Damage assessment by Non-Smooth Contact Dynamics method of the iconic crumbling of the clock tower in Amatrice after 2016 center Italy seismic sequence

*Francesco Clementi¹, Gabriele Milani², Valentina Gazzani¹, †Marina Poiani¹ and Stefano Lenci¹

¹Department of Civil and Building Engineering, and Architecture (DICEA), Polytechnic University of Marche, Ancona, Italy
²Department of Architecture, Built Environment and Construction Engineering (ABC), Politecnico di Milano, Milan, Italy

*Presenting author: francesco.clementi@univpm.it
†Corresponding author: m.poiani@pm.univpm.it

Abstract

The dynamics of the medieval civic clock tower of Amatrice (Rieti-Italy) has been studied by means of the Non-Smooth Contact Dynamics (NSCD) method, implementing a discrete element numerical model in the LMGC90© code. Schematized as a system of rigid blocks, undergoing frictional sliding and plastic impacts, the tower has exhibited a complex dynamic, because of the geometrical non-linearity and the non-smooth nature of the contact laws. Numerical simulations are performed with the aim of comparing the numerical result and the observed damage after the seismic sequence of the Central Italy earthquakes.

Keywords: NSCD method, nonlinear dynamics, masonry towers, damage assessment

Introduction

The seismic events which hit Central Italy on 24th August, 26th and 30th October 2016, and 18th January 2017, have caused casualties and major damage mostly to buildings and architectural heritage of the Italian regions of Marche, Lazio, Abruzzo, and Umbria. The mainshock occurred on August 24th at 3:36 am (local time) with an epicenter close to Accumoli (Rieti province) and with a magnitude $M_w=6.2$; it was followed, at 4.33 am, by an aftershock with an epicenter close to Norcia (Perugia province) and with a magnitude $M_w=5.5$. These events caused a total of 299 fatalities, 386 injured and about 4800 homeless [1,2]. Most of the victims were in the areas of Amatrice, Accumoli, and Arquata del Tronto. In these municipalities, heavy damage and collapse of residential buildings were reported. On 26th October, there were two strong aftershocks, the first at 07:10 pm with $M_w$ 5.6 and the second at 09:18 pm with $M_w$ 6.1. The earthquake of 30th October, which happened at 07:40 am, with a $M_w$ 6.5, is the largest event in terms of released energy occurred in Italy since the $M_w$ 6.9 in 1980 in the event of the Irpinia earthquake. The events of 26th and 30th October did not cause any victim thanks to the evacuation of people from damaged and vulnerable houses after the previous seismic events. It has also to be considered that the October epicenters are located close to Norcia municipality, where many buildings had been strengthened after the 1997 earthquake. Nevertheless, the impact of the seismic events of 26th and 30th October 2016 and 18th January 2017 was distributed on a larger portion of territory extending northwards in the Marche Region respect to the earthquake of 24th August that had a very destructive impact on a restricted area included in the above-listed municipalities. Many small towns and villages, which have survived to the first earthquake, were heavily damaged during the 30th October earthquake [3–5]. Finally, on
18th January 2017 took place a new sequence of four strong shocks of \( M_w = 5 \), with a maximum equal to \( M_w = 5.5 \), and epicenters located between the municipalities of Montereale, Capitignano and Cagnano Amiterno. All these earthquakes are indicated in Fig. 1 with the relative intensity map.

Figure 1. The maps of ground shaking (http://shakemap.rm.ingv.it/shake/index.html) of the four main shocks of the central Italy sequence 2016-2017.

Symbol of the damage and destruction done by a long sequence of the strong earthquake of 2016-2017 is the Amatrice civic clock tower which will be investigated in this paper. An advanced numerical model is here utilized to have an insight into the modalities of progressive damage and the behavior of the structure under strong non-linear dynamic excitations, namely the Non-Smooth Contact Dynamic (NSCD) method. A full 3D detailed discretization is adopted. The tower is schematized as a system of rigid blocks, undergoing frictional sliding and perfect plastic impacts. The structure exhibited a complex dynamic behavior, because of the geometrical non-linearity and the non-smooth nature of the contact laws.
From the numerical results, both the role played by the actual geometries and the insufficient resistance of the constituent materials are envisaged, showing a good match with actual crack patterns observed after the seismic sequence. The numerical analyses provide a valuable picture of the actual behaviour of the structure, thus giving useful hints for the reconstruction.

The Amatrice clock tower

Symbol of the city of Amatrice (Fig. 2a), the Civic Tower rises in the city center namely Cacciatori del Tevere square, underlining the crossroads of the two main streets of the city, Via Roma and Corso Umberto I (Fig. 2b). In the opposite side of the tower there is the Town Hall, intentionally positioned to symbolize the centralization of civic power. In the area behind the tower, still on the same square, it overlooks another small treasure of Amatrice i.e. the Church of San Giovanni. There are few historical data about the Civic Tower, its origins are placed back to medieval times, as early as 1293 was mentioned in ancient documents. These sporadic historical informations tell us that the tower was originally connected to the Church of Santa Lucia, demolished in 1684 by the feudal "lord" Alessandro Maria Orsini who wanted to give a wider space to the street of the main course, expanding the square. On this occasion, the base of the tower was reinforced and a small annex was added, leaning against it on two sides, to obtain the main entrance to the belfry.

![Image: Amatrice (Rieti, Italy) (a), and location of the civic clock tower inside the historic center (b).](image)

The last and probably the only consolidation intervention was carried out on the tower dates back to the early 80s. Already requested in the previous decades it was solicited in 1979, when, following the earthquake of the Alta Valnerina (central Italy), significant damage was noticed to the tower. In 1985 the original bell of 1494 was replaced as the latter, because it had undergone a crack during the restoration phases. The original one is preserved in Madonna di Porta Ferrata Church and a lighter bell has been inserted in the tower so as to avoid the high oscillation of the tower as in the past.

Geometric survey

The civic tower of Amatrice has a rectangular plan of 4.00 m x 5.30 m and a height of about 25 m. At the base, there is a small overhang leaning only two walls, to north elevation with a depth of 1.5 m, and to east elevation with a depth of 0.60 m.

The annex houses the staircase leading to the entrance of upper floors. In its vertical development, there are three distinct areas, identified by decorative frames and folds that externally show the reduction of the wall thickness. The first floor is located at about 9 m in height and it is composed of smoothed stones on the outer side, falling 15 centimeters in the
wall thickness, while the second frame is located at a height of about 19 meters and it marks the passageway from the tower to the belfry. In its highest part, there is the bell which develops longitudinally for just over 5 m. It consists of 4 regular piers, with dimension 0.90x0.80 m², on which rest round arches that make up the single-hole openings on the four sides of the tower. The belfry has a single symmetry, so the openings of the belfry are the same on the opposite sides (see Fig. 3). Finally, the tower presents a pavilion wooden roof.

![Figure 3. Main fronts of the civic tower of Amatrice.](image)

**The model**

In this section the principal peculiarities of the Non-Smooth Contact Dynamics method, the modelling simplifications on which it is based, and its aptitude to reproduce the dynamics of large three-dimensional ancient masonry structures are highlighted. As the deepening of the NSCD theory goes beyond our purposes, an exhaustive description of the method is reported in [6,7]. The problem parameters and the seismic excitation applied to the base of the civic tower are also briefly reported.

*Non-smooth Contact Dynamic method*

The dynamics of a system of rigid bodies is governed by the equation of motion and by the frictional contact conditions. To describe the frictional contact laws, we must introduce some basic definitions. In the following, the notation adopted in [6] is used (scalars, vectors and tensors are explicitly declared, and italic letters are used for all of them). Given two arbitrary bodies $B_i$ and $B_j$, let $P_i$ and $P_j$ (Fig. 4a) be the points of possible contact on the boundaries of $B_i$ and $B_j$, respectively, and let $n$ be the outer unit vector, orthogonal to the boundary of $B_i$ in
We define \( g = (P_j - P_i) \cdot n \) the gap between \( P_i \) and \( P_j \) (a dot means scalar product), \((\dot{u}_n; \dot{u}_t)\) the normal and tangential velocities of \( P_j \) with respect to \( P_i \), and \((r_n, r_t)\) the normal and tangential reactive forces of \( B_i \) on \( B_j \).

The contact conditions are:

1. The Signorini’s law of impenetrability (Fig. 4b)
   \[
   g \geq 0, \quad r_n \geq 0, \quad gr_n = 0, \quad (1)
   \]
   which, in the case of contact \( g = 0 \), is equivalent to the following Kuhn–Tucker conditions [7]
   \[
   \dot{u}_n \geq 0, \quad r_n \geq 0, \quad \dot{u}_n r_n = 0, \quad (2)
   \]
   written in term of relative normal velocity.

2. The dry-friction Coulomb’s law (Fig. 4c), that governs the behavior in the tangential direction
   \[
   \left\{ \begin{array}{l}
   r_t < \mu r_n \rightarrow \dot{u}_t = 0 \\
   |r_n| \leq \mu r_n, \quad |r_t| = \mu r_n \rightarrow \dot{u}_t = -\lambda \frac{r_t}{|r_t|},
   \end{array} \right. \quad (3)
   \]
   with \( \mu \) the friction coefficient, and \( \lambda \) an arbitrary positive real number.

If \( q \) is the vector of the system configuration parameters (unknown translations and rotations of each body), and \( p \) is the global vector of reaction forces, the equation of motion can be written as follows

\[
M \ddot{q} = f(q, \dot{q}, t) + p, \quad (4)
\]

where \( M \) is the mass matrix, and \( f \) is the vector of external forces.

The local pairs \((\dot{u}_n; \dot{u}_t)\) and \((r_n, r_t)\), characteristic of each contact, are related to the global vectors \( q \) and \( p \), respectively, through linear maps which depend on \( q \) (see [1] for details).

Since the contact laws (1) - (3) are non-smooth, velocities \( \dot{q} \) and reactions \( p \) are discontinuous functions of time. They belong to the set of bounded variation functions, i.e. functions which, at each time, have finite left and right limits. Since the accelerations are not defined when the velocities are discontinuous, Eq. (4) is reformulated in integral form (see [6,7] for the mathematical details), and solved numerically using a time-stepping approach, alternative to the event-driven method (we point out that time-stepping approaches are more appropriate than event-driven schemes when problems with many contacts are handled). The time is discretized into time intervals, and, within each time interval \([t_i, t_{i+1}]\), the equation of motion is integrated over the interval as follows

\[
M(q_{i+1} - q_i) = \int_{t_i}^{t_{i+1}} f(q, \dot{q}, t) dt + \bar{p}_{i+1}, \quad (5)
\]

Where \( \bar{p}_{i+1} \) is the impulse in \([t_i; t_{i+1}]\). The primary variables of the problem are the velocity vector \( \dot{q}_{i+1} \) and the impulse vector \( \bar{p}_{i+1} \) at the instant \( t_{i+1} \). In the NSCD method, the integrals in (5) are evaluated by means of an implicit time integrator. The overall set of global Eq. (5) and
local contact relations (1) and (2), where the reactions are approximated by the average impulses in $[t_i, t_{i+1}]$, are condensed at the contact local level, and then they are solved by means of a non-linear Gauss-Seidel by block method.

![Diagram](image)

**Figure 4. The interaction between two bodies (a) Signorini’s law (b) Coulomb’s law (c)**

The numerical model, which implements the NSCD method, is based on some modelling simplifications which deserve some comments in perspective of its application to ancient buildings. Regarding the contacts between bodies, the model does not account for elasto-plastic impacts governed by restitution laws on velocities (Newton law) or impulses (Poisson law) [8], or energetic impact laws [9], although originally the Contact Dynamics method proposed by Moreau [7] considered the Newton law. The relations (1) imply perfectly plastic impact, i.e., the Newton law with restitution coefficient equal to zero. A perfectly plastic impact law makes impossible to describe, for instance, bouncing phenomena, and, furthermore, overestimates the energy dissipated during impacts. However, in the case of systems of bricks or stones, the restitution coefficient has low values, and thus bouncing phenomena are secondary, and they can be neglected. More sophisticated impact laws would bring to more accurate models, such as that proposed in [10] for the dynamics of masonry arches, but they drastically increase the model complexity and result impracticable for large systems with many impacts, like that considered here.

Furthermore, the deformability of blocks is neglected. This is a reasonable approximation since the expected operating compressive stresses at the base of the masonry walls of the tower are reasonably low. Deformable blocks have been considered in [11,12] for two-dimensional systems. Since deformability drastically increases the computational complexity, practically it cannot be applied to large three-dimensional structures, like the tower of this study. On the other hand, simplified two-dimensional schemes rule out a crucial aspect of the dynamics of box-shaped structures such as houses, churches, and towers that is, the interaction between adjacent walls laying on different planes (for instance the façade and the longitudinal walls in churches), which mutually exchange considerable inertia forces. Since we are interested in the dynamical interactions between different parts of the civic clock tower, we consider three-dimensional schemes but we neglect blocks deformability. It follows that the numerical results obtained depict an overall picture of the tower dynamics and describe the failure mechanisms of the whole tower, due to blocks rocking and sliding, but, obviously, they do not give a description of the stresses and strain distributions within each block.

Since experimental data are not available, the friction coefficients were selected from standard values reported in the literature. The values of $\mu$ range from 0.3 to 1.2, according to different combinations of units and mortars [13]. As a first attempt, we assume the value $\mu = 0.5$ for the interface block/block, and $\mu = 0.9$ for the interface block/foundation in order to observe, mainly, the dynamics of the tower without interaction with the foundation. Furthermore, it is important to underline that, in real old masonry buildings, the degradation of the mortar over
time contributes to deteriorate the friction coefficient confirming the hypothesis of the first attempt. Finally, it is important to observe that damping, a fundamental aspect of continuum models, is not considered here, and only friction and perfectly plastic impacts contribute dissipating energy.

Discretization scheme and analysis settings

The geometrical complexity of Amatrice civic clock tower requires some geometrical simplifications, being impossible to reproduce the real layout of the masonry walls, made by brick fragments and ashlars of small size.

![Image of blocks discretization](a)

![Image of view inside the thickness of the walls](b)

**Figure 5. Blocks discretization for the geometrical scheme of the tower in LMGC90® (b) view inside the thickness of the walls in LMGC90®.**

With the final aim of confirming what occurred following the seismic shocks, it was decided to use a punctual mapping of the masonry as it is possible to see from Fig. 2 with the discrete approach using the code LMGC90®. The size of the blocks (Fig. 5a) used is directly taken up by the reliefs of the facades, while the internal wall texture has been hypothesized avoiding the addition of several transversal connections (i.e, *diatoni*) between the two-external leaf. Only in the presence of very small and irregular ashlars, typically at the top of the belfry and in the annex walls, we have used larger dimensions than relief, simply merging up to five (small) adjacent blocks. As it is possible to observe in Fig. 5b, where the numerical model is reported, the two-leaf masonry is modelled at the best of possibilities. Obviously, the rounded geometry of the blocks has been regularized using straight vertical and horizontal surfaces to avoid further computational burdens. Finally, the numerical model is composed of 2899 rigid blocks with different geometries.

Concerning the seismic loading, the accelerations of Amatrice (Italy) of the 24th August 2016 earthquake have been considered. During that earthquake in Amatrice (AMT) a Peak Ground Acceleration (PGA) of 850.804 cm/s² and a Peak Ground Velocity (PGV) of 43.549 cm/s have been registered (see the website: http://itaca.mi.ingv.it). In the numerical simulations, accelerations are applied to FE model and velocities are applied to DE model at the base where the tower is laid. The three velocities components in the three main coordinate directions are determined by direct integration of the accelerations in a time interval of 28 s,
during which the maximum amplitudes are attained, without the use of correlation method (Fig. 6). In the simulations, the time step $dt = 0.005$ s has been used.

**Figure 6. Velocities of the shock of 24th August 2016 applied to the foundation in the three coordinate directions.**

*Preliminary results of numerical analyses*

The first numerical results are reported in Fig. 7 for different time steps. With the above data, the tower collapses at the bell cell, where the overturning mechanisms is favored by the presence of non-regular and small-sized materials. The bell blocks motion occurs in the first 10 s of the seismic excitation, during which the largest acceleration peaks are attained. For time instants larger than 12.5 s, the tower stays at rest, since the acceleration peaks of the seismic excitation are not sufficiently large to activate other blocks sliding. Further damage can also be read from Fig. 7a in the enlargement area at the base of the tower, where there are significant cracks along the perimeter walls, without showing any collapses.

This agrees with the real damage shown by the tower following the shock of 24th August 2016, after that the bell tower was seriously damaged in correspondence of the bell cell (Fig. 7b). The non-activation of the two mechanisms - instead numerically obtained - are certainly due to the presence of an interaction with the roofs which, to a certain extent, have prevented the start of a tilting mechanism of the bell tower columns and of the perimetral wall at the base of the tower.

Then the subsequent earthquakes have led to the collapse of the bell cell already seriously compromised (Fig. 7b).

As can be observed from the Fig. 7a, the NSCD method totally differs from a continuum approach, and it stands as complementary to this latter. While the NSCD accounts for an
accurate description of the motions induced by the inertial masses, a continuum approach
describes stress and strain distributions [3,14–16]. The combination of both two methods can
bring to a complete comprehension of the mechanical response of such complex structures to
seismic loadings, but it will be left for future works.

![Numerical analysis damage with the NSCD method for the friction coefficient $\mu=0.5$ at different time steps](image)

Figure 7. Numerical analysis damage with the NSCD method for the friction coefficient $\mu=0.5$ at different time steps (a) the increment of real damages after the seismic sequences (b).

Conclusions

The Non-Smooth Contact Dynamics method, implemented in the LMG90©, has revealed a
powerful tool to explore the complex dynamical behavior of an ancient brick-made building,
as the civic clock tower of Amatrice (Rieti, Italy). Indeed, it combines modelling simplicity
and great predictive capabilities. Its simplicity comes from the following fundamental simplifying assumptions: (i) block rigidity (often a feasible assumption for masonry structures); (ii) simple contact laws, such as the Signorini’s law, which supposes that impacts between blocks are purely plastic, and the dry friction Coulomb’s law for the tangential relative motions between blocks; (iii) absence of any damping, according to which the kinematic energy of the system is dissipated only by impacts and friction. As a result, the mechanics are governed by only one material parameter, the friction coefficient. This is an advantage for modelling ancient building, for which the determination of the mechanical properties (like the material moduli) is always uncertain and variable. Despite its simplicity, the model can predict a large variety of behaviors.

The numerical results have given a deep insight into the seismic vulnerability of the considered civic clock tower, pointing out the same portions most damaged during the earthquake of 24th August 2016. In fact, the belfry and the enlargement at the base are the most damaged portions following the main shock.

References