Surface discretization based on bionic patterns in search of structural optimization

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Abstract

The division of surface becomes an important element in the pragmatics of creating structural elevations that simultaneously function as an external barrier. The discretization of freely formed surfaces seems to be particularly interesting. Most of the existing solutions are concerned with the shaping of elevation panels. However, in structural searches a greater interest is placed in algorithmic generation of surface divisions, where the use of appropriate method becomes a significant factor in surface discretization. This enables the pursuit of synergistic solutions which optimize both the architectural and structural parameters. As a result one can expect to obtain optimized structural surfaces shaped according to the chosen criteria. In this regard, nature provides an interesting field of research for effective patterns. Biological structures shaped under the influence of acting loads are optimized through evolution on the basis of, among others, minimisation of material and energy consumption. A number of forms observed in the natural world provide interesting examples for observing morphogenetic processes. The implementation of biological patterns in the design of load-bearing structures affects the design of structural forms in contemporary architecture in an interesting way. The article presents the results of model tests conducted on selected grid systems based on structures found in nature.

Keywords: bionic, structural surfaces, structural optimization, discretization of surface, architecture optimization

Introduction

In the design of modern architecture, mathematical methods and algorithms play an increasingly important role, enabling, among others, imitation of patterns taken from nature. The pursuit of synergistic solutions becomes an important element of bionic exploration, especially in the interdisciplinary design environment. In the search for structural solutions in architecture, shape optimization as well as surface discretization methods are interesting research areas. Contemporary digital modeling tools provide new opportunities for the rationalization of technical solutions through the design of the tectonics of architectural forms – thus realizing the idea of cooperation between architecture and structural design. This paper analyzes different variants of canopies' surface division with the use of bionic patterns. The research was aimed at finding effective solutions for the design of structural surfaces.

Methods of structural form's surface discretization

In adopting the method of surface discretization of structural forms it is important to know regular tessellations [1], which are used to determine the division of geometrically complex surfaces and to obtain planar elements. Tessellations can be achieved as a result of geometric transformations of surfaces such as translations, rotation and reflection, and glide reflection - when the surface or space is filled with a repeating geometric motif. The characteristic roof surface of the Sydney Opera House is an example of polygonal tessellation. The spectacular project by Jorn Utzon (created in the years 1956-1973) was the first in which segmental approximation of spherical surfaces with different radii was applied. Structural divisions occurring in nature are characterized by a more complex structure, and what is important, it is possible to find interesting patterns in terms of rationalization of material consumption. The forms observed in the natural world constitute an interesting model of morphogenetic processes - biological development, the aim of which is to create a structure in the context of existing environmental conditions, material, adaptation to live loads, etc. An important factor in the development of organic structures is the minimization of material and energy consumption, which is another significant analogy to the concept of sustainable development in architecture. Parametric modeling, which appeared alongside the possibility of implementing bionic patterns in the field load-bearing structures' design, simultaneously allows to generate diverse variants with the same boundary conditions and to verify the solutions on the basis of the adopted criteria. The Finite Element Method (FEM), which is an advanced numerical method of solving systems of differential equations, uses digital tools in the design and discretization of the surface. Equations of this type most often describe the phenomena and processes known in nature, and in recent years, along with the development of biomimicric trends, are used more and more often in architectural design. In the case of grid structures, FEM divides the structure into finite elements by means of nodal points - in the newly created record, certain geometrical, physical and mechanical features (so-called shape functions) are simultaneously assigned, thus creating a digital model of the structure [3]. Interesting examples of surface optimization using FEM can be found in the projects of the Italian architectural studio Studio Fuksas (Massimilian and Doriana Fuksas). One of the most interesting objects in which the designers used digital triangulation is the 'My Zeil' Shopping Center in Frankfurt located in the PalaisQuartier complex (which has the additional functions of an office and a hotel). The curvilinear surface constituting both the roofing of the building and its frontal elevation was subject to tessellation. From the main entrance to the shopping center there is a characteristic "dent" in the elevation, whose geometry, through the shape of an irregular tube, transforms into a waving roof of commercial spaces. An organic roof with an area of approximately 6000m² was made of a triangular steel mesh (made up of around 3,200 triangles) filled with glass and metal panels. The Knippers Helbig office from Stuttgart was the author of the structure. Another example of the use of digital tools supporting the optimal creation of grid structures by imitating nature can be seen in the Dynamic Relaxation Method. By using Newton's second law, Dynamic Relaxation generates a catenary model for a given point grid - it is possible to search for the optimal number of nodes, and to find the most effective position. Examples of the use of Dynamic Relaxation in surface discretization can be found in Foster & Partners' projects, among others: the

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[1] Gawell E., Nowak A., Rokicki W., *„Aperiodic tesselations in shaping the structural surfaces in the contemporary architecture"*, The Journal Biuletyn of Polish Society for Geometry and Engineering Graphics, Volume 26 (2014), p. 47-54, Publishing House.: Polskie Towarzystwo Geometrii i Grafiki Inżynierskiej (Polish Society of Geometry and Engineering Graphics)
"So far, 17 periodic tessellations have been described, usually composed of regular polygons such as: equilateral triangle, a square, a hexagon, an octagon and a decagon."

Smithsonian Institution in Washington, or at the *British Museum's Great Court* in London. The more irregular divisions built on the basis of structural patterns found in nature are Voronoi diagrams, describing the optimal division of the surface into cells (or planes on convex polygons). The characteristic, polygonal Voronoi fields are created for the indicated points, taking into account their location and value (eg size). One Voronoi cell is created around each point as a polygon adhering to neighboring cells, and the points around which the diagram is created are exactly in the center of gravity of each cell[2]. In addition, the Voronoi Diagram is a dual graph for Delaunay triangulation - in effect, the Voronoi polygon vertices are the centers of circles described on the triangles that form the triangular grid. As a result of the algorithm based on the Voronoi diagram it is possible to create various solutions in the field of surface discretization. Such a method of surface division was used for the *Landesgartenschau Exhibition Hall* project, which was implemented as a research project in the Institute for Computational Design, the Institute of Building Structures and Structural Design and the Institute of Engineering Geodesy at the University of Stuttgart. The design was shaped like the skeleton of a sea urchin, and the structural surface divisions were optimized using numerical analyzes.

Own research for selected grid models

The search for the geometrical shape of structural grids is an important factor in the design of architecture, both reflecting the intended artistic effect as well as rationalizing technical solutions. From the point of view of the solid's tectonics the surface grid division affects the visual reception of the form. Determining the metrics of the distribution is crucial for the realization of the curves describing the form such as in the case of free-form architecture. Increasing or decreasing the density of the grid is fundamental to structural optimization – too many elements cause larger deformations and increase stress due to own weight etc. Obtaining effective structural grid divisions should be addressed interdisciplinarily when analyzing paradigms from two disciplines: architecture and construction. Such phenomena can be observed naturally in the development processes of living organisms. Thanks to the algorithmization of tools in the design process such a pattern can be adopted in the creation of eco-efficient solutions for contemporary architecture. A recently popularized example can be observed in the use of Delaunay triangulation in the construction of grid structures (by using multi-variant solutions for the indicated boundary parameters). Choosing one grid system becomes a difficult task, requiring the ability to rationalize technical solutions in search for artistic effects. The results of the grid structure analysis are described below its construction was modeled on a bionic model. It assumed that the catenary was set on rectangular plan with the 30.0m x 26.0m dimensions, with three supports – the minimal number of supports guaranteeing stability of the structure while reducing any unnecessary geometry (Fig. 1). The supports were set with the assumption that the proportions between the individual support and cantilever spans were fixed. The supports were positioned in such a way that the lines passing through them are always in the 1:3:1 ratio, so the cantilevers are 6.0 m long and the distance between them is 18.0 m (according to the scheme – Fig. 2). Due to the varying length of the bars, the roof surface was discretized and the divisions were based on Delaunay triangulation. (Fig. 3). The idea of search for the curvilinear forms was based on a catenary model, by carrying out transformations in the third dimension for each of the metric variants. The adopted five-step curvature of the surface was dependent on the variable height in the

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[2] On the basis of the Voronoi diagram, two-dimensional space is divided so that for a given set of ***n*** points, the plane is divided into ***n*** areas in such a way that each point in any area is closer to a specific point from the set of ***n*** points than from the other ***n-1*** points. Voronoi cells, being an intersection of half-planes, are convex polygons whose collection breaks the two-dimensional Euclidean space creating an optimal net built of nodal points.

proportion of the curvature height to the support span equal to 1/8, 1/6, 1/5, 1/4, and 1/3 (Fig. 3). Models were generated in the Rhinoceros program with the Grasshopper and Kangaroo2 plugins. Determining the basic permutation for planar systems required an assumption regarding the divisions (due to the maximum bar length) – the transformations of the curvilinear variants occurred only in the third dimension, while maintaining the XY axis division of the structure. Additionally, for all examined systems, the location and the fixing of supports (restraints) were assumed to be constant. In addition, homogeneous THEX profiles (hexagonal tubes), TRON (round tubes) and TREC (rectangular tubes) made of S335 steel were used. Due to the initial assumptions, it was assumed that the analyzed structure variants will be homogenous, hence each construction bar will have the same cross-section. To obtain comparable results, the cross-section database was limited to closed sections. Because all variants have a similar geometry (after projecting onto a flat plane), the maximum permissible deformation of elements equal to 17.9cm was assumed. Due to the fact that the geometry of the analyzed structures is a model study, the analyzes were carried out by taking into account the own mass of the structure, wind and snow load according to EC, and the assumed load of 1.0 kPa for the coating material.

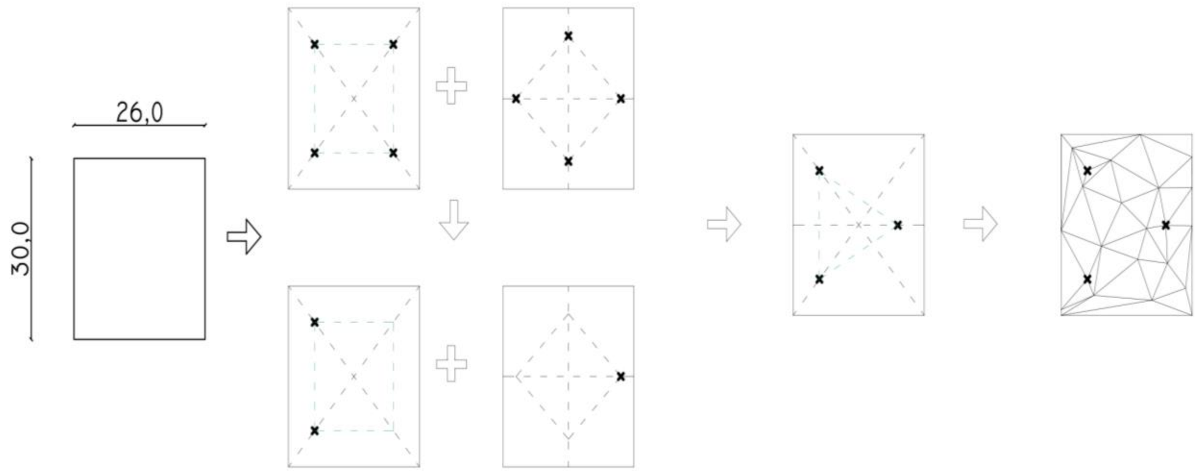


Fig. 1. The principle of setting the roofing supports

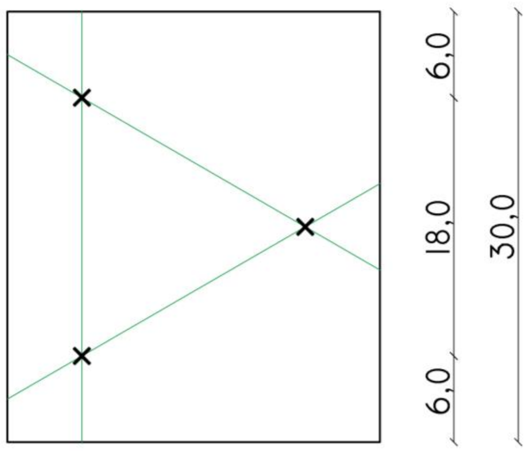


Fig. 2. Scheme of supports' placement – top down view of the model

Planar grid structures – discretization of the Surface

The use of triangular divisions allowed for the creation of a geometrical model, divided into the simplest elements connected at the nodes. Computer aided engineering calculations were used for the structural analyzes using the finite element analysis. Defining the grid metric was the first step of research. We analyzed 3 random planar variant solutions with various degrees of grid density (Fig.3):
- ***Variant 1*** – maximum bar length = 3,5m
- ***Variant 2*** – maximum bar length = 4,0m
- ***Variant 3*** – maximum bar length = 4,5m

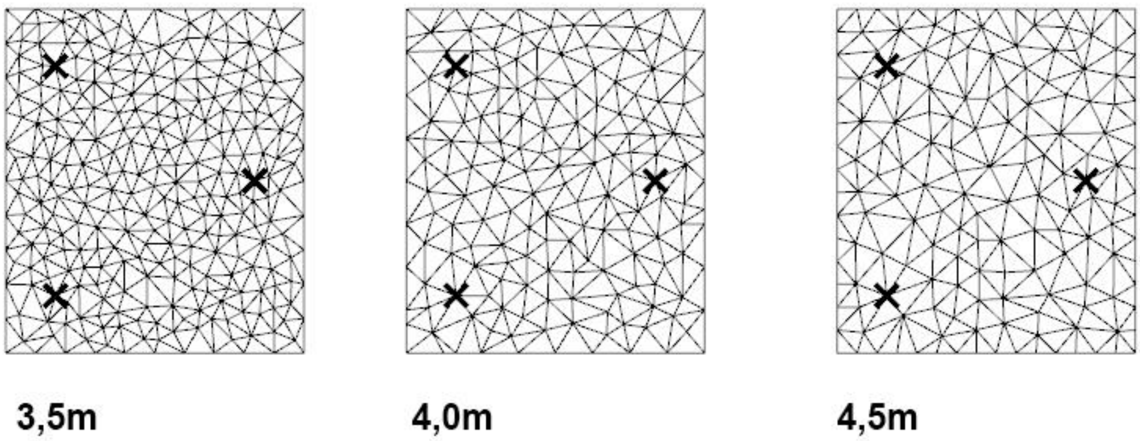


Fig. 3. Juxtaposition of the planar variants determining the grid metrics for curvilinear divisions

The planar systems achieved poor computational results and were thus only used to guide further research. From the 3 analyzed planar variants, the one with the bar length not exceeding 4,5m proved to be the most effective. Thus, further geometrical deformations were made by using this metric.

Space grid structures – shape optimization

In the search for structural efficiency due to the accepted minimum mass criterion, the surface was transformed using Dynamic Relaxation in the Grasshopper / Kangaroo2 plugin. Spatial models were created as a result of moving the center point of the plane on the Z axis (the center of gravity between the supports), while maintaining a constant horizontal projection of structural divisions. In the analyzed variants, the ratio of height to support span was assumed as constant:
- ***Variant 4*** – 1/8 ratio
- ***Variant 5*** – 1/6 ratio
- ***Variant 6*** – 1/5 ratio
- ***Variant 7*** – 1/4 ratio
- ***Variant 8*** – 1/3 ratio

By creating catenary models with different heights for the selected planar layout (with a maximum bar length of 4.5m), 5 curvilinear solutions were obtained (Fig.4). It was noticed that as the proportion between the height and the span of curvature between the supports increased, the total mass of the structure decreased (Tab.1). Although the curvature of the surface resulted in an increase of the total length of the bars, the mass of the most effective

Variant 8 (1/3) was smaller by 45% in relation to the heaviest Variant 4 (1/8). In addition, the bar cross-section has also decreased (the difference between the two extremes variants is 33%), making the structure more attractive in terms of aesthetics, which affects the reception of architecture (Fig. 6a, c). Due to the popularization of formative building technologies, such as the printing of construction and building elements, the parameter calculated as the ratio of total weight to the square meter area [kg/m^2] is significant (Tab.1). Curving the surface up to a 1/3 ratio would make it possible to create twice as effective structures as by using the 1/8 ratio.

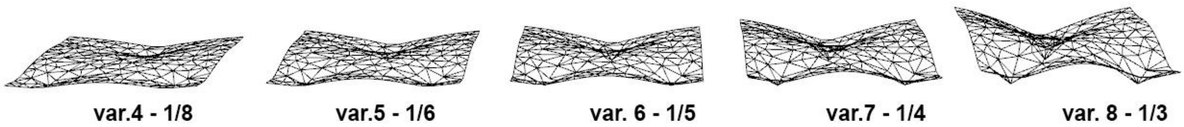


Fig. 4. Analyzed variants of space grid structures

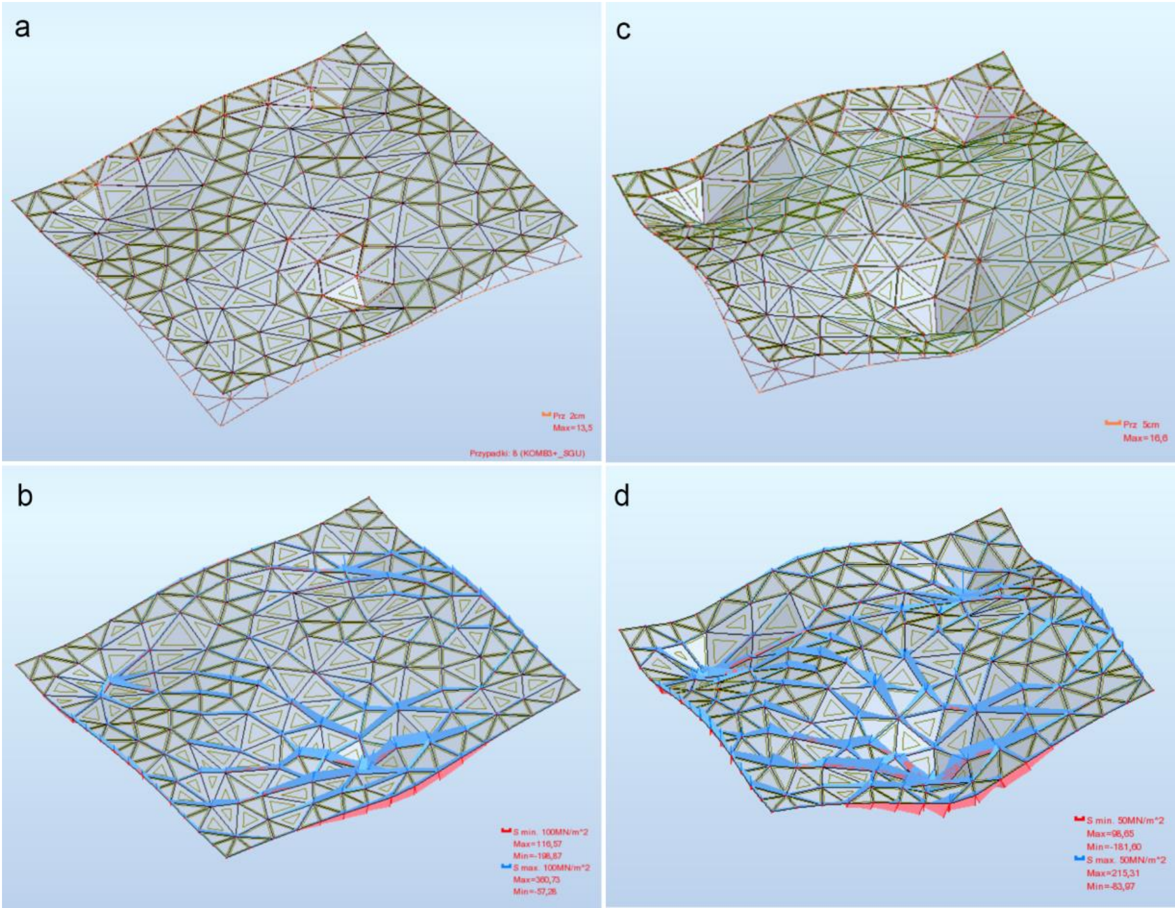


Fig. 5. Analysis for two extreme variants; ***Variant 4*** (1/8): ***a*** - deformation chart; ***b*** - stress diagram; ***Variant 8*** (1/3): ***c*** - deformation chart; ***d*** - stress diagram

Tab. 1. Calculation results for spatial structures when the projected bars do not exceed 4,5m in length.

	Types of structure		Profile				
	lenght of single planarized bar [m]	height of centre point/supports span	Type/Dimensions[mm]	max deformation [cm]	total lenght of elements [mm]	total weight [kg]	kg/m2
var. 4	4,5	1/8	TRON 406x8	13,5	1160	91 185	110,61
var. 5	4,5	1/6	TRON 355x8	13,7	1166,98	80 030	95,96
var. 6	4,5	1/5	TRON 323x7,1	15,6	1173,14	65 075	77,25
var. 7	4,5	1/4	TRON 323x6,3	14,4	1184,44	58 446	68,18
var. 8	4,5	1/3	TRON 273x6,3	16,6	1207,01	50 014	56,39

According to the assumptions adopted in the study, the maximum permissible deformation of 17.9 cm was met for all the variants. In each of the systems, the largest bending occurred on the cantilevers. However, differences in the work of individual bars can be seen in the stress diagram - curvature of the surface caused an increase in the number of compressed rods in the support zones forming structures similar to arboral supports [2]. At the same time, it is worth mentioning that the curvilinear deformations of the roof also cause changes in the reactions at the fixed supports.

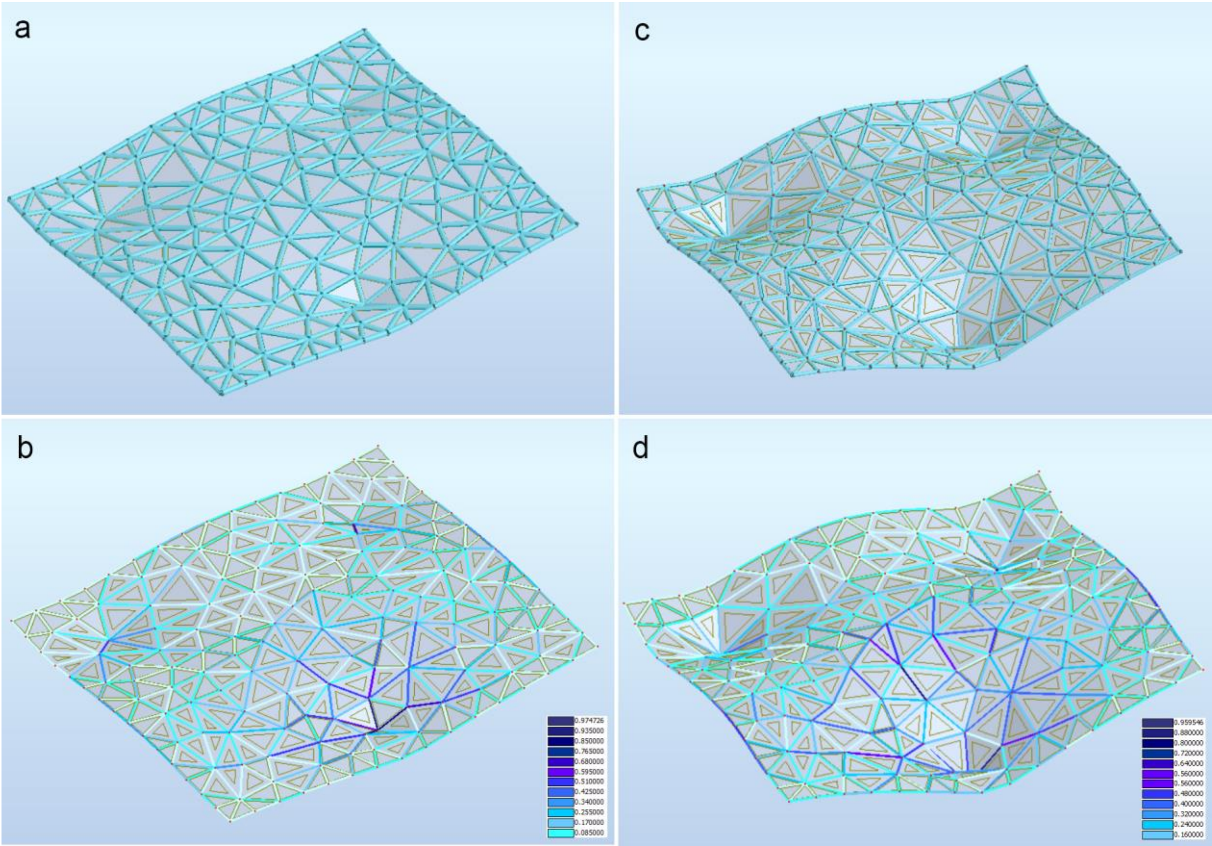
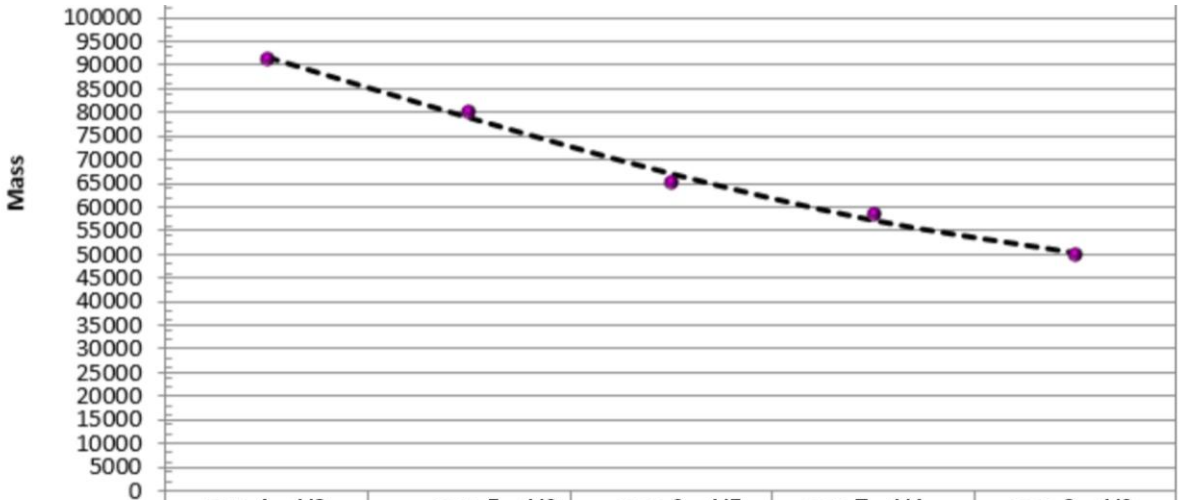


Fig. 6. Analysis of two extreme variants; ***Variant 4*** (1/8): ***a*** - shape of the roof together with the sketch of the profile; ***b*** – graph of limit state ratio for individual bars; ***Variant 8*** (1/3): ***c*** - shape of the roof together with the sketch of the profile; ***d*** – graph of limit state ratio for individual bars

As a result of approximation with the third degree polynomial, the least squares method was used for the given five variants. The curve determined in this way convinces us that for the assumed boundary conditions the optimum was not found. However, continuing the optimization process by increasing the degree of surface curvature on the one hand will lead to a change in the structure of the system (which may be desirable), and on the other will affect the aesthetics and functionality of the architectural form. So in the pursuit of optimal grid structures in architecture, making rational decisions through compromise, also in the context of designing the structure and determining the direction of optimization, is an important element.



	var. 4 - 1/8	var. 5 - 1/6	var. 6 - 1/5	var. 7 - 1/4	var. 8 - 1/3
y_i	91185	80030	65075	58446	50014
$p^3(x_i)$	91502,92858	78758,28572	66982,61144	57174,28574	50331,9286

Fig. 7. Polynomial approximation diagram for ***$p^3(xi)$*** and ***yi*** values (mass) for individual curvilinear variants

Summary

Nowadays we are observing a growing interest in bionic solutions. Digital tools supporting architectural design are more often equipped with methods of surface discretization using laws governing the processes occurring in nature. Biomimicry in architecture fosters the development of digital technologies, thanks to which it is possible to map chaotic systems, while the ways of reproducing the undeniable beauty of nature are gradually discovered and described by means of discrete mathematics.
Structural optimization is an integral part of shaping of the architectural vision, and the development of knowledge and the standardization of generative design methods have a significant impact on the way the architectural form is designed - its shape and the structure of the building's surface. Due to dynamically developing digital processes of design optimization the designers are not only aware of its logical use in bionic structures but also are able to use generative modeling methods to determine rational criteria for more detailed searches.
Delaunay triangulation is one of the bionic methods of surface division which finds application in architecture in the design of shell elements such as roofs, elevations, walls, etc. In architectural and construction optimization Delaunay triangulation can be used as an

alternative way of search for rational, and at the same time, tectonically innovative structural solutions. An important element of the optimization of the structural surfaces is the determination of the metric describing the density of the grid - so that the structure on one hand meets the visual effect intended by the architect, and on the other hand it remains a rational solution in the context of construction and building logic.

References

[1] Gawell Ewelina, Nowak Anna, Rokicki Wiesław, *„Aperiodic tesselations in shaping the structural surfaces in the contemporary architecture"*, The Journal Biuletyn of Polish Society for Geometry and Engineering Graphics, Volume 26 (2014), Publishing House.: Polskie Towarzystwo Geometrii i Grafiki Inżynierskiej (Polish Society of Geometry and Engineering Graphics)

[2] Md Rian Iasef, Sassone Mario, *Tree-inspired dendriforms and fractal-like branching structures in architecture: A brief historical overview,* "Science Direct", < https://doi.org/10.1016/j.foar.2014.03.006 >, [access: 28.06.2018]

[3] Sieczkowski Jeremi Michał, „Podstawy komputerowego modelowania konstrukcji budowlanych", Publishing House: Oficyna Wydawnicza Politechniki Wrocławskiej, Wroclaw 2001, ISBN 83-7085-573-3

[4] Tarczewski Romuald, *Topologia form strukturalnych; naturalne i tworzone przez człowieka prototypy form konstrukcyjnych w architekturze [Topology of structural forms; natural and manmade prototypes of structural forms in architecture]*, Publishing House: Oficyna Wydawnicza Politechniki Wrocławskiej, Wroclaw 2011, ISBN 978-83-7493-660-6