Anomalous Diffusion in Periodic Phononic Crystals

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Abstract

In recent years, several theoretical and experimental studies have shown that transport mechanisms in complex and inhomogeneous media are not always governed by purely propagating or diffusive processes. Several examples are available in different areas of applications that show evidence of either anomalous or hybrid transport regime. As an example, the occurrence of anomalous transport mechanisms has been observed in solute transport in soil and porous materials, in polymer translocation, in the movement of lipids on model membranes, and in diffusion of light in random media. These phenomena are generally governed by non-Fourier or non-Fickian diffusion models and are characterized by heavy-tailed distributions. On the other hand, hybrid transport mechanisms (that involve simultaneous propagation and diffusion) have been observed in the propagation of electromagnetic or acoustic waves traveling through inhomogeneous media such as engineered material (e.g. metamaterials), soil, stratified fluids, and porous materials.
The theoretical and analytical modeling of the phenomena mentioned above share an important common limitation: hybrid transport processes cannot be accurately described by integer order continuum models that typically account only for a single transport modality. On the other hand, direct numerical simulations can provide some insight but become soon impractical due to the size of the computational problem and to the coexistence of multiple spatial and temporal scales.

More recently, anomalous transport has also been observed in disordered media and metamaterials. In these studies, several unconventional regimes of propagation that deviate from classical diffusion have been identified. These regimes include sub-diffusion (typically related to Anderson localization) or super-diffusion (typically related to Lévy-flights).

In the present study, we investigate the propagation of sound in perfectly periodic acoustic metamaterial composed of sound-hard cylinders in air. Metamaterials represent a perfect example of inhomogeneous media in which the transport of the acoustic field encounters a variety of non-conventional mechanisms characteristic of many other processes. In particular, we use a combination of deterministic and stochastic models to explore theoretically and numerically the contributions of these non-conventional mechanisms to the overall sound transport process. This study reveals that anomalous transport mechanisms, can originate even in perfectly periodic media (that is in the absence of disorder and/or random properties), in case of anisotropic bandgaps. The analysis of the field quantities, specifically the acoustic intensity, revealed a key aspect associated with the anomalous transport, i.e. the occurrence of heavy-tailed distributions. These distributions are also indicator of fractional dynamic behavior.

In relatively recent years, fractional differential calculus has provided a rigorous foundation for the development of mathematical models able to capture anomalous transport mechanisms as well as a way to capture the progressive transition between different transport modalities. The results of our work suggest that also systems that are not intrinsically fractional (like, for example, viscoelastic or fractal systems) may exhibit fractional dynamics due to their inhomogeneous nature. These effects may stem from self-similarities of geometric properties such as those typical of metamaterials, or spatial distribution of materials properties such as the index of refraction. Numerical results will show that the acoustic diffusive response characteristic of highly scattering periodic media can be effectively described by fractional diffusion equations whose fractional order can be extracted from an analysis of the heavy tails of the acoustic intensity distribution.

Keywords: **Anomalous diffusion, Hybrid transport, Phononic Crystals, Fractional Continuum Models**