Two-stage method applied in calculations for statically indeterminate truss of larger span

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Abstract

The paper presents application of two-stage method to calculations of the force values acting in members of selected type of statically indeterminate truss having longer span than trusses considered in previous scientific articles. The calculations are carried out in two stages. The two-stage method applies principles of calculus of vectors as well as rules of superposition. At each stage of this method it is calculated a statically determinate truss system, shape of which is determined by reduction of the appropriate number of members from the basic statically indeterminate truss. The number of deleted members equals the statically indeterminacy of the basic truss. Thus in both stages there are calculated the statically determinate trusses. The final values of forces acting in the members of the basic truss are resultants of forces calculated for the counterpart members in each stage. Basic shape of the statically indeterminate truss has horizontal layer of members located in the middle of construction depth. The basic truss is subjected to symmetrical and nonsymmetrical type of the load. There are discussed differences between results obtained for these two types of the load and between force values calculated in the two-stage method and by application of a suitable computer software.

Keywords: Truss, Statically indeterminate system, Superposition method, Calculus of vectors, Cremona's method, Approximate solution.

Introduction

Truss systems are applied for a long time in the structures of roofs and floors. In comparison to bending beams of the same clear span and the same conditions of load the trusses are more lightweight. Members of the truss are connected together by means of theoretically articulated joints, due to which the members are subjected to act only of axial forces. The assumption that members are connected in articulated nodes is one of the basic requirements of the truss system. This assumption makes possible the application of principles of the calculus of vectors to the methods of calculation of values of forces in statically determinate trusses, like for instance the Cremona's method. Some analytical methods are used for this purpose, Ritter's method is one of them [1]-[4]. More complex trusses are mostly the statically indeterminate systems and they are also more efficient structures than the statically determinate trusses. In both systems the basic structural assumptions remain the same but due to their complexity, however the way of force distribution inside statically indeterminate trusses also depends on the stiffness of members joined in each particular node [5,6]. Therefore the processes for calculation of forces in members of these systems are also more complex. In such cases the distribution of forces in a single node depends, among others, on mutual ratios of the stiffness of members connected in it. There are numerous other factors having influence on force distribution between members especially in statically indeterminate systems, which have to be taken into consideration in various methods of the static calculations [7]-[11]. The degree of difficulty of the calculation procedure increases enormously with the growing number of members and nodes of the structure. It is especially important for the space trusses also called the space structures or the space frames [12]. At present the new numerical methods used into modern software make the calculation processes of complex statically indeterminate trusses very fast and enormously efficient. In certain types of the structural analysis it is not necessary to take into consideration the exact values of forces acting in the truss members.

Concept of the two-stage method and subject of calculations

The two-stage method was worked out during initial static analyses of a simple tension-strut truss, being initially a statically indeterminate system. If the basic tension-strut structure is overloaded some of its members are excluded from the force transmission and the remaining structure become a statically determinate system. The number of excluded tension members is equal to the statically indeterminacy of the basic truss system. Then the reduced structure can be calculated by one of very simple methods, for instance by help the Cremona's method. The new calculation procedure should keep rules of the calculus of vectors, together with principle of superposition and the below given basic conditions of equilibrium of coplanar force systems:

$$\sum_{i=1}^{n} F_{ix} = 0 \qquad (1)$$

$$\sum_{i=1}^{n} F_{iy} = 0 \qquad (2)$$

$$\sum_{i=1}^{n} M_{i} = 0 \qquad (3)$$

Calculations of the force values are carried out in two stages. In each stage the considered truss is shaped on pattern of the basic truss by removing of number of members equal to degree of statically indeterminacy of the basic truss. Moreover in each stage the load forces are of the half values and they have to be applied to the suitable nodes. Final values of forces acting in particular members are resultants of forces calculated in the both stages for appropriate members and having corresponding localizations into the truss area.

The point of the two-stage method is described in papers [13,14,15], where are presented results of calculations of simple forms of trusses having relatively very small clear spans. The paper presents outcomes obtained by application of the two-stage method to defining values of forces acting in the statically indeterminate truss of longer clear span than trusses calculated previously. The basic truss system is subjected to symmetrical and nonsymmetrical type of load. The concentrated load forces are applied to suitable nodes of the top chord. Static analysis of the assumed form of the basic truss is undertaken in order to estimate accuracy of the force values calculated in two-stage method for trusses of complex shapes and larger clear spans.

Methods, results of calculations and comparison analyses

Some characteristic results gained by the application of the two-stage method can be closely recognized after analyses of the force values calculated for truss of geometry shown in Fig. 1.

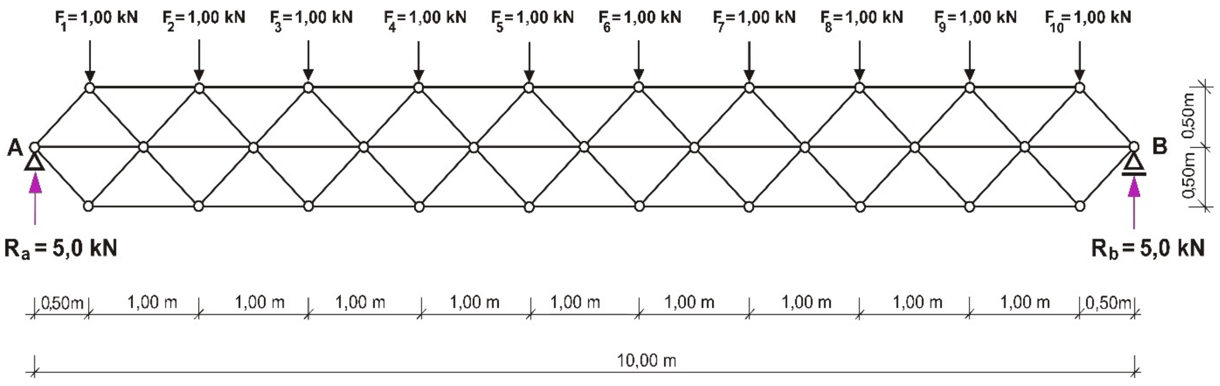


Figure 1. Static scheme of the basic truss subjected to symmetrical load

The basic truss has clear span of 10.00 meters, its construction depth equals 1.00 meter and it is loaded by concentrated forces, each of value equal to 1.00 kN. Here are consider the two ways of the force application. The first one is a symmetric loading, while concentrated forces are applied to each node of the upper chord. The second one is called asymmetric, when these forces will be applied to half the number of the upper chord nodes, only on one side of the truss. The basic truss is calculated by application of the two-stage method and these results are compared with outcomes gained by usage of the software Autodesk Robot Structural Analysis Professional 2017, designed for the precise calculation of force values in the statically indeterminate systems. It has been assumed that the truss consists of steel tubular members having diameter of 30.00 mm, the thickness of the section equals to 4.00 mm and the steel material has the Young's modulus equal to 210 GPa.

Analysis of basic truss loaded in symmetric way

The condition for the inner statically determinacy of the plane truss is as follows:

$$p = 2 \cdot w - 3 \tag{4}$$

where symbol "p" defines number of members, while "w" determines number of nodes. The considered shape of the truss system shown in Fig.1 is created by number of nodes w = 16, what implies that the statically determinate truss created by number of members p = 68, which are connected together by means of number of nodes w = 31. A statically determined truss consisting of this number of nodes has to be created by a number of members determined by the equation below:

$$59 = 2 \cdot 31 - 3 \tag{5}$$

It implies that the considered truss is the nine-fold indeterminate system (68 – 59 = 9). Therefore in each stage of the two-stage method one should remove nine appropriate members. In the first stage there were deleted nine members from the top chord of the basic truss. Because the truss is of symmetric form and moreover it is symmetrically loaded the calculation process can be limited only to half of the truss. The load forces are of halve values, they equal to 0.50 kN and they are applied to the top chord nodes. The values of forces calculated in the first stage by the application of Cremona's method is presented in Fig. 2.

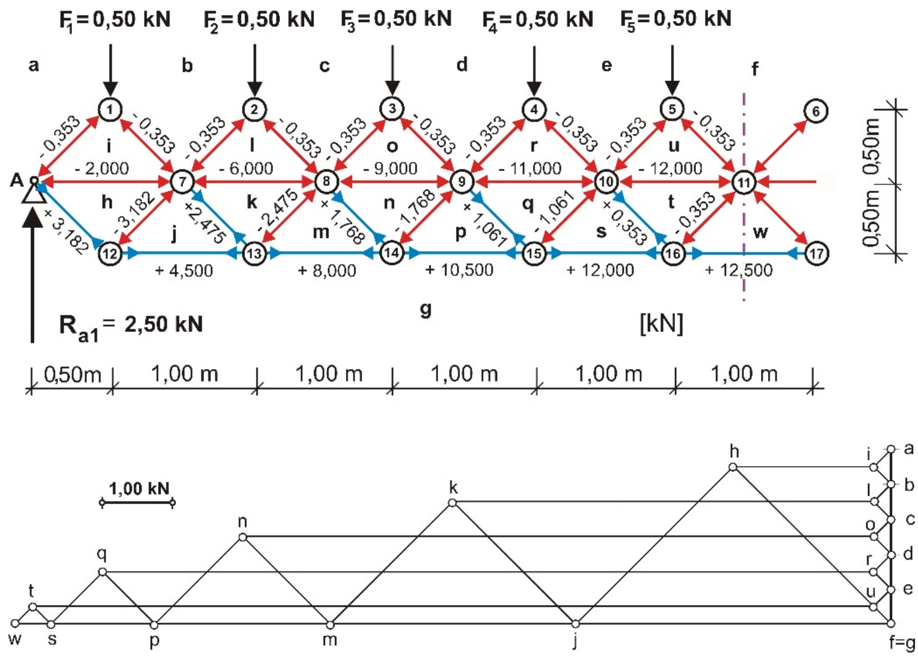
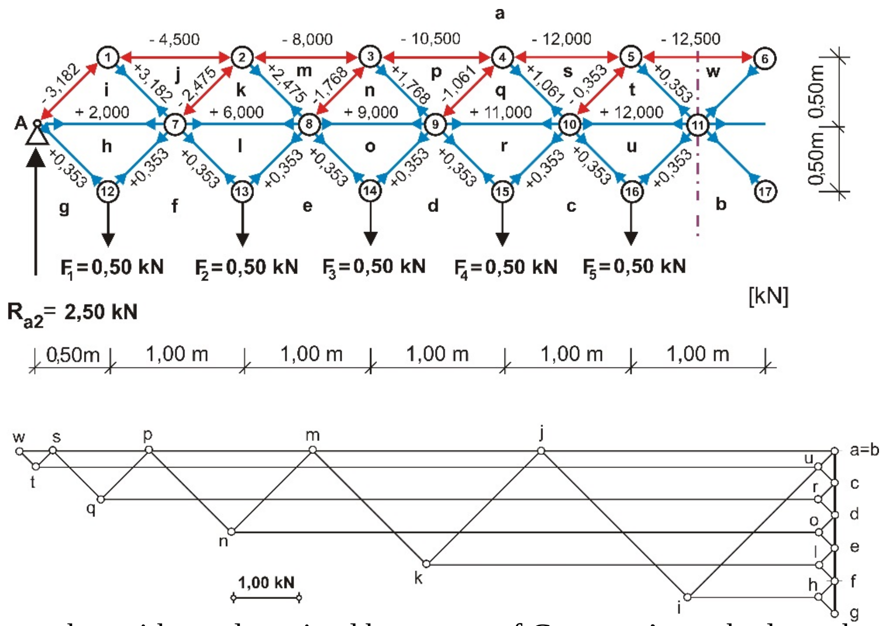


Figure 2. Results calculated in the first stage of calculations for the basic truss loaded in symmetric way together with Cremona's polygon of forces

According to previously explained rules in the second stage of this method one should eliminate nine members from the bottom chord of the basic truss. From analysis of the geometry of this truss and the general conditions of equilibrium it follows, that load forces of values equal 0.50 kN have to be applied to the bottom chord nodes. Scheme of the calculated



truss system together with results gained by means of Cremona's method are shown in Fig. 3.

Figure 3. Results calculated in second stage for basic truss loaded in symmetric way together with Cremona's polygon of forces

Resultant values of forces in members of the truss are calculated by the application of the two-stage method are shown in Fig. 4a. The results calculated for the same truss by the application of Autodesk Robot Structural Analysis Professional 2017 are presented in Fig. 4b.

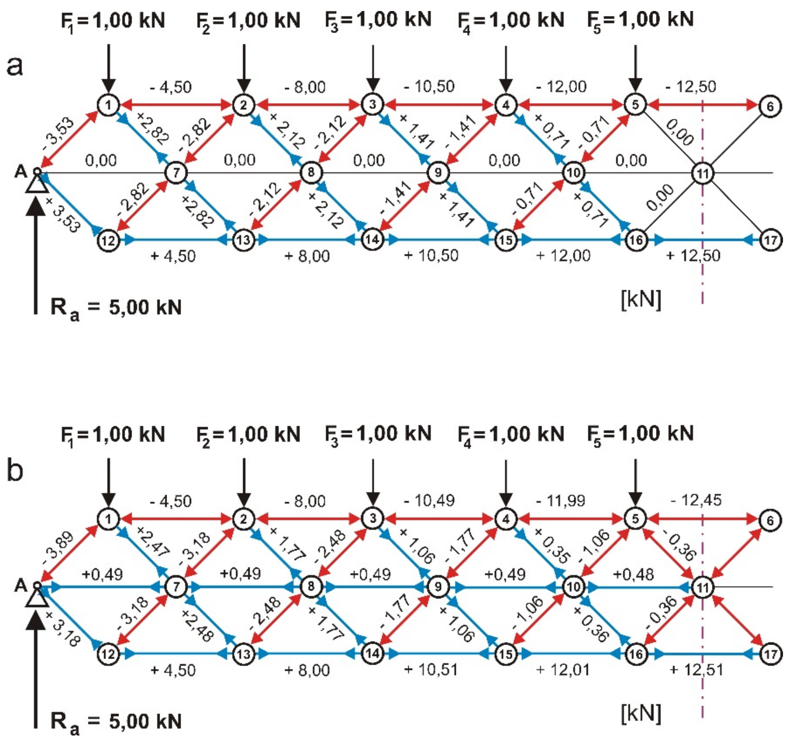


Figure 4. Final values of forces calculated for the symmetrical loaded truss, a) in the two-stage method, b) by application of computer software

Let us compare outcomes gained by usage of various methods for the same members. For instance, the value of the force acting in the member placed between node No 5 and node No 6 calculated in the first stage of the two-stage method equals zero, see Fig. 2, because it has been removed in this stage. In the second stage the value of the compression force is determined as -12.50 kN, which is why the final, resultant force value in this member is equal to -12.50 kN, see Fig. 4a. By application of the computer software the compression force value in member located between nodes No 5 and No 6 is defined as -12.45 kN, see Fig. 4b. The difference in relation to the smaller force value is rather small and it comes to 0.40 %. Bigger differences of the force values are noticed in other members like, for example, in the member connected to the support and placed between the support node A and No 1. In the first stage of calculations the compression force has value of -0.353 kN, see Fig. 2, while in the second stage its value is calculated as equal to -3.182 kN, see Fig. 3. Therefore the final value of the compression force defined in this member by means of the two-stage method equals -3.53 kN, see Fig. 4a. The value of the compression force calculated by the application of Autodesk Robot Structural Analysis Professional 2017 in the same member is equal to -3.89 kN, see Fig. 4b. The real difference between these two values equals 0.36 kN, which gives the relative difference coming to 10.1 %. A bigger relative differentiation can be noticed between values of forces defined in both compared methods for members of the middle layer located e.g. between nodes No 9 and No 10. In the two-stage method the force acting in this member is of zero value, while calculated by means of the computer software it is a tension force of value equals +0.49 kN. From initial analysis of the gained results follows the conclusion, that the biggest relative differences of forces appear in members subjected to act of relatively small absolute values.

Analysis of basic truss loaded in asymmetrically way

The statically indeterminate truss system shown in Fig. 5 has the same geometric and structural parameters like the truss structure presented in Fig. 1, what implies that it is a nine-fold indeterminate system, but it is now loaded in an asymmetric way. In this case the truss is loaded by five concentrated forces, each of 1.00 kN value, applied only to five successive nodes located only on the right site, in close vicinity to the support node B. In the first stage of the two-stage method nine members of the top chord have to be removed from the basic truss in order to make the truss a statically determinate system. The concentrated forces are of half value, equal to 0.50 kN, are applied to the same nodes of the upper chord.

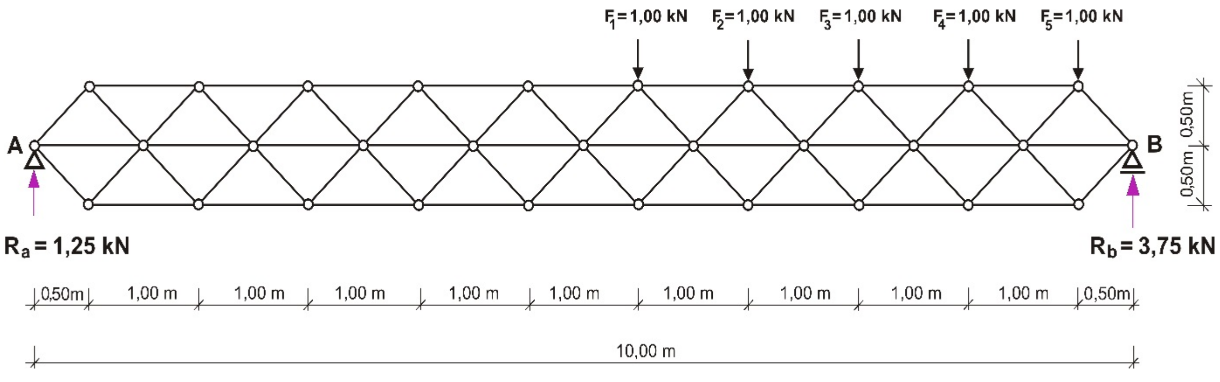


Figure 5 . Static scheme of the basic truss subjected to asymmetric way of load

Results of this stage of the calculations together with a suitable Cremona's polygon of forces are shown in Fig. 6. In the second stage of the two-stage method there are removed also nine members, but this time from the lower chord. Like previously the load forces of half values are applied to the corresponding nodes of the lower chord. All values of forces calculated in the second stage of calculation of the asymmetric loaded truss are presented in Fig. 7. The final results, being resultants of the force values calculated in both stages for the counterpart members, are shown in Fig. 8a. The procedure for determining the force values in particular truss members is the same as described above. The results defined for the same truss by the application of the computer software Autodesk Robot Structural Analysis Professional 2017 are presented in Fig. 8b.

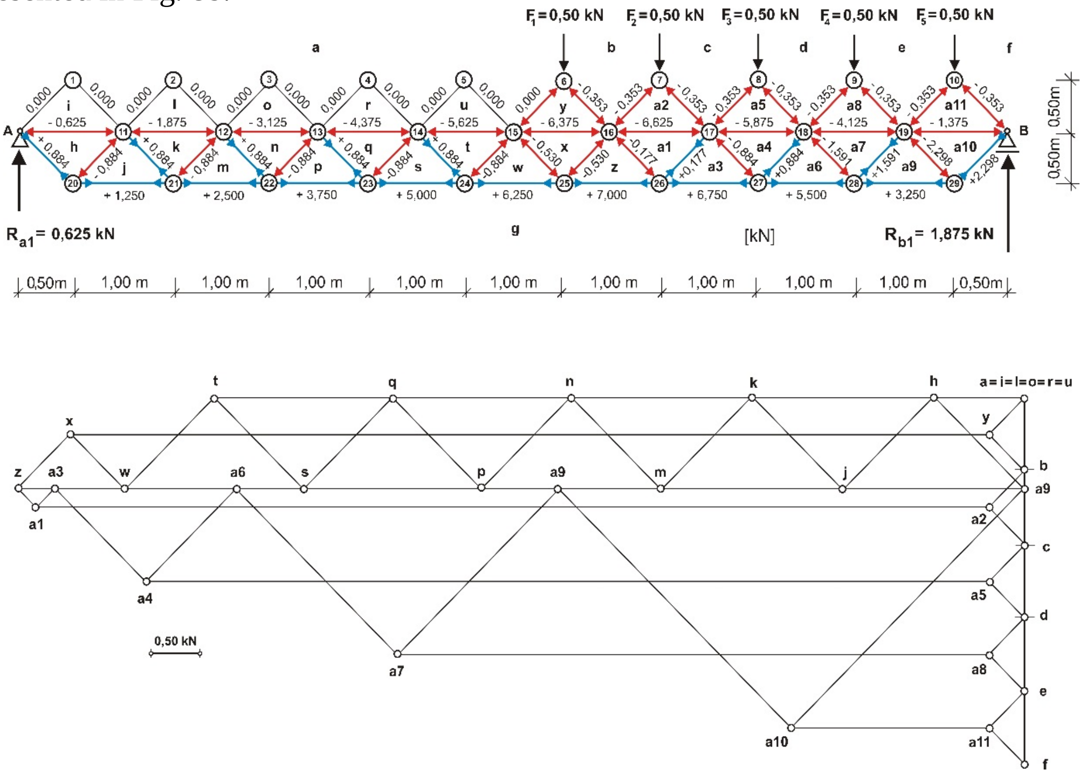


Figure 6. Results calculated in the first stage for the asymmetrically loaded truss together with Cremona's polygon of forces

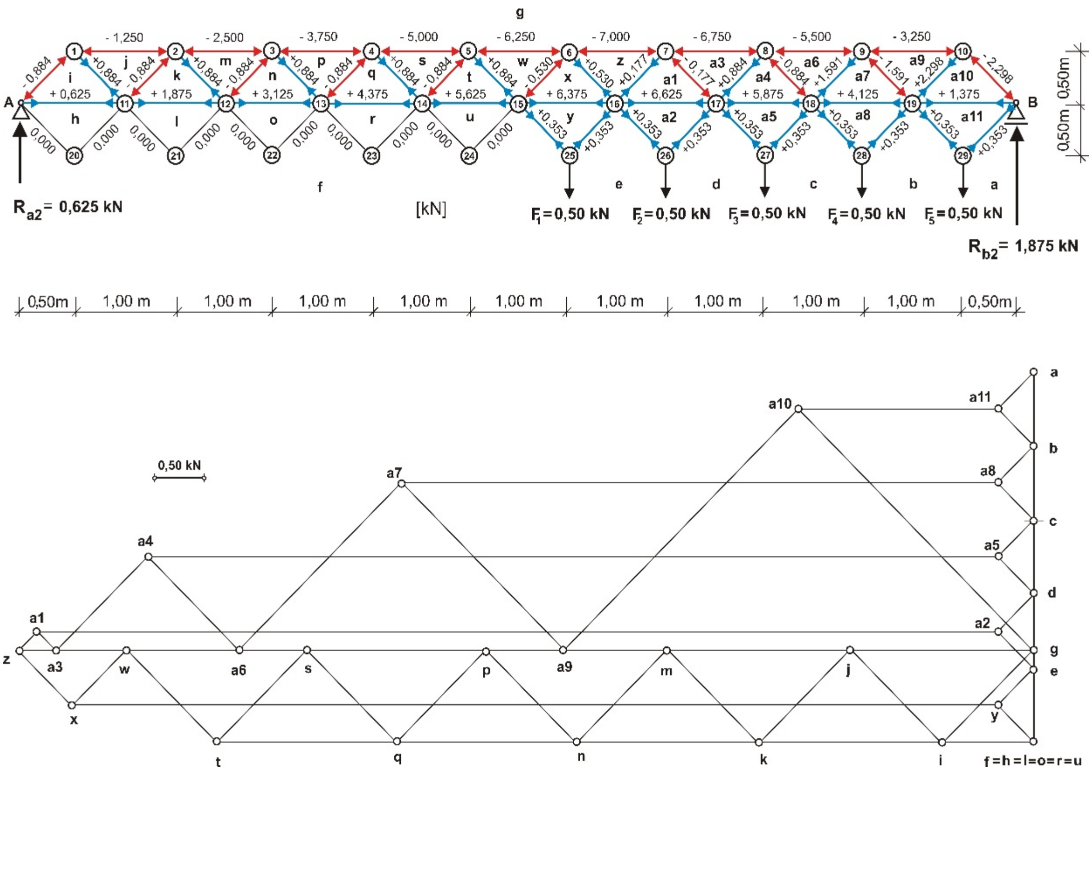


Figure 7. Results calculated in the second stage for the asymmetrically loaded truss together with Cremona's polygon of forces

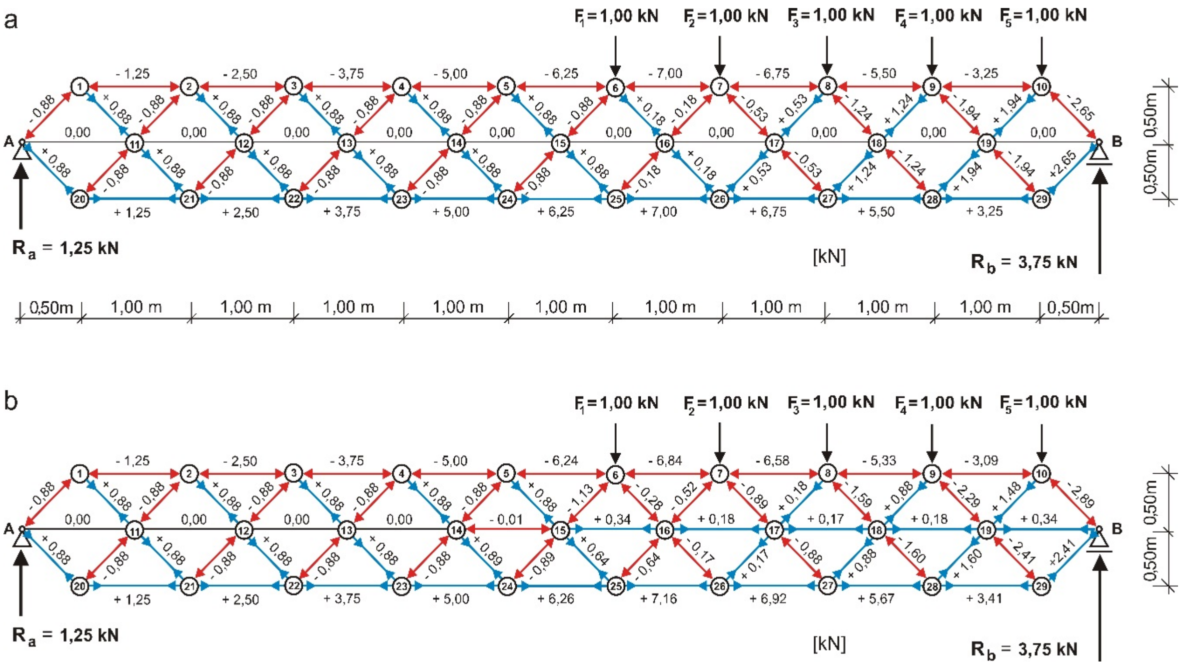


Figure 8. Final values of forces calculated for the asymmetrically loaded truss, a) in the two-stage method, b) by application of computer software

Differentiation of force values in members of the outer chords in the considered truss is rather small. For instance the force value calculated in the two-stage method in compression member of upper chord located between nodes No 6 and No 7 equals -7.00 kN, while by application of the computer software it is equal to -6.84 kN. The difference in amount of 0.16 kN constitutes towards the smaller value only 2.33 % of its basic dimension. One can notice some differences between the calculated force values acting in members of the middle chord spaced between support nodes A and B. Forces calculated in both of the compared methods in such members placed closely to support A are of the same value equals 0.00 kN. One should notice that these members are located quite far from the area, where the applied outer load forces are located. One can observe substantial differences in the members of the middle chord located closely to the support node B in the sector, where there are only the applied load forces. For example the force value calculated in the two stage method in the member placed between nodes No 15 and No 16 equals zero, while by application of the computer software it is estimated that the tensile force has a value of +0.34 kN. The differences can be considered as significantly big, but one has to be aware that they refer to very small values of forces. A similar force differentiation can be noticed in values calculated in cross braces. In the area located in the vicinity of the support A, forces defined in both the compared methods are of identical values. In the zone located close to support B differences of the estimated force values are evident. The tensile force calculated in the member placed between node No 17 and No 8, by means of the two-stage method, is of the value +0.53 kN, while by application of the computer software Autodesk Robot Structural Analysis Professional 2017 it is defined to equal +0.18 kN. However the difference is only equal to 0.35 kN of the absolute value, but the relative difference is of 194% in relation to the smallest force value. In spite of this observation one should be aware that the biggest relative differentiation appears only for the smallest force, close to the zero value.

Conclusions

This paper presents examples of the application of the two-stage method of calculation of statically indeterminate trusses together with a comparison of the results gained by the application of a commonly used computer software package. The investigated plane truss has two outer chords and a middle chord consisting of horizontal members, while its clear span is ten times bigger than its construction depth. The basic truss is subjected to symmetrical and asymmetrical way of load. The two-stage method is an approximate method of calculation of statically indeterminate trusses because it incorporates processes of calculations, which among others do not take into consideration different stiffness of members connecting in particular nodes. From the comparison of results obtained from both of the compared methods follows that in general the results obtained by means of the two-stage method are very compatible with results gained by application of the exact calculations made by means of Autodesk Robot Structural Analysis Professional 2017. The differentiation of the force values calculated in both methods in the same members are subjected to the biggest forces is relatively small, it is around couple of percent. The differences are considerably big for members, where values of calculated forces are very small, being close to the zero value. One should to point out, that members subjected to such small forces are designed by applying suitable rules or requirements of building codes, which implies that areas of their cross-sections are sometimes many times bigger than it directly follows from the calculated values and forces and strength of a material. Due to its simplicity and due the sufficient approximation of obtained results the two-stage method can be applied not only in the initial structural design of the statically indeterminate trusses. It can be adapted for an appropriate and new type computer software what will be not complex because of the simplicity of elementary calculation procedures needed for this purpose. Accuracy of the two-stage method can be significantly improved by application of a set of suitable coefficients, which can define ways of the force distribution according to differentiation of stiffness of members connecting in the same node. One should expect that principles of the method can be relatively easily adapted for the calculation procedures of space trusses.

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