Dynamics of mistuned rotor - analytical and numerical calculations

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Abstract

This study considers forced vibrations of the rotor system. The structure under consideration consist of a rigid hub and three straight elastic composite beams oscillating in the rotor plane. In the performed analysis it is assumed one or two blades of the system is de-tined due to slight differences in their geometry comparing to a nominal design. Therefore, the different synchronisation phenomenon scenarios are observed depending on the de-tuning magnitude and mutual ratios. The studies are performed by numerical and analytical methods, namely by direct simulations and harmonic balans method. The obtained results of these two approaches showed a very good agreement.

Keywords: **rotating structure, synchronisation phenomenon, composite beams, de-tuned rotor**

1. Model and equations of motion

The rotor model comprises three slender straight elastic beams made of composite material and clamped to the rigid hub of radius R_0 and inetria J_h (see Fig. 1). The hub is allowed to oscillate or rotate about a vertical axis CZ_0 and its temporary position is represented by an angle $\psi(t)$ measured with respect to an inertial reference frame (X_0, Y_0, Z_0). Also the external torque T is applied to the hub, which is excitation of a whole rotor. Each beam is described by length (l_i), width (d_i) and thickness (h_i) of cross-section, where $i = 1, 2, 3$. As it was mention before, one assumes that one or two of the beams are detuned by geometric differences.

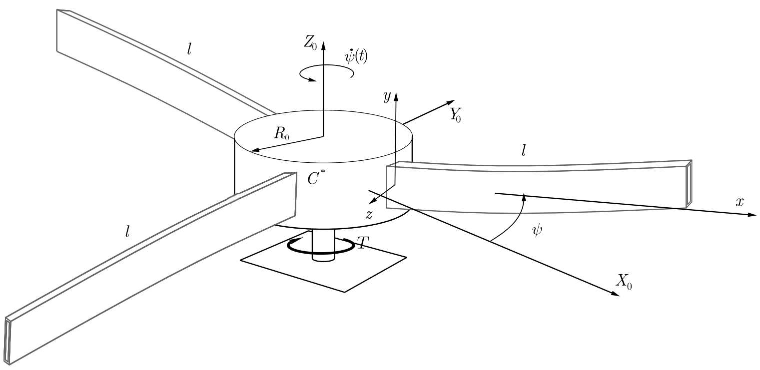

Figure 1: Model of the rotating hub with three clamped elastic beams

The Hamilton's principle of least action and Galerkin's projection method have been used to get the set of ordinary differential governing equations [2]:

$$
\begin{aligned}
&(J_\mathrm{h}+J_\mathrm{b1}+J_\mathrm{b2}+J_\mathrm{b3}+\alpha_\mathrm{h12}q_1^2+\alpha_\mathrm{h22}q_2^2+\alpha_\mathrm{h32}q_3^2)\ddot{\psi}\\
&+\alpha_\mathrm{h11}\ddot{q}_1+\alpha_\mathrm{h21}\ddot{q}_2+\alpha_\mathrm{h31}\ddot{q}_3+\alpha_\mathrm{h13}q_1\dot{q}_1\dot{\psi}+\alpha_\mathrm{h23}q_2\dot{q}_2\dot{\psi}+\alpha_\mathrm{h33}q_3\dot{q}_3\dot{\psi}+\zeta_\mathrm{h}\dot{\psi}=\mu(t)\\
&\ddot{q}_1+\alpha_{12}\ddot{\psi}+\alpha_{14}q_1\dot{q}_1\dot{\psi}+(\alpha_{11}+\alpha_{13}\dot{\psi}^2)q_1+\zeta_1\dot{q}_1=0\\
&\ddot{q}_2+\alpha_{22}\ddot{\psi}+\alpha_{24}q_2\dot{q}_2\dot{\psi}+(\alpha_{21}+\alpha_{23}\dot{\psi}^2)q_2+\zeta_2\dot{q}_2=0\\
&\ddot{q}_3+\alpha_{32}\ddot{\psi}+\alpha_{34}q_3\dot{q}_3\dot{\psi}+(\alpha_{31}+\alpha_{33}\dot{\psi}^2)q_3+\zeta_3\dot{q}_3=0
\end{aligned}
\tag{1}
$$

The generalized coordinates of the flexural-torsional deformation for each beam are denoted as q_i $(i=1,2,3)$. The viscous damping coefficients are arbitrary included and are represented by ζ_h and ζ_i for the hub and for each beam, respectively. All α_{ij} coefficients are the result of Galerkin's projection and have been calculated on the basis of the physical laboratory model.

First equation from Eq. (1) represents the dynamics of the hub, while the last three describe the motion of each individual beam. The terms J_h and $J_{\mathrm{b}i}$ are dimensionless mass moment of inertia of the hub and each blade, respectively. All of them are expressed as a magnitude of the inertia of the first beam. Furthermore, $\dot{\psi}$ is the dimensionless angular velocity of the hub. The term present on the right hand side of the first equation $\mu(t)$ is considered as a excitation of the rotor (external torque), expressed as an arbitrary periodic analytical function or chaotic input.

Based on the ordinary differential equations of motion (see Eq.(1)) analytical and numerical calculations have been performed. The harmonic balans method has been used to plot resonance curves, which present the dynamics of whole rotor. In the following step a series of numerical simulations in the Dynamics software have been done, like in the previous research of authors [1]. The analytical and numerical results have been compared. Moreover, time series plots have been prepared during the numerical simulations. Based of them the synchronisation phenomenon has been analysed.

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References

[1] Szmit Z., Warminski J. (2018) Synchronization phenomenon in the de-tuned rotor by regular or chaotic oscillations. *AIP Conference Proceedings* **(1922)**, 100013.

[2] Warminski J., Latalski J., Szmit Z. (2018) Vibration of a mistuned three-bladed rotor under regular and chaotic excitations. *Journal of Theoretical and Applied Mechanics* **56**(2), 549-566.