Boundary Element Method and Statistical Linearization solution of nonlinear fractional diffusion equations

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Abstract

Nonlinear fractional diffusion equations are used for describing the evolution of several natural phenomena, such as the systems where particles move randomly in a space while exposed to the action of a random excitation allowing long jumps [1]. In this context, the problem is described by the anomalous diffusion equation. The main feature of this class of equations is the presence of the fractional Laplacian, which is an operator generalizing the classical Laplacian operator [2], and the inclusion of nonlinear terms rendering the estimation of the system response a daunting task.
In this paper, two approaches for determining the response of this system are described: a numerical approach based on the Boundary Element Method (BEM); and an approximate analytical approach based on the Statistical Linearization (SL) approach. The BEM method is used for estimating numerically the system response in the time domain. It is implemented by utilizing the integral representation of the classical Poisson equation solution, whose unknown, time-dependent, constants are determined by the BEM approach at every time instant. The SL approach is proposed for estimating an approximate analytical solution of the system response. In this context, the approximate response is represented via mode expansion of the linear diffusion problem, which allows deriving a system of nonlinear differential equations which is then replaced by a linear system equivalent to the nonlinear one in a mean square sense. Relevant numerical results show that the approximate solution is in agreement with the numerical approach.

Keywords: **anomalous diffusion, fractional laplacian, BEM, statistical linearization**

References

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