Approximation of the parallel robot working area using the method of nonuniform covering

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Abstract

The work is devoted to optimization of the joint working area of 6-DOF relative manipulation mechanism, which include flat 3-RRR mechanism that rotates around the z axis and translates along the x and y axes of the lower platforms, and a tripod that provides movement of the tool along the z axis and its rotation around the x and y axes. The method of nonuniform covering is used to build the working area. As a result of the work of the method, external and internal approximations are obtained, given as a set of parallelepipeds. The paper presents the results of a computational experiment. The results of modeling the working areas of each of the relative manipulation modules and other parallel mechanisms, such as the flat 2-DOF DexTAR robot, are presented. The joint work area at moveable coordinate system is obtained. Moveable coordinate system is located in center of lower mechanism. The algorithms are implemented in C ++ using the Snowgoose library.

Keywords: **Approximation, relative manipulation, working area, parallel robot.**

Introduction

The paper considers a parallel robot consisting of two mechanisms of relative manipulation (Fig. 1):

1. Upper mechanism is a tripod, which provide move along the z axis and rotate around the x and y axes.
2. Lower planar 3-RRR mechanism, which provide a rotation around the z axis and translational movement along the x and y axes.

Thus, the mechanism has 6 degrees of freedom. The mechanism will be designed to perform machining operations of details and other operations if the working tool is fixed to the mobile platform.

The main problem for parallel mechanisms is the small size of the working area and the presence of the singularity zones.

The task is approximation of the working area on the basis of optimization algorithms.

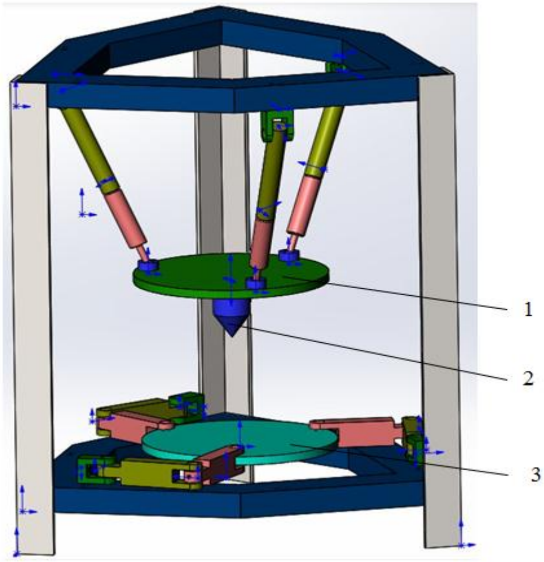


Figure 1. A 3D-model of the robot: 1 – the tool installation module, 2 –the tool, 3 – the detail machining module.

DexTAR working area approximation based on the method of nonuniform covering

The paper suggests the use of the method of nonuniform covering (Fig. 2) to solve the problem of approximation of the working area. The method is based on works [1,2]. This method allows approximating the solutions set of the equalities or inequalities systems describing the robot working area. The external and internal approximation sets are constructed. The internal approximation set is included in the set of solutions of the inequalities system. Both sets are represented as unions of n-dimensional parallelepipeds. Mathematical transformation of the coupling equations of some robots allows you to reduce the dimension, and hence the computation time. Using the parallelepipeds of large dimension avoids significant mathematical transformations, and then project them onto the coordinate axes necessary for the imaging.

Interval estimates using the developed Snowgoose library in C++ can be used to find the maximum and minimum of functions in parallelepipeds. The grid approximation method is used for cases of multiple occurrence of variables where errors can affect the result.

Method is development and tested on models of robots with 2 and 3 degrees of freedom within the framework of the Russian Science Foundation project, the agreement number 16-19-00148.

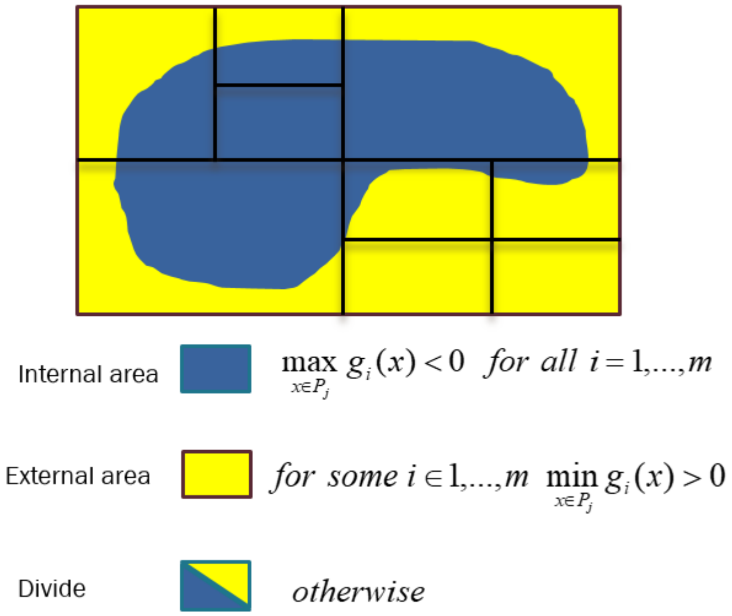


Figure 2. Illustration of the method.

The method was successfully applied to the 2-DOF robot DexTAR (Fig. 3). It is a planar parallel four-link mechanism, controlled by two engines. Result is presented in [3].

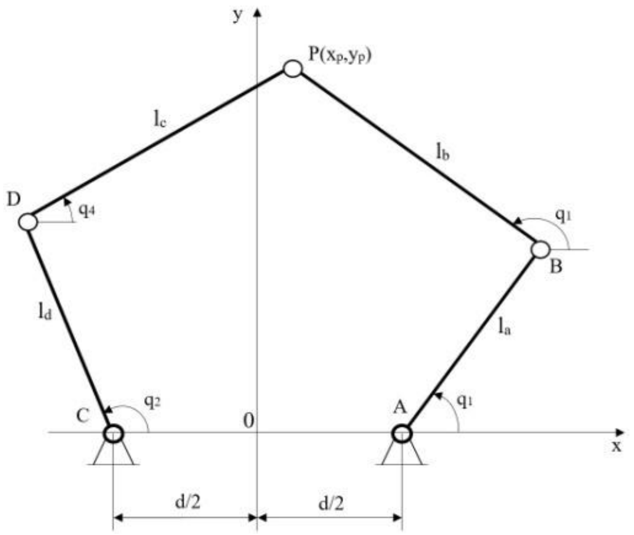


Figure 3. DexTAR robot scheme

Equations describing the motion:

$$\begin{cases} x_p = l_b \cdot \cos q_3 + l_a \cdot \cos q_1 + \frac{d}{2}, \\ x_p = -\frac{d}{2} + l_d \cdot \cos q_2 + l_c \cdot \cos q_4, \\ y_p = l_a \cdot \sin q_1 + l_b \cdot \sin q_3, \\ y_p = l_d \cdot \sin q_2 + l_c \cdot \sin q_4. \end{cases} \tag{1}$$

Equations have 6 variables:

- Input coordinates: q_1, q_2
- Output Coordinates: x_p, y_p

- Intermediate coordinates: q_3, q_4

Interval estimates are used to search for extrema. Under conditions of a single occurrence of variables in the expression, the interval estimates coincide with the extrema of the function on the parallelepiped, i.e. cannot be improved. However, the calculation time was 39 minutes 40 seconds, which is due to the large dimensionality of the problem (6-dimensional parallelepiped). Results of simulation is presented on Fig. 4, 5.

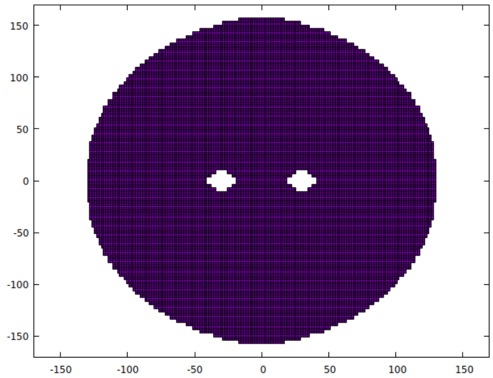


Figure 4. Results of simulation DexTAR working area

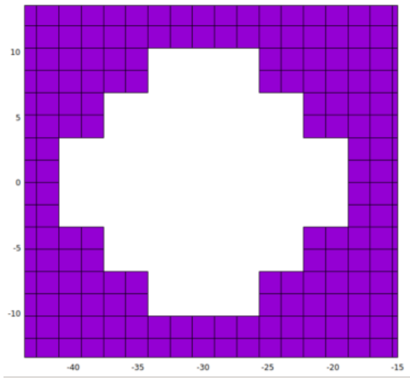


Figure 5. Singularity zone on an enlarged scale

Dimensionality of the problem was lowered and the grid approximation method was applied, due to the multiple occurrence of variables in expressions. The Fig. 6 shows the results of modeling the working area, Fig. 7 is the singularity zone on an enlarged scale. The calculation time, decreased by 10.35 times, while the accuracy of calculations increased by 10 times.

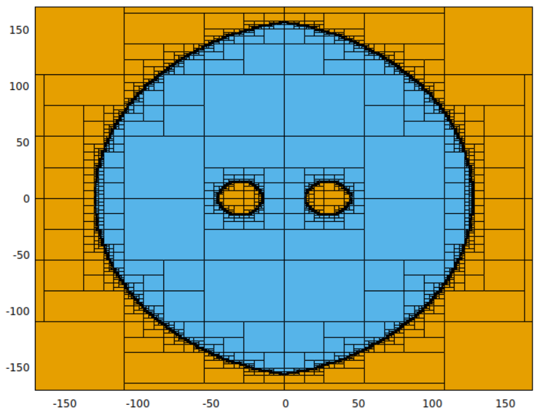


Figure 6. Results of simulation DexTAR working area

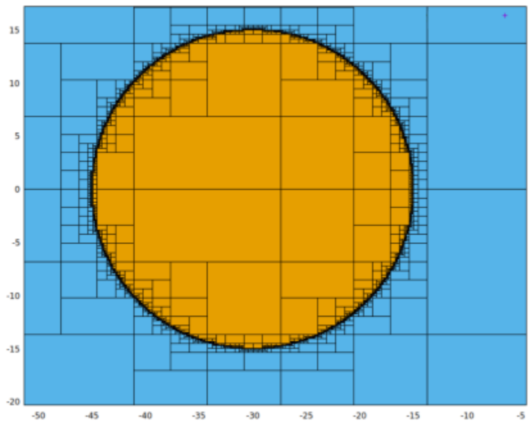


Figure 7. Singularity zone on an enlarged scale

Modeling the working area of the relative manipulation mechanism

Consider the construction of the working area of the robot with relative manipulation mechanisms (Fig. 1). Definition the working area of the upper mechanism - the tripod (Fig. 8) was developed within the framework of the Russian Science Foundation project, the agreement number 16-19-00148. The tripod consists of three link of variable length, which are connected by rotational hinges to the base and spherical hinges to the working platform. The base and the working platform are equilateral triangles. As a result of changing the lengths of the rods, the working platform moves along the z axis and rotates along the x and y axes.

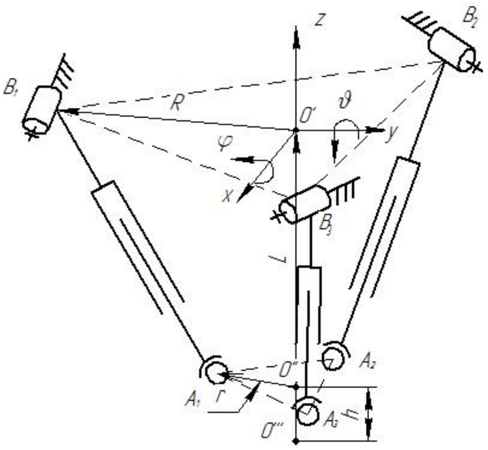


Figure 8. Tripod scheme

The center of the tripod mobile platform, in addition to the basic three basic degrees of freedom, has small displacements, which can be found by the following formulas [4]:

$\varphi = Tan^{-1}(\frac{sin\psi sin\theta}{cos\psi + cos\theta})$, where φ – rotation angle around z axis, θ – around y axis, ψ – around x axis

$y = -rcos\psi sin\varphi$, where y – displacement on y axis, r – radius of moveable platform

$x = \frac{r}{2}(cos\theta cos\varphi + sin\psi sin\theta sin\varphi - cos\psi cos\varphi)$, where x – displacement on x axis.

The rotation matrix for the transition from the coordinate system located in the center of the fixed platform to the coordinate system located in the center of the moving platform:

$$R_p = \begin{bmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\psi & -sin\psi \\ 0 & sin\psi & cos\psi \end{bmatrix} \begin{bmatrix} cos\varphi & -sin\varphi & 0 \\ sin\varphi & cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2)$$

$$R_p = \begin{bmatrix} cos\theta cos\varphi + sin\psi sin\theta sin\varphi & -cos\theta sin\varphi + sin\psi sin\theta cos\varphi & sin\theta cos\psi \\ cos\psi sin\varphi & cos\psi cos\varphi & -sin\psi \\ -sin\theta cos\varphi + sin\psi cos\theta sin\varphi & sin\theta sin\varphi + sin\psi cos\theta cos\varphi & cos\theta cos\psi \end{bmatrix} = \begin{bmatrix} Rp11 & Rp12 & Rp13 \\ Rp21 & Rp22 & Rp23 \\ Rp31 & Rp32 & Rp33 \end{bmatrix} \quad (3)$$

The main criterion for determining the occurrence of a point in the working area is the occurrence of each of the link lengths in the permissible range. The generalized formula for link lengths:

$$L_i = \sqrt{(x_{Ai} - x_{Bi})^2 + (y_{Ai} - y_{Bi})^2 + (z_{Ai} - z_{Bi})^2} \quad (4)$$

The links lengths can be determined as follows:

$$L_1 = \sqrt{(x + r \cdot Rp11 - R)^2 + (z + r \cdot Rp31)^2}, \quad (5)$$

$$L_2 = \left(\left(x + 0{,}5r\left(Rp11 + \sqrt{3}Rp12\right) - 0{,}5R\right)^2 + \left(y - 0{,}5r\left(Rp21 + \sqrt{3}Rp22\right) + \frac{\sqrt{3}}{2}R\right)^2 + \left(z - 0{,}5r\left(Rp31 + \sqrt{3}Rp32\right)\right)^2\right)^{0{,}5}, \quad (6)$$

$$L_3 = \left(\left(x - 0{,}5r\left(Rp11 - \sqrt{3}Rp12\right) + 0{,}5R\right)^2 + \left(y - 0{,}5r\left(Rp21 - \sqrt{3}Rp22\right) - \frac{\sqrt{3}}{2}R\right)^2 + \left(z - 0{,}5r\left(Rp31 - \sqrt{3}Rp32\right)\right)^2 \right)^{0{,}5}, \tag{7}$$

where z – moving along the z axis.

Given a list of parallelepipeds $\mathbb{P}_1$, which initially includes one parallelepiped Q_1 in the moving coordinate system, which is guaranteed to include the work area. During the execution of the algorithm (Fig. 9), the link length functions are calculated. If at least one of the calculated lengths is out of the allowable range at all points of the parallelepiped, then this parallelepiped is not included in the working area and added in the list of external approximations. If in all points of a parallelepiped all the functions of the link lengths are in the permissible range or the diameter of the parallelepiped $\mathrm{d}(Q_i)$ is smaller than the accuracy of the approximation δ, then this parallelepiped enters the working area and added in the list of inner approximation. In other cases, the parallelepiped is divided in half along the maximal length of the edge and 2 parallelepipeds are added to the end of the list $\mathbb{P}_1$ for further consideration. The following geometric parameters were chosen for simulation:

$$L_{i,min} = 70\ mm$$
$$L_{i,max} = 130\ mm$$
$$R = r = 50\ mm$$

Results of simulation is presented on Fig. 10, 11.

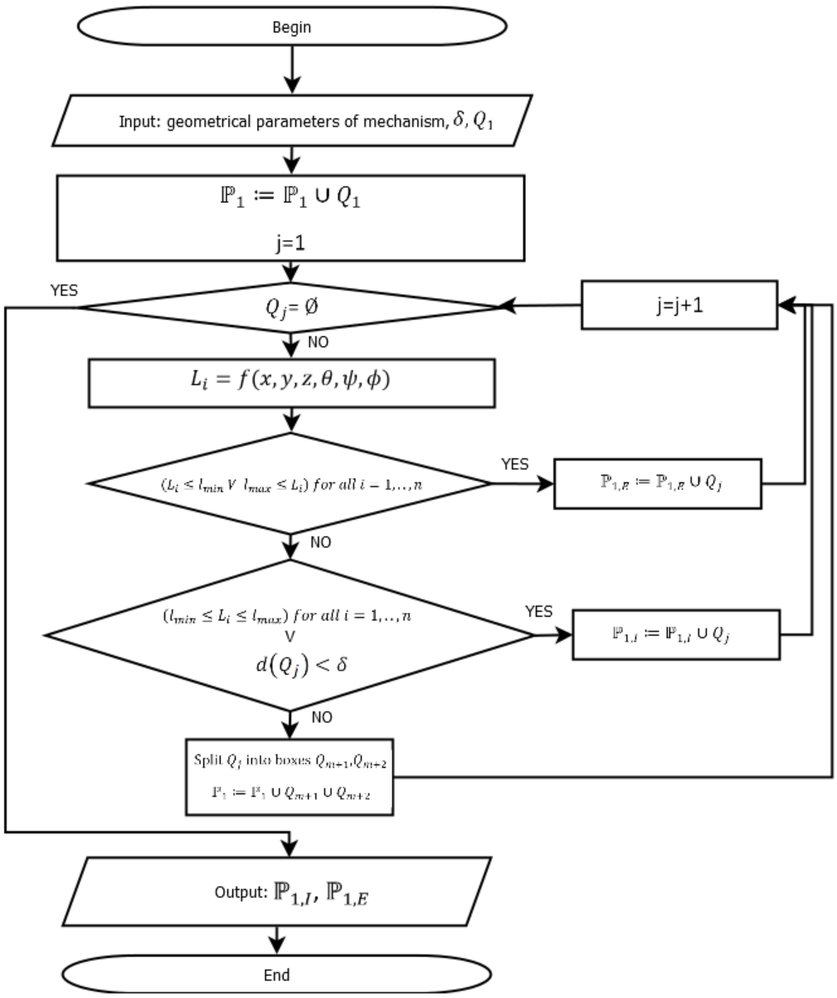


Figure 9. Algorithm for tripod

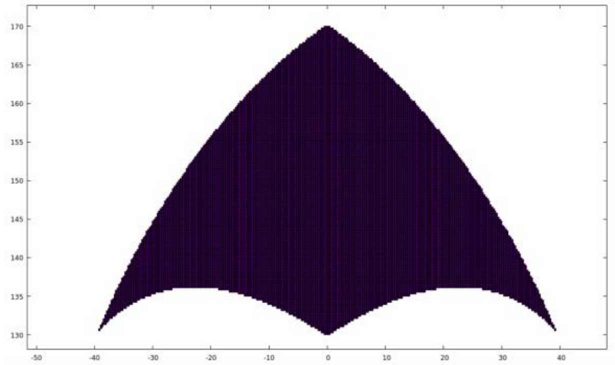


Figure 10. The working area in projection on *yz* plane

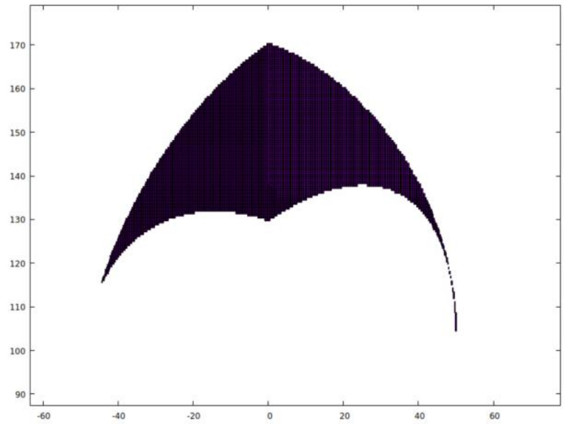


Figure 11. The working area in projection on *xz* plane

Definition the working area of the lower mechanism - a planar robot (Fig. 12) was developed within the framework of the Russian Science Foundation project, the agreement number 17-79-10512. This mechanism consists of three chains containing three rotational kinematic pairs O_i, A_i, B_i (i=1,2,3). The axes of rotation of all pairs are parallel to each other and perpendicular to the plane in which the mechanism moves. Rotary pairs A_i are fixed on a base, and their position is given by the coordinates x_i, y_i in a fixed rectangular coordinate system. The position of the output link of the mechanism is given by the position of the point D and is described by the coordinates x and y as well as the angle of rotation φ of this link with respect to some initial position. The displacement of the output link is carried out due to the rotation of the driving (input) pairs A_i. The angles of rotation θ_i of these pairs are generalized coordinates for the given mechanism. R and r – the radii of the circles described the triangles $O_1O_2O_3$ and $B_1B_2B_3$ respectively.

The geometry of the output link, i.e. the mutual arrangement of points C_1, C_2, C_3 and D is given by angles γ_i as well as distances C_iD .

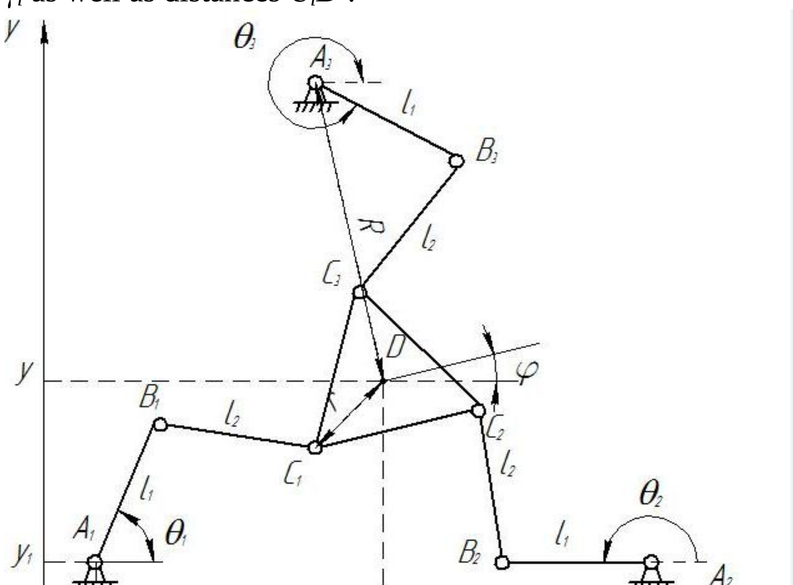


Figure 12. Scheme of a planar 3-RRR mechanism

The criterion of a planar mechanism is the permissible range of rotation angles of drive pairs. The formulas to calculate the angles θ_i are:

$$\theta_i = \arcsin\left(\frac{l_{2,i}^2 - l_{1,i}^2 - \left[b_{i,\cos}\right] - \left[b_{i,\sin}\right]}{\sqrt{\left[a_{i,\sin}\right]^2 + \left[a_{i,\cos}\right]^2}}\right) - \phi, \quad (8)$$

$$\left[a_{i,\sin}\right] = 2\left[y_i l_{1,i} - y l_{1,i} - l_{1,i} l_{3,i} \sin(\gamma_i + \varphi)\right], \quad (9)$$

$$\left[b_{i,\cos}\right] = \left[\begin{array}{l} x^2 + 2x l_{3,i} \cos(\gamma_i + \varphi) - 2x x_i + l_{3,i}^2 \cos^2(\gamma_i + \varphi) - \\ -2x_i l_{3,i} \cos(\gamma_i + \varphi) + x_i^2 \end{array}\right], \quad (10)$$

$$\left[a_{i,\cos}\right] = 2\left[x_i l_{1,i} - x l_{1,i} - l_{1,i} l_{3,i} \cos(\gamma_i + \varphi)\right], \quad (11)$$

$$\left[b_{i,\sin}\right] = \left[\begin{array}{l} y^2 + 2y l_{3,i} \sin(\gamma_i + \varphi) - 2y y_i + l_{3,i}^2 \sin^2(\gamma_i + \varphi) - \\ -2y_i l_{3,i} \sin(\gamma_i + \varphi) + y_i^2 \end{array}\right]. \quad (12)$$

Planar mechanism algorithm (Fig. 13) is developed. It is similar to the tripod algorithm. The following geometric parameters were chosen for simulation:

$$l_{1i} = l_{2i} = l_{3i} = r = 50\ mm,$$
$$R = 100\ mm,$$
$$\theta_{O,min} = -45°,$$
$$\theta_{O,max} = 45°.$$

Results of simulation is presented on Fig. 14.

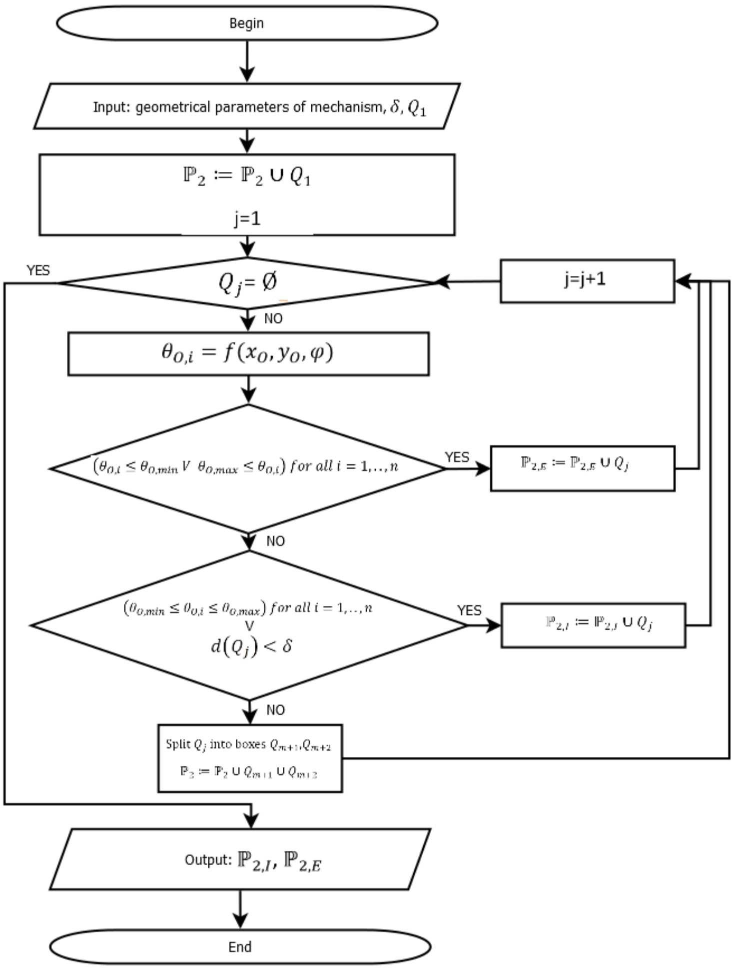


Figure 13. Algorithm for planar 3-RRR.

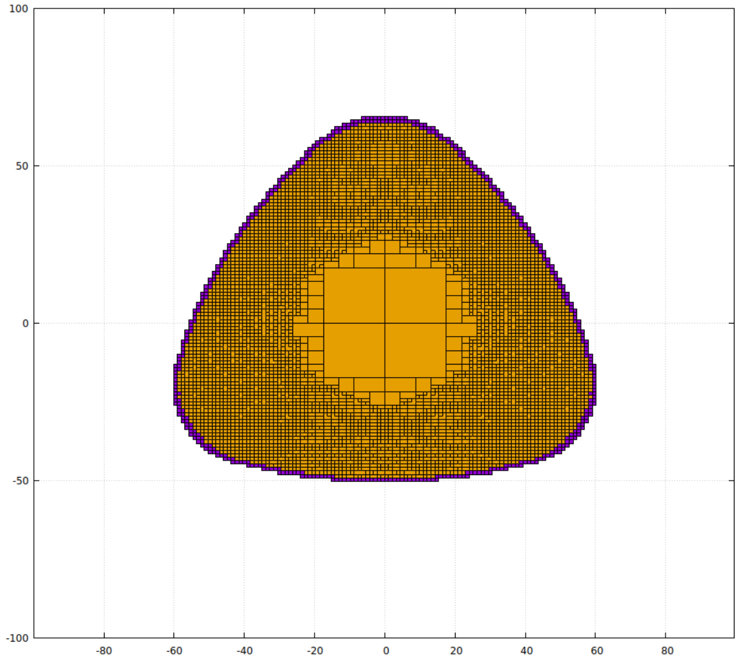


Figure 14. Results of simulation

Consider the scheme of a complicated mechanism which include the tripod and flat 3-RRR mechanism.

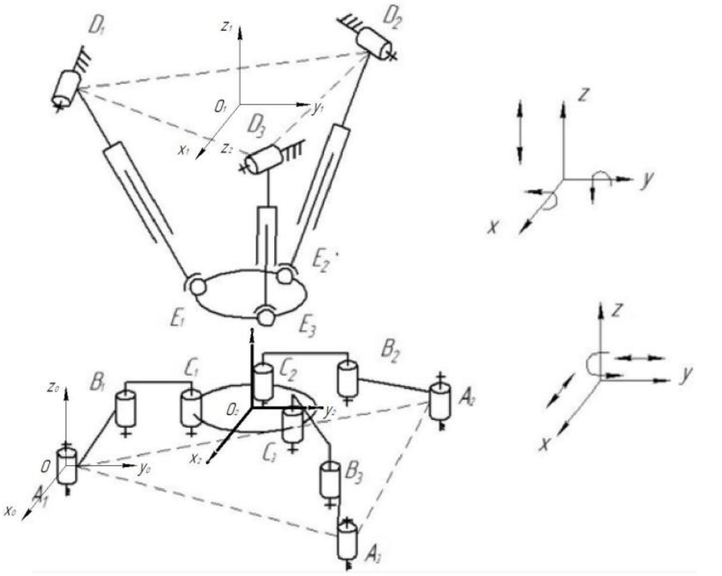


Figure 15. Kinematic scheme of the robot with relative handling modules

Development of an algorithm for approximating a joint working area

Algorithm for approximating a joint working area in a moving coordinate system is developed within the framework of the Russian Science Foundation project, the agreement number 17-79-10512. The algorithm is described below:

1. 2 fixed coordinate systems: $x_0y_0z_0$ is located in the center of the joint A_1, $x_1y_1z_1$ in the center of the upper fixed platform O_1.
2. The moving coordinate system $x_2y_2z_2$ of the planar mechanism is placed in the center of the planar mechanism platform O_2.
3. Let`s use the working area parallelepiped lists of the upper mechanism $\mathbb{P}_{1,I}$ and the lower mechanism $\mathbb{P}_{2,I}$.
4. Given a list of parallelepipeds $\mathbb{P}$, which initially includes one parallelepiped Q_1 in the moving coordinate system, which is guaranteed to include the work area.
5. Apply a grid A_1 to this parallelepiped.
6. Apply the grid A_2 to the parallelepipeds from list $\mathbb{P}_{2,I}$ of the flat mechanism.
7. For each of the grid A_1 nodes of the parallelepiped Q_i, calculate the rotation and displacement matrices of the transition from the coordinate system $x_2y_2z_2$ to the coordinate system $x_1y_1z_1$, This matrices have variable values equal to the coordinates of the grid A_2 nodes in the coordinate system $x_0y_0z_0$.
9. If all nodes of the grid A_1 after the transition do not belong to the working area $\mathbb{P}_{1,I}$, then the parallelepiped Q_i is added to the $\mathbb{P}_E$ list of parallelepipeds of the external approximation.
10. If all nodes of the grid A_1 after the transition belong to the working area $\mathbb{P}_{1,I}$ or the diameter of the parallelepiped $d(Q_i)$ is smaller than the accuracy of the approximation δ, then the parallelepiped Q_i is added to the list $\mathbb{P}_I$ of parallelepipeds of internal approximation.
11. In other cases, the parallelepiped is divided in half along a larger edge and added to the end of the list $\mathbb{P}$.
12. Repeating items 5-11 is performed before check the last parallelepiped in list $\mathbb{P}$.

Results of simulation with projection on different plane is presented on Fig. 17.

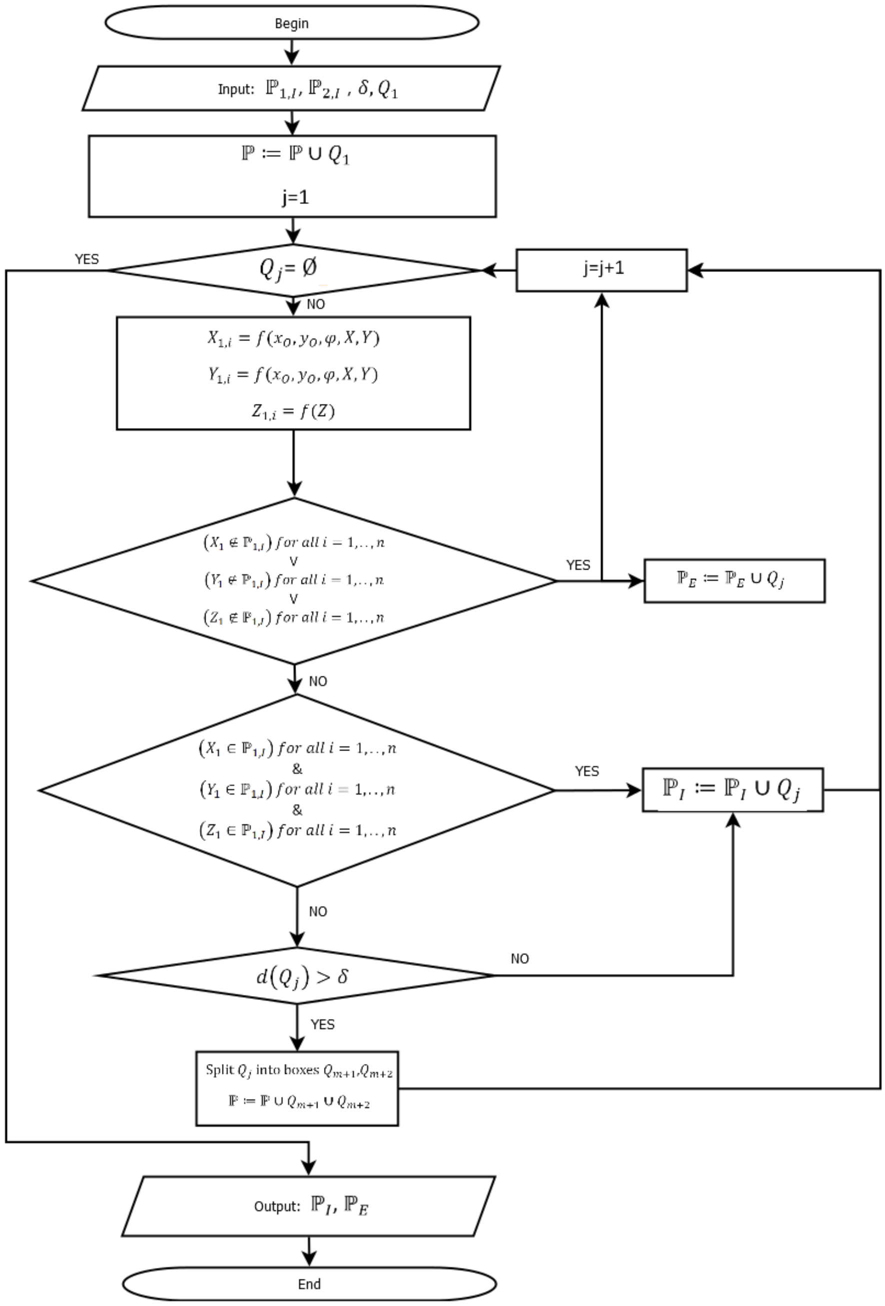
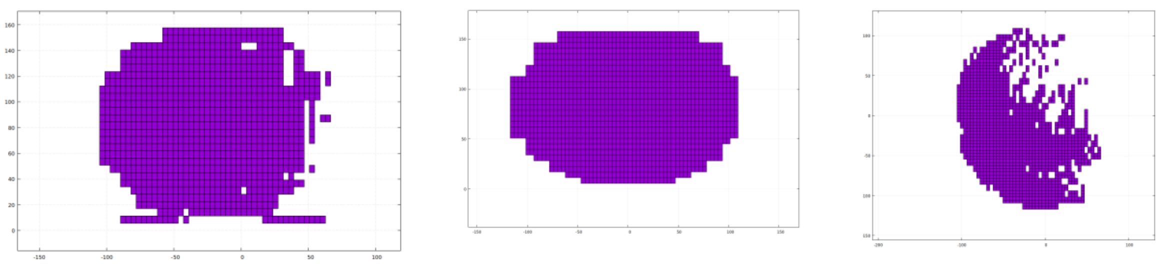


Figure 16. Algorithm for joint working area



a) b) c)

Figure 17. The working area in projection on a) *xz* plane, b) *yz* plane, c) *xy* plane

Conclusions

The applied algorithm showed its efficiency. The computation time for accuracy of approximation $\delta = 6$mm and the dimension of the grid for enumerating the calculation of functions 64x64x64 on a personal computer was 2 hours and 45 minutes. The accuracy of the approximation ($\delta = 6$ mm) of given complicated robot with 6 degrees of freedom in comparison with planar mechanisms with three degrees of freedom ($\delta = 0.006$-0.06 mm) is reduced in 100-1000 times, which is due to increase in the dimensionality of the problem and significant computing resources are required to increase accuracy. The resulting work area has fuzzy boundaries, which may be due to insufficient computational power and the presence of singularity zones.

Conclusions

This work was supported by the Russian Science Foundation, the agreement number 16-19-00148, the agreement number 17-79-10512

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