numerical simulations. Then the enhanced data are used to train and validate the candidate formulas in the guide of a special artificial intelligence (AI) algorithm, which is adapted from general AI algorithm to fit the characteristics of aerodynamic heating data. The proposed KBL method has been applied to two data sets of aerodynamic heating from different wind tunnels of different models. The study indicate that the KBL method is promising for detecting more universal laws of aerodynamic heating.

The basic idea of KBL is using specialized AI algorithm to replace conventional analytical method of manual deduction. This does make sense, and will be an inevitable trend in the near future. However, current AI algorithm still needs improving, and the data sets are not merged together. Furthermore, to get a formula that really matters, the training data set needs more data from more wind tunnels of more test models. These tasks are left for future studies.

References

[1] Anderson, J. Jr. (2006), *Hypersonic and High-Temperature Gas Dynamics*, 2nd edn, American Institute of Aeronautics and Astronautics, Inc., Reston, Virginia.

[2] Brunton, S. L., Proctor, J. L. and Kutz, J. N. (2016) Discovering governing equations from data by sparse identification of nonlinear dynamical systems, *Proceedings of the National Academy of Sciences of the United States of America*, 113(15), 3932-3937.

[3] Chen, C., Luo*, C., Jiang, Z. (2018) Block building programming for symbolic regression, *Neurocomputing* 275 (31), 1973-1980.

[4] De Jesus, and Hagan, M. T. (2007), Back propagation algorithms for a broad class of dynamic networks, *IEEE Transactions on Neural Networks* 18(1), 14-27.

[5] Eckert, R. G. (1955), Engineering Relations for Friction and Heat Transfer to Surface in High Velocity Flow, *Journal of the Aeronautical Sciences* 22(8), 585-587.

[6] Fay, J. A. and Riddell, F. R. (1958), Theory of Stagnation Point Heat Transfer in Dissociated Air, *Journal of the Aeronautical Sciences* 25(2), 78-85.

[7] Horn, J. M., De Jesus, O.and Hagan, M. T. (2009), Spurious Valleys in the Error Surface of Recurrent Networks - Analysis and Avoidance, *IEEE Transactions on Neural Networks* 20(4), 686-700.

[8] Koza, J. R. (1992) *Genetic programming: on the programming of computers by means of natural selection*, MIT Press, Cambridge, Massachusetts.

[9] Lee, J.-H., Kim, E. T., Chang, B.-H., Hwang, I.-H., and Lee, D.-S. (2009) The accuracy of the flight derivative estimates derived from flight data, *International Journal of Aerospace and Mechanical Engineering* 3(10), 1317-1323.

[10] Lees, I. (1956) Laminar Heat Transfer over Blunt Nosed Bodies at Hypersonic Flight Speeds, *Jet Propulsion* 26 (4), 259-269.

[11] Lesin, A. B. and Lunev, V. V. (1994). Heat transfer peaks on a blunt-nosed triangular plate in hypersonic flow, *Fluid Dynamics* 29(2), 258-262.

[12] Li, S. X. (2007) *Hypersonic flow characteristics around typical configurations*, National Defence Industry Press, Beijing. (in Chinese)

[13] Luo, C., Zhang, S.-L. (2012) Parse-matrix evolution for symbolic regression, *Engineering Applications of Artificial Intelligence* 25, 1182-1193.

[14] Luo, C., Hu, Z., Zhang, S.-L., Jiang, Z. (2015) Adaptive space transformation: An invariant based method

for predicting aerodynamic coefficients of hypersonic vehicles, *Engineering Applications of Artificial Intelligence* 46, 93-103.

[15] Mason, E. A., and Saxena, S. C. (1958). Approximate Formula for the Thermal Conductivity of Gas Mixtures, *Physics of Fluids* 1(5), 361-369.

[16] Morelli, E. A., DeLoach, R. (2003) *Wind tunnel database development using modern experiment design and multivariate orthogonal functions*, AIAA paper, 2003-0653.

[17] Nicoli, A., Imperatore, B., Marini, M., Catalano, P., Pizzicaroli, A., and Perigo, D. (2006) *Ground-to-flight extrapolation of the aerodynamic coefficients of the VEGA launcher*, AIAA paper, 2006-3829.

[18] Pulliam, T. H. (1986) Artificial dissipation models for the Euler equations, *AIAA Journal* 24(12), 1931-1940.

[19] Schmidt, M., and Lipson, H. (2009). Distilling free-form natural laws from experimental data. *Science* 324, 81-85.

[20] Vaglio L. R. (1960) Turbulent Heat Transfer on Blunt Nosed Bodies in Two-dimensional and General Three-dimensional Hypersonic Flow, *Journal of the Aerospace Sciences* 27(1): 27-38.

[21] Vlasov, V. I., and Gorshkov, A. B. (2001) Comparison of the Calculated Results for Hypersonic Flow Past Blunt Bodies with the OREX Flight Test Data, *Fluid Dynamics* 36(5), 812-819.

[22] Vlasov, V. I., Gorshkov, A. B., Kovalev, R. V. , and Lunev, V.V. (2009) Thin triangular blunt-nosed plate in a viscous hypersonic flow, *Fluid Dynamics* 44(4), 596-605.