Simulations of Dynamic Concrete Fracture with Brazilian Splitting Test

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Abstract

Concrete fracture is a phenomenon that material science deals with since its very beginning. With increase of computational capacity, linear models previously used for the description of material behavior in the vicinity of stress concentrators are being replaced with complex models using nonlinear material laws. Such models can be continuous, but for the better description of material behavior at the scale, where heterogeneities are recognized, discrete formulations are usually adopted.

The contribution presents simulations of concrete fracture in dynamic regime using rigid body spring network. The material is represented by interconnected rigid cells of convex polyhedral shape. In the dynamic regime, for relatively slow loading rates, use of implicit time integration scheme for solving system of equations is quite common. For faster loading rates, the explicit scheme is usually used, however, in the presented contribution, the implicit scheme is conveniently used as well. The time step length is then not restricted by the stability of the system, but only by the desired accuracy. Model is applied to dynamic simulations of Brazilian splitting discs experimentally investigated in literature.

Keywords: **Concrete Fracture; Implicit Dynamics; Discrete Modeling; Meso-scale; Strain Rate**

Introduction

It is well known that response of a tested specimen is highly dependent on loading rate. To obtain material properties such as tensile, flexural or compressive strength, the loading rate is usually of order 10^{-4} m/s. Using higher loading rate may affect the resulting values of obtained material properties. It is due to change in mechanism of failure. For a quasi-static loading, damaged volume of material is relatively small, in a case of tensile failure, the initial micro-cracking finally localizes into one relatively narrow highly damaged region – macro-crack. With increasing loading rate, the energy accumulated in a material volume is too high to be consumed by one crack only and multiple cracking occurs. The total damaged volume is then larger and damage is diffused in a wider region.

Attempts to describe this phenomenon of strain-rate dependency can be found in literature [1,7,8]. The relative increase (with respect to quasi-static rate) of material characteristic (strength, fracture energy, elastic modulus) is described by Dynamic Increase Factor (DIF). However, this phenomenon is associated not only with the increase in material resistance, but also with material inertia. General use of DIF is complicated, since it is not applicable to different geometry than the one it was obtained at.

It is therefore convenient to use the model that already incorporates the influence of inertia. Strain rate dependency of the material properties can then be investigated.

The contribution presents simulations of dynamic tests on Brazilian Splitting Discs. The simulations are calculated using discrete particle model. The time dependent response is obtained solving equations of motion using implicit time integration scheme according to

Newmark [6]. The performance of the mathematical model is compared with experimental results from literature [5].

Particle model

When modeling heterogeneous materials behavior, discrete approach is particularly convenient. It is because of its ability to represent internal material structure of concrete. The material volume is discretized into polyhedral particles that are assumed to be ideally rigid and their interaction is prescribed at their contact facets. The model particles then represent larger concrete/mortar aggregates with surrounding cement paste. The particle shape is obtained by Voronoi tessellation applied on a set of points randomly placed in the volume domain within prescribed minimum distance $l_{\min}$, which is related to maximum aggregate size. Each particle has 3 translational and 3 rotational degrees of freedom. Omitting direct representation of smaller mineral aggregates because of the computational reasons affects the model response, however, phenomena occurring below the resolution of the model are captured by nonlinear constitutive law.

The model is adopted according to [2] using further simplifications from [3]. In its basic version, only 4 parameters are used for material description; Elastic modulus E_0 and tangential to normal stiffness ratio α for elastic behavior and parameters of tensile strength f_t and fracture energy for tensile failure G_f in nonlinear regime. Other parameters such as compressive strength etc. are derived from these according to recommendations in [2]. It is important to note that all of these parameters are applied at meso-scale and their values differ from overall macro-scale material properties. The approximate relation between meso-scale elastic parameters and macro-scale Youngs modulus and Poissons ratio can be obtained from principle of virtual work [4]

$$E_0 = \frac{E}{1-2\nu} \qquad \alpha = \frac{1-4\nu}{1+\nu} \tag{1}$$

As has been stated, constitutive law is applied at the contacts of discrete particles. The contact behavior is dependent on straining direction in both elastic and nonlinear regimes. After reaching the elastic limit, the damage model is applied to describe loss of integrity of material. The stress-strain relation depends on damage parameter D

$$\mathbf{s} = (1-D)E_0\boldsymbol{\alpha}\mathbf{e} \qquad D \in \langle 0,1 \rangle \tag{2}$$

Here $\mathbf{s}$ and $\mathbf{e}$ are meso-scopic stress and strain vectors respectively with elements corresponding to normal and two tangential directions and $\boldsymbol{\alpha}$ is diagonal matrix with the first diagonal element 1 (for normal direction) and remaining diagonal elements α. Initially, parameter D is equal to zero. After reaching the elastic limit, the parameter increases up to 1, which indicates stress free crack. The evolution of damage is crucial part of the nonlinear behavior of the model. It takes into account combination of normal and tangential straining. Since its description is quite complex, the interested reader is referred to [3].

Dynamics

The solution of equations of motion $\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{F}$ is provided by an unconditionally stable time integration scheme according to Newmark [6]. **M, C** and **K** stay for mass, damping and stiffness matrix respectively, **F** is a loading vector. The damping matrix is not taken into account, since the system is damped in inelastic regime by the energy dissipation at contact facets. The solution is based on dynamic equilibrium at the end of each time step (time $t{+}\Delta t$). The values of accelerations $\ddot{\mathbf{u}}$ and velocities $\dot{\mathbf{u}}$ at this time are estimated as numerical derivatives of displacements $\mathbf{u}$

$$\ddot{\mathbf{u}}_{t+\Delta t} = \frac{1}{\beta \Delta t^2}\left(\mathbf{u}_{t+\Delta t} - \mathbf{u}_t\right) - \frac{1}{\beta \Delta t}\dot{\mathbf{u}}_t - \left(\frac{1}{2\beta} - 1\right)\ddot{\mathbf{u}}_t \tag{3}$$

$$\dot{\mathbf{u}}_{t+\Delta t} = \dot{\mathbf{u}}_t + \Delta t\left(1-\gamma\right)\ddot{\mathbf{u}}_t + \gamma \Delta t \ddot{\mathbf{u}}_{t+\Delta t} \tag{4}$$

where β and γ are parameters of the Newmark method, which, to keep the solution unconditionally stable, need to be kept within limits $\gamma, \beta \in \langle 0,1 \rangle$ and $2\beta \geq \gamma \geq 0.5$. Omitting damping and substituting these into equation of motion, following system is obtained

$$\left(\mathbf{K} + \frac{1}{\beta \Delta t^2}\mathbf{M}\right)\mathbf{u}_{t+\Delta t} == \mathbf{F}_{t+\Delta t} + \mathbf{M}\left(\frac{1}{\beta \Delta t^2}\mathbf{u}_t + \frac{1}{\beta \Delta t}\dot{\mathbf{u}}_t + \left(\frac{1}{2\beta} - 1\right)\ddot{\mathbf{u}}_t\right) \tag{5}$$

Each grain contributes to global mass matrix by 6x6 matrix. This particle mass matrix can be further divided into 4 smaller 3x3 submatrices. Upper left, relating forces and translations is diagonal containing value of particle mass at each diagonal element. Lower right, connecting rotations with moments consists of moments and products of inertia. Since the Voronoi cell center does not necessarily need to be particle center of gravity, static momentum needs to be taken into account to obtain balance, which is included in upper right and lower left submatrices.
No strain rate dependency at the level of constitutive equation was used for simulations presented in this contribution, it is assumed that the meso-level model correctly accounts for inertia even within the fracture process zone. Viscous effects due to presence of water are neglected.

Simulations

Simulations presented in this section were calculated using mathematical models corresponding to experimental series on Brazilian splitting tests reported in [5]. Simplified models of Brazilian discs were supported (respectively loaded) by line of boundary aggregates as shown in Fig. 1. The loading is applied by increasing deformation under prescribed loading rate. It certainly does not fully correspond to experimental loading, which was applied by Hopkinson bar setup. Loading by pressure (force) wave would be more realistic, but in our case, we can exactly specify the desired strain rate by prescribing it.
The experiments were performed on concrete and mortar discs with diameter D = 70 mm and with thickness T = 30 (further referred to as thin) and 55 mm (further referred to as thick). Model geometry was set accordingly. Two materials (concrete and mortar) were used in experimental study, therefore also two sets of material parameters were used for the modeling. Model parameters used in calculations are listed in Tab. 1. Parameter $l_{\min}$ was set according to maximum aggregate size used in experiments. The material fracture parameters were estimated according to maximum load obtained by quasi-static tests on thin discs. For quasi-static simulations, the prescribed displacement rate of $1.67\ 10^{-6}$ m/s was applied. Value of material meso-scale tensile strength was set to the value of macro-scopic tensile strength of the material and meso-level fracture energy was then found by fitting the model response to match the peak load only, since the post-peak behavior is hard to capture in the case of splitting test. The fit was performed separately for both simulated materials – concrete and mortar.

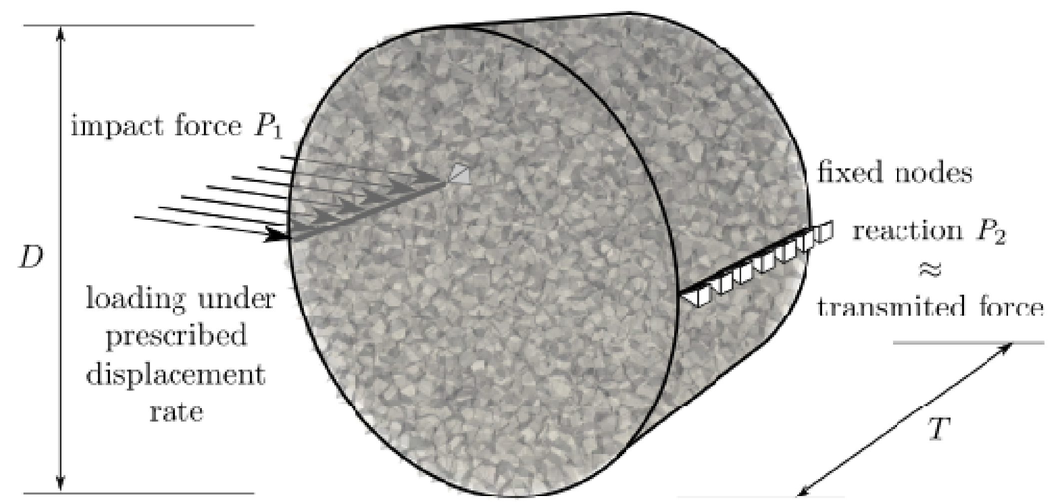

Figure 1 Model setup.

Table 1 Material parameters used from simulations of concrete and mortar discs.

	E_0 [GPa]	α	f_t [MPa]	G_f [N/m^2]	ρ [kg/m^3]	$l_{\min}$ [mm]
concrete	44	0.237	2.64	9.93	2400	7.50
mortar	40	0.237	3.47	40.66	2020	2.36

In the case of Brazilian splitting disc tests, the material tensile strength f_{tu} is estimated according to eq. (6), P_1 and P_2 are impact and transmitted force respectively, that are different in the case of dynamic loading, but equal in the case of quasi-static loading

$$f_{tu} = \frac{P_1 + P_2}{\pi TD} \tag{6}$$

After finding the best possible set of parameters to match the quasi-static response on thin specimens, the response of thick specimens was calculated. Lower strengths were obtained for thick discs, which corresponds to experimental data. Values of peak load and tensile strengths obtained by the model are listed in Tab. 2 together with experimental values. Note that rows corresponding to thin specimens are from the fitted response.

Table 2 Peak load and tensile strength obtained by the model compared to experiments.

		experiments [5]		simulations	
		peak load [kN]	strength [MPa]	peak load [kN]	strength [MPa]
concrete	thin	8.71	2.64	***fit*** 8.71	2.64
	thick	15.71	2.60	15.24	2.52
mortar	thin	11.44	3.47	***fit*** 11.44	3.47
	thick	20.26	3.35	23.01	3.80

The dynamical simulations were performed under wide range of displacement rates in correspondence to the experimental test. On the graph in the left part of Fig. 2 load-displacement curves are plotted for strain rates up to 150 s^{-1}. Significant delay in transmitted force can be observed as well as different increase of *impact* force P_1 compared to the increase in *transmitted* force P_2. Greater increase in the first one is mainly caused by inertia.

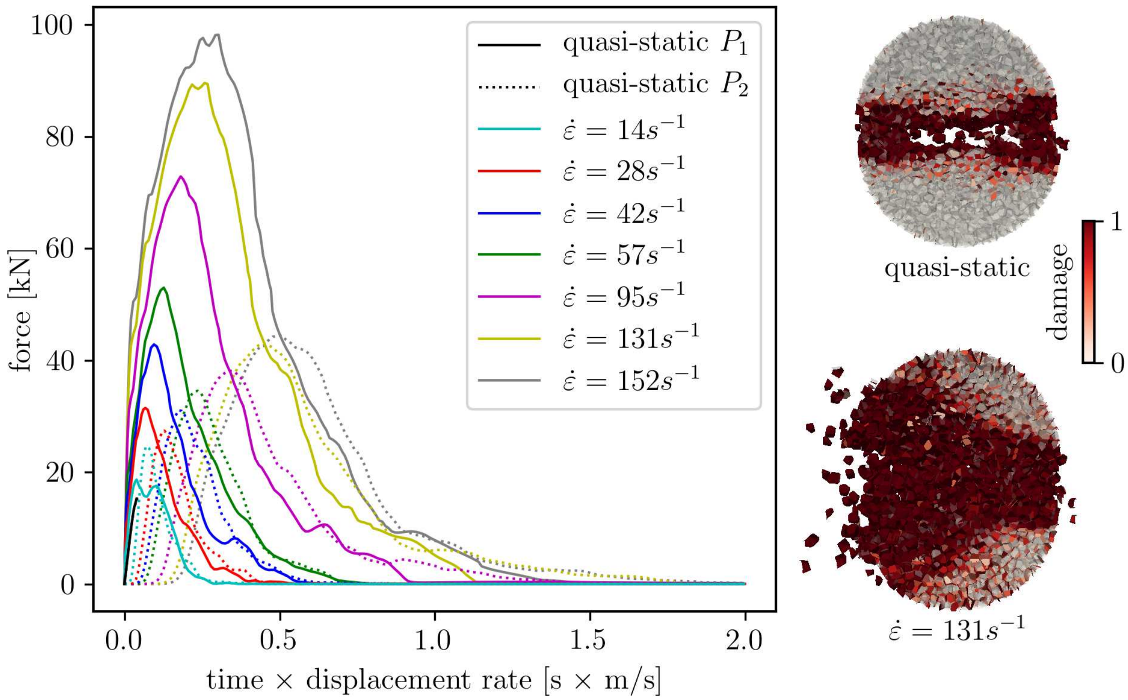

Figure 2 Force vs. displacement curve for thin mortar specimens (left) damaged volume obtained by simulation with 10× magnified displacements (right).

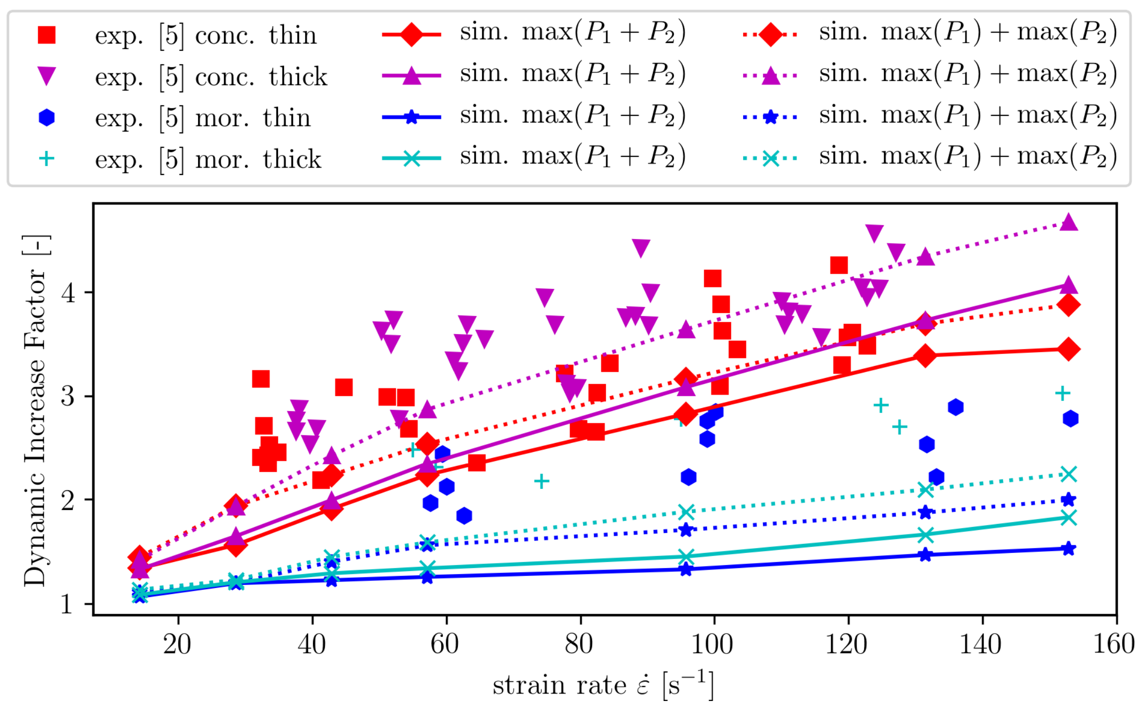

Figure 3 Dynamic increase factor of material tensile strength obtained by the model compared with experiments [5].

The smaller increase in *transmitted* force P_2 can be attributed to the change in material resistance due to greater strain rate.
Right part of Fig. 2 shows crack patterns for quasi-static and fast loading rate. In these pictures, displacements are 10× magnified. Particle facets are colored according to value of damage *D*. It can be clearly seen that in case of quasi-static loading, the model predicts the specimen to be split into two parts with relatively localized crack path, whereas in case of fast rate, the damaged volume is much greater, compare with [5].
Fig. 3 displays relation between material dynamic tensile strength according to Eq. (6) and strain rate obtained by the discrete particle model compared to the experimental data from [5]. Value of DIF is calculated by dividing dynamic and static strength. Since forces evolve in time, it is important to note the difference between $\max(P_1 + P_2)$ and $\max(P_1) + \max(P_2)$, even though the latter gives us results closer to the experimental evidence, the former should be taken into account for DIF estimation. The difference is not only in case of different material, but also for different thickness of the specimen, which corresponds to experimental evidence. However, even though the trends in increasing value of DIF are present, desired match is not obtained.

Conclusion

Discrete particle model was used for simulations of Brazilian splitting discs loaded under various strain rates. No strain rate dependency of constitutive law was imposed. It has been shown that the model is able to predict the increase in material strength, however, the desired match is not obtained. Authors believe that it is caused by the effects under the resolution of the model, thus finer resolution should be used for investigation of this phenomena. If course resolution is kept, the omitted part of the strain rate effect should be incorporated in the constitutive law phenomenologically.

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