Analysis of the results obtained from the application of the two-stage method with calculations of some statically indeterminate trusses

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Abstract

The paper presents results of calculations of forces in members of selected types of statically indeterminate trusses carried out by application of the two-stage method of computations of such structural systems. The method makes possible to do the simple and approximate calculations of the complex trusses in two stages, in each of which is calculated a statically determinate truss being an appropriate counterpart of the basic form of the statically indeterminate truss structure. Systems of the statically determinate trusses considered in the both stages are defined by cancelation of members, number of which is equal to the statically indeterminacy of the basic truss. In the paper are presented outcomes obtained in the two-stage method applied for two different shapes of trusses and carried out for various ways of removing of appropriate members from the basic trusses. The results are compared with outcomes gained due to application of a suitable computer software for computation of the same types of trusses and for the same structural conditions.

Keywords: Truss, Calculus of vectors, Superposition method, Statically indeterminate system, Cremona's method, Approximate solution.

Introduction

Values of forces acting in members of the statically indeterminate systems are computed by the application of various methods like for instance, the force method, the displacement method, the iteration methods like the method of successive approximations, and the finite elements method, etc. Procedures of these methods are nowadays adapted in numerous and various types of computer software [1]-[7]. Concept of computational procedure used in the two-stage method is described in previous papers [8]-[10] and it is in simplified way shown in Fig. 1.

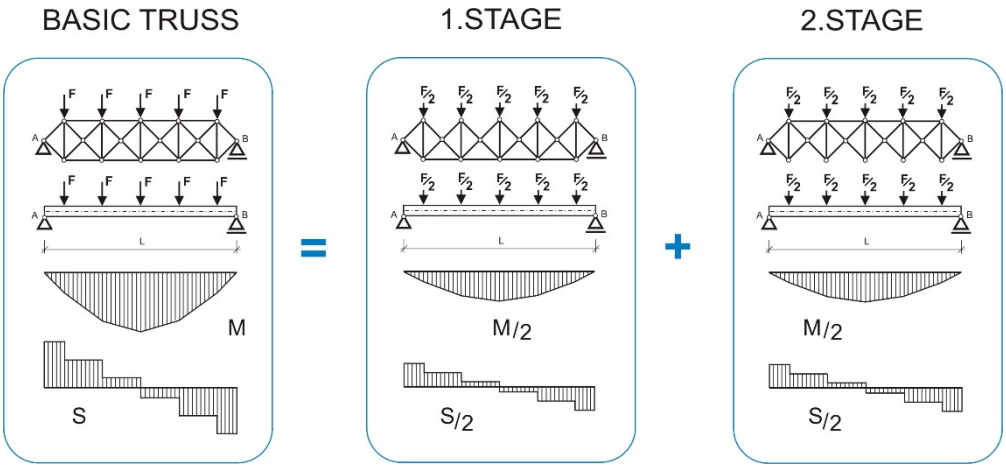

Figure 1. Concept of two-stage method of calculation of statically indeterminate trusses

The point of the two-stage method is to define shapes of the statically determinate trusses calculated in the intermediate stages, which shapes are appropriately compatible with the basic system. It is done after cancelation suitable members from the geometry of the basic truss, number of which equals the degree of statically indeterminacy of the basic truss system. Then in each stage of this method it is calculated the statically determinate truss loaded by forces having half values of the load forces applied to the basic truss. Final values of forces acting in a member of the basic truss is the resultant of forces calculated in both stages acting in the members having appropriate positions in the calculated trusses. Because in the two-stage method there are not taken into account differences between stiffness of members joined in the same node, that is why it is an approximate method of calculation of the statically indeterminate trusses.

The two-stage method uses in general the principles of geometry of strains as well as the static equilibrium regarding reactions and the generalized internal forces. These principles are based on the background of statics of the rigid body therefore they can be applied for solutions of statically determinate systems. Methods used for calculations of statically indeterminate systems apply concepts of virtual work [11]-[14] what causes, that they give the exact values of forces really acting in members of such complex structural systems.

Subject of static calculations and analyses

Although the two-stage method gives the approximate results but on basis of conclusions of previous research one can state that its accuracy is good enough for the engineering practice. In the initial analyses there have been considered simple forms of trusses similar to the geometry schemes shown in Fig. 1. These trusses have been mostly loaded by concentrated forces applied in symmetric way along their lengths only to the top chord or only to the bottom chord. Moreover in the first stage members of the top chord were canceled, while in the second stage members of the bottom chord were rejected. Subject of the current research refers to the same truss geometry but the investigated trusses are loaded in way presented in Fig. 1a and in Fig. 1d. Moreover it is assumed, that in each stage are canceled members at the same time form the top chord and from the bottom chord. There are defined following types of structural configurations of the calculated trusses.

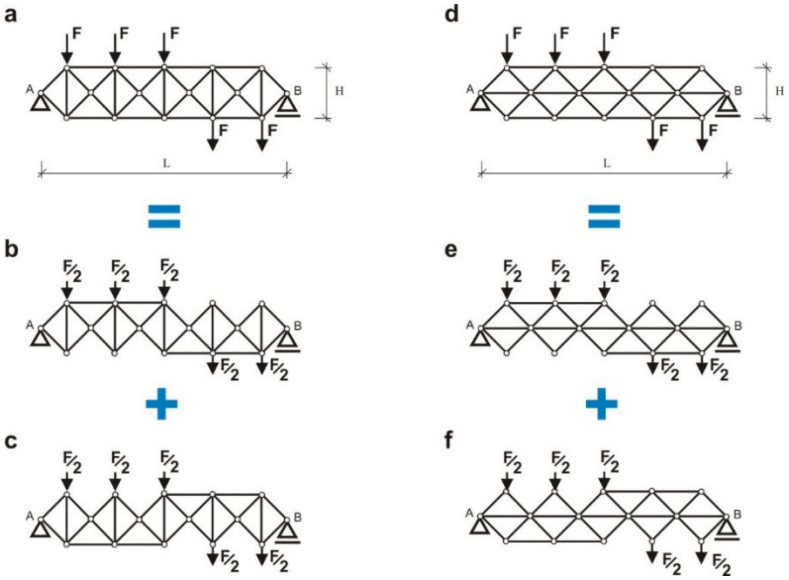

Figure 2. Basic static schemes of two groups of calculated trusses

If the truss has some vertical members and in each stage two members are excluded from the top and bottom chord then this structural configuration, shown in Fig. 2b and in Fig. 2c, is marked by symbol V 2/2. When the truss has in its chords horizontal members and like previously two members of external chords are deleted in each stage then that type of structural configuration is marked by symbol H 2/2, compare Fig. 2e and Fig. 2f. In this

research there are considered also other subtypes of such configurations, in which three members of the top chord are excluded and only one member of the bottom chord is rejected in suitable stages. These structural configurations are marked respectively by symbols V 3/1 and H 3/1. Various types of considered static schemes of the calculated trusses are assumed purposely in order to recognize more precisely the features of two-stage method and exactness of obtained results. Values of forces calculated in the two-stage method for the presented truss systems are compared with outcomes gained by application of a computer software for the same static and structural conditions for all the calculated truss systems. Basic structures are of 5.00 m clear span and their construction depth is equal to 1.00 m.

The both compared types of truss systems consists of the same number of members and nodes. Number of nodes is defined by symbol "w", while symbol "p" defines number of members. The condition for the inner statically determinacy of the plane truss is defined as follows:

$$p = 2 \cdot w - 3 \tag{1}$$

The considered truss systems are built by number of nodes w = 16, what implies that the statically determinate truss created by means of this number of nodes has to be constructed by the following number of members:

$$29 = 2 \cdot 16 - 3 \tag{2}$$

The basic truss systems are in each case built by the number of members p = 33, what means that the calculated structures are the fourfold statically indeterminate systems. It implies that in order to create the statically determined system it is necessary to delete 4 appropriate members from area of the basic truss. Static systems of considered trusses calculated according to the rules of the two-stage method in its stages, for the structural configuration marked by symbols V 2/2 and H 2/2, are shown in Fig. 2. Final values of the forces calculated for the both basic system will be resultants of forces defined in each stage for members of suitable positions. The concept of the assumed calculation method is compatible with rules of calculus of vectors, principle of superposition and with the three fundamental conditions of equilibrium presented below:

$$\sum_{i=1}^{n} F_{ix} = 0 \tag{3}$$

$$\sum_{i=1}^{n} F_{iy} = 0 \tag{4}$$

$$\sum_{i=1}^{n} M_{i} = 0 \tag{5}$$

It is assumed that the both basic trusses are subjected to the same type of load. In this investigation three concentrated forces are applied to the nodes of the top chord located in the close vicinity to the support node A, while remaining two concentrated forces are applied to the bottom chord located closer to the support node B, compare Fig. 2. It is assumed that the unit load forces F have value equal to 1.00 kN.

The same static systems of basic trusses have been subjected to the static calculation carried out by the application of the Autodesk Robot Structural Analysis Professional 2017. The computer software is used for the exact calculation of the force values acting in members of the statically indeterminate systems. Static calculations were made by assumption that the truss consists of steel tubular members having diameter of 30.00 mm, the thickness of the section equals to 4.00 mm and the steel material has the Young's modulus equal to 210 GPa.

Values of forces calculated for structural configurations V 2/2 and V 3/1

The truss system with vertical members is the subject of the first group of static calculations carried out by means of the two-stage method for two selected types of rejection of members, which was shortly discussed above. In the first case two members of are appropriately deleted from the top and from the bottom chords of the basic truss system. Results obtained in the first stage of calculations, together with Cremona's polygon of forces, are presented in Fig. 3.

Outcomes of second stage of computations are shown in Fig. 4. Final values of forces calculated for the truss configuration V 2/2 are presented in Fig. 7a.

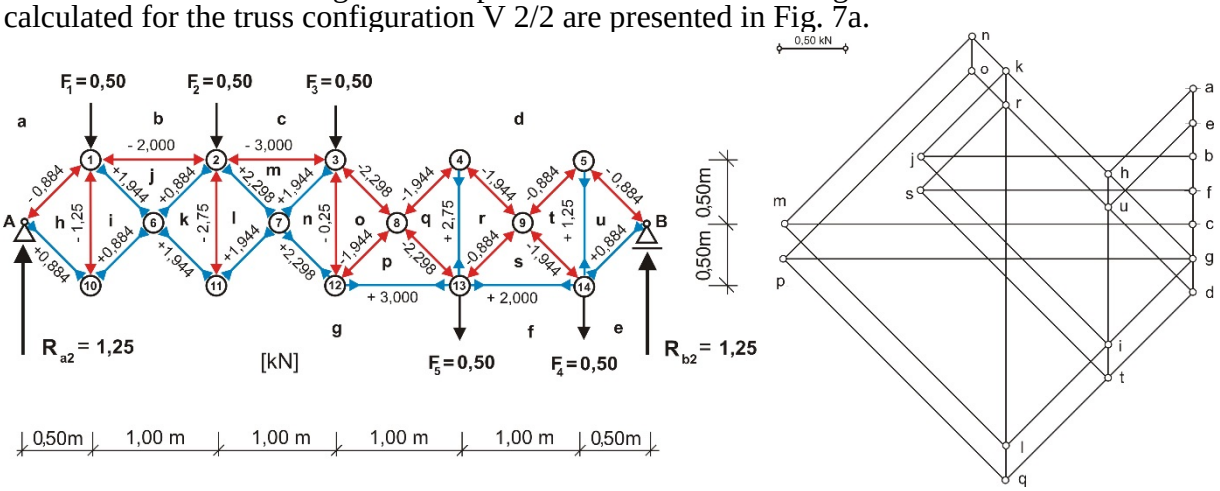

Figure 3. Values of forces determined in the first stage of calculations for the truss type configuration V 2/2 together with Cremona's polygon of forces

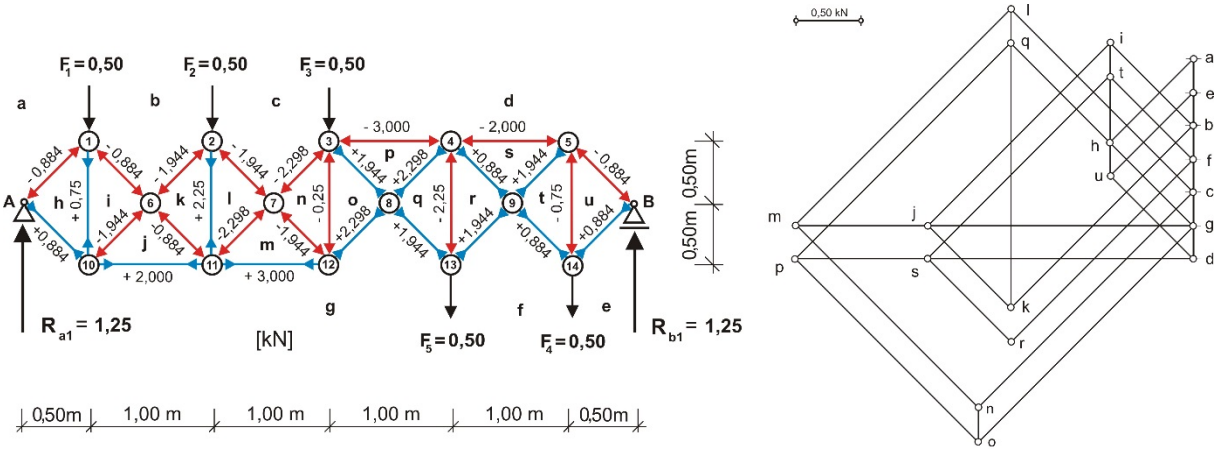

Figure 4. Values of forces defined in the second stage of calculations for the truss type configuration V 2/2 together with Cremona's polygon of forces

The same shape of truss system is now calculated in the two-stage method for structural configuration denoted by symbol V 3/1. Results of such calculations are shown in Fig. 5 and in Fig. 6. The final force values computed for this configuration are also presented in Fig. 7a.

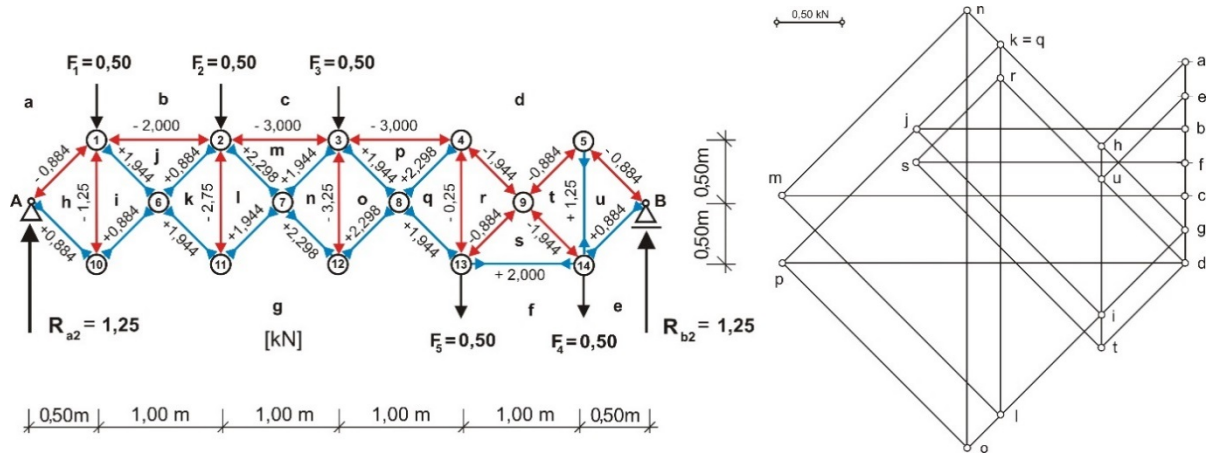

Figure 5. Values of forces determined in the first stage of calculations for the truss type configuration V 3/1 together with Cremona's polygon of forces

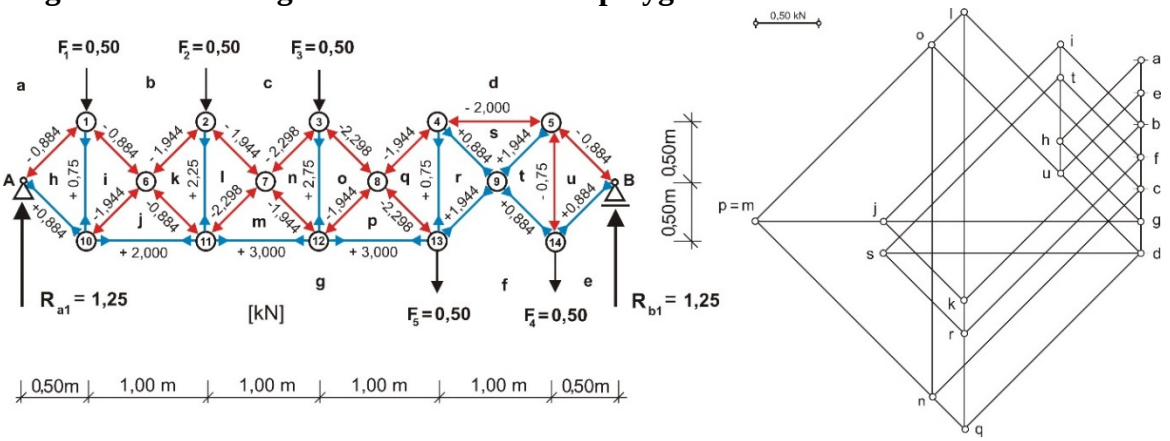

Figure 6. Values of forces determined in the second stage of calculations for the truss type configuration V 3/1 together with Cremona's polygon of forces

After application of computation procedures appropriate to the two-stage method values of forces defined in the same members of the basic statically indeterminate truss are of the same values, what does not depend on the considered types of configurations. It implies that they are exactly equal, when in process of calculation are removed 2 corresponding members from external chords, type configuration V 2/2, and when from the top chord are deleted 3 members and 1 member from the bottom chord, which configuration is marked by symbol V 3/1. Truss of the same static scheme, compare Fig. 1a, has been calculated by application of the computer software Autodesk Robot Structural Analysis Professional 2017. Outcomes of the computer calculations are presented in Fig. 7b.

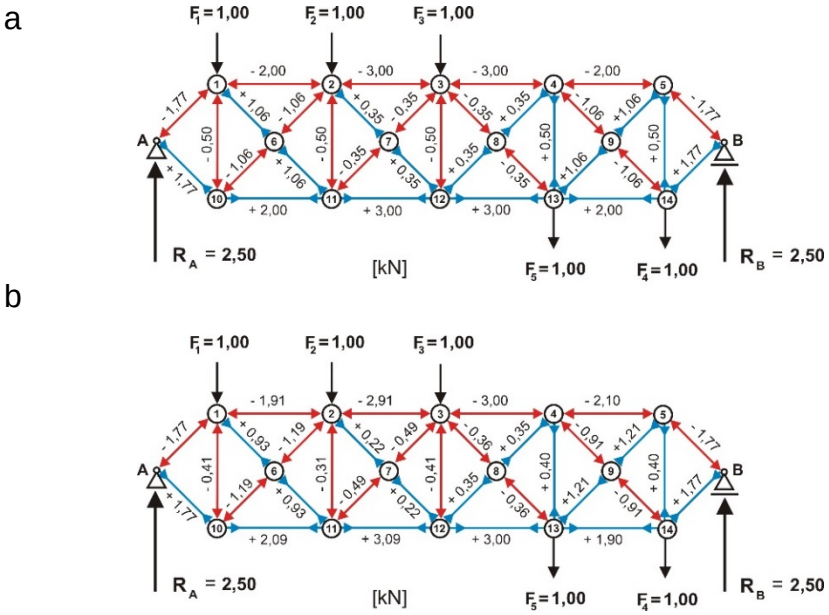

Figure 7. Values of forces in members for the truss type configuration V 2/2 together calculated, a) in the two-stage method, b) by means of computer software

Differentiations of the force values calculated in the both compared methods in members of external chords are rather small. For example the compression force defined in the two-stage method in member situated between nodes 4 and 5 equals -2.00 kN, while by application of the computer software it is equal to -2.10 kN, which constitutes about 5 % of the biggest value. Bigger differentiations one can notice between force values defined in vertical members and in the cross braces. For instance the force value calculated in the two-stage

method in vertical member located between nodes 2 and 11 equals -0.50 kN, while by the computer software it is equal to -0.31 kN. Similar bigger differentiations one can notice only between values of forces, calculated in the both methods, however having the very small absolute values.

Values of forces calculated for structural configurations H 2/2 and H 3/1

The second group of calculations has been carried out by application of the two-stage method for the statically indeterminate truss having static system shown in Fig. 2d. In Fig. 8 and in Fig. 9 are presented results gained for the truss schemes, where in both the stages are removed two members from external chords of the basic truss, truss configuration denoted H 2/2. Values of forces defined for configuration marked by symbol H 3/1 are shown in Fig. 10 and in Fig. 11.

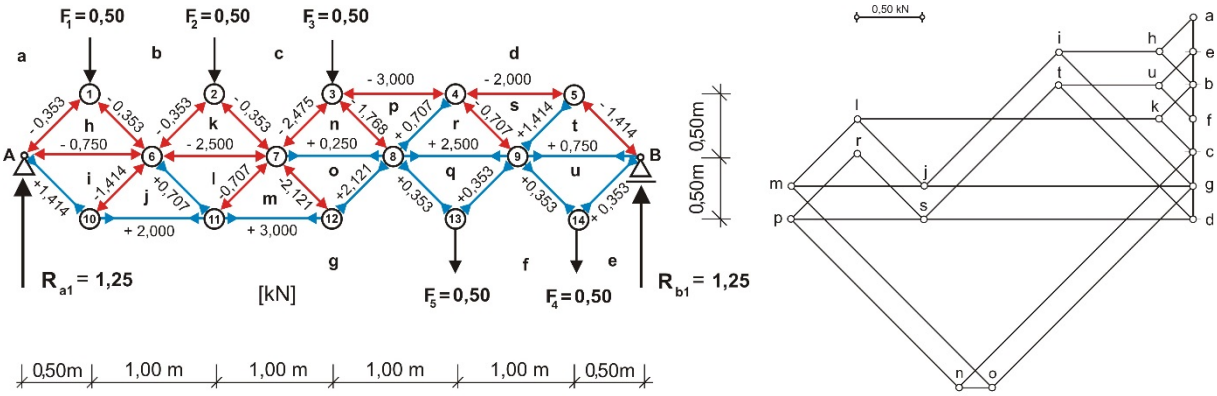

Figure 8. Values of forces determined in the first stage of calculations of the truss type configuration H 2/2 together with Cremona's polygon of forces

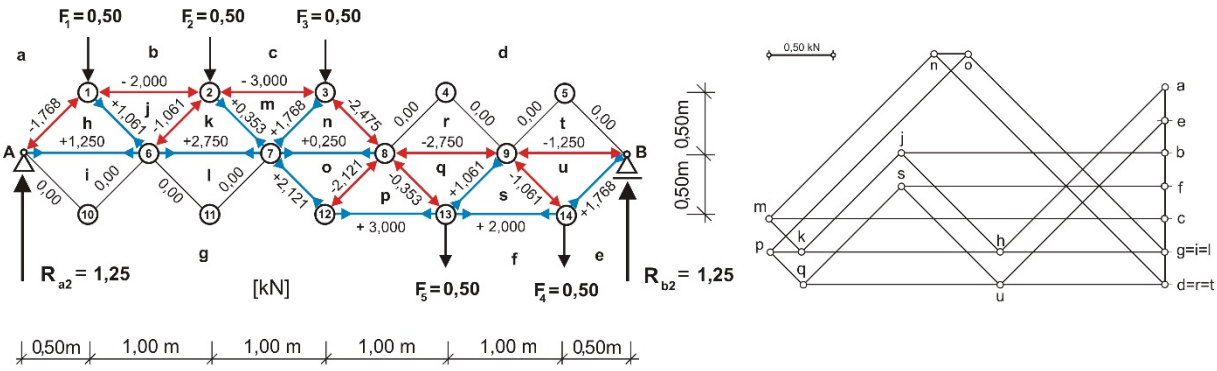

Figure 9. Values of forces determined in the second stage of calculations of the truss type configuration H 2/2 together with Cremona's polygon of forces

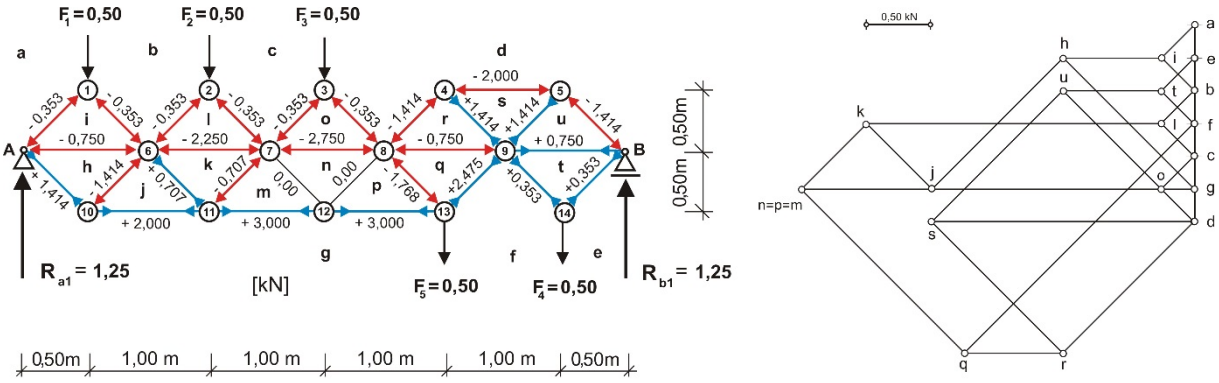

Figure 10. Values of forces determined in the first stage of calculations of the truss type configuration H 3/1 together with Cremona's polygon of forces

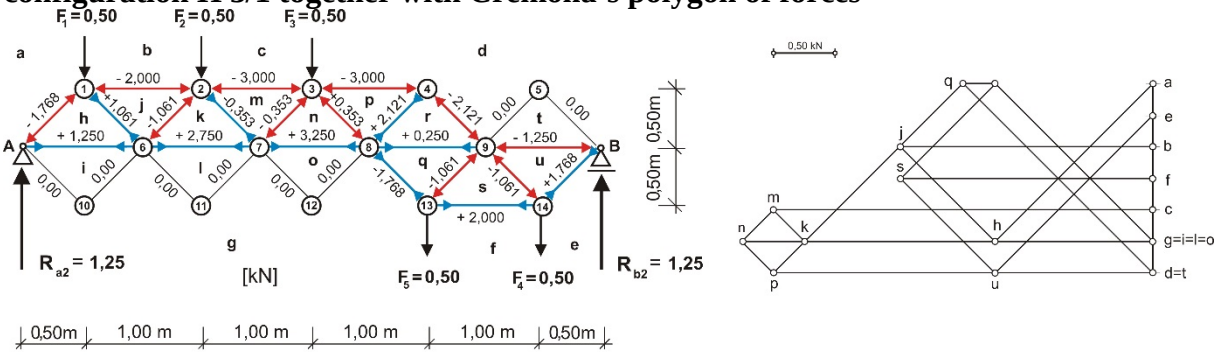

Figure 11. Values of forces determined in the second stage of calculations of the truss type configuration H 3/1 together with Cremona's polygon of forces

All final results of the calculated statically indeterminate truss are presented in Fig. 12. Values of forces defined in members of the basic truss by means of the two-stage method using procedure of cancelation of two members from each of the external chords, type of configuration H 2/2, are shown in Fig. 12a. Results obtained in this method for the procedure of excluding three appropriate members from the top chord and cancelation of single member from the bottom chord, type of configuration H 3/1, are shown in Fig. 12b. Values of forces determined by application of suitable computer software are presented in Fig. 12c.

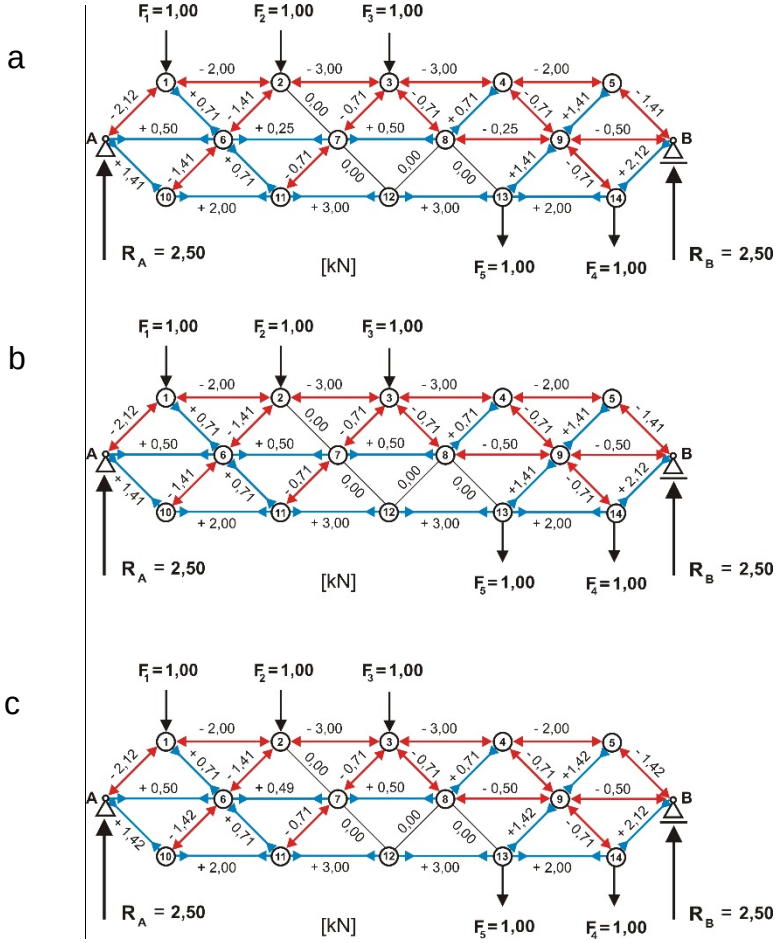

Figure 12. Values of forces defined in members of the H truss system by application of two-stage method, a) for the type configuration H 2/2, b) for the type configuration H 3/1, c) calculated by application of computer software

From analysis of all results obtained in compared calculations carried out in the two-stage method for various types of rejection of members from the outer chords follows, that values of forces defined in members of these chords and in cross braces of the basic truss are mostly of the same or of nearly the same values, compare Fig. 12a and Fig. 12b. Moreover they are also mostly of the same values of forces, which are calculated by application of the Autodesk Robot Structural Analysis Professional 2017, see Fig. 12c. Slightly bigger differentiation one can notice in values of forces defined in members of the middle chord. For instance value of force determined in member located between nodes 6 and 7 by application of the two-stage method equals +0.25 kN for the configuration H 2/2, see Fig. 12a, while in the same member for the configuration H 3/1 it is equal to +0.50 kN, see Fig. 12b. In general one can state, that results obtained in the two-stage method for the configuration H 3/1 are very similar to values of forces calculated in the basic statically truss by help of the computer software.

Results of all calculations, presented above, can testify the usefulness of the two-stage method for calculations of statically indeterminate trusses. It belongs to a group of recently developed methods of approximate solutions of such systems [15] invented on basis of various types of basic principles.

Conclusions

Values of forces obtained by application of the two-stage method for the calculation of the statically indeterminate trusses are approximated but mostly very similar to the force values determined by use of the exact methods applied in computer software. Bigger differences one can notice in values of very small forces, but they can be considered of small importance in the engineering practice. Accuracy of these results in a very small degree depends of the way

of the necessary cancellation of selected members from area of the basic truss in order to create the intermediate forms of statically determinate systems computed in the both stages. Other characteristics of the two-stage method will be subjects of the next research.

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