Numerical Investigation of Vortex-induced Vibration (VIV) of a Flexible Cylinder in Combined Flow

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Abstract

In actual oil exploration process, vortex-induced vibration (VIV) is the main source of structural fatigue damage of the risers. In this paper, VIV of a flexible cylinder experiencing combined uniform and oscillatory flow is investigated numerically. All investigations are carried out by the in-house CFD code viv-FOAM-SJTU, which is developed basing on the pimplyDyMFOAM solver attached to the open source OpenFOAM. The effects of flow ratio on VIV are concentrated, while the flow ratio α is defined as the proportion of the uniform flow velocity in the total velocity. Main parameters of the cylinder are as follows: the mass ratio $m^* = 1.53$, the diameter $D = 0.024$, the length $L = 4$ and the Keulegan–Carpenter (KC) number KC=178. The flow ratio varies from 0 to 1 with an interval of 0.2. The modal analysis method and the wavelet analysis method are used to study the effect of flow ratio to VIV response of the cylinder in combined flow.

Keywords: vortex-induced vibration (VIV); viv-FOAM-SJTU solver; strip method; flow ratio

Introduction

Vortex-induced vibration (VIV) of a flexible cylinder in steady flow has been investigated extensively during the past decades through experimental and numerical methods, such as Chaplin et al[3][4], Lie and Kaasen[9], Willden and Graham [13][14] and Yamamoto et al[16]. Chaplin et al[3] carried out benchmark experiments of VIV of a long flexible vertical tension cylinder in stepped flow. The modal analysis method is used to obtain modal weights of each vibration mode and determine the dominant vibration mode of the cylinder. Lie and Kaasen[9] also used the modal analysis method to analyze the vibration feature of a flexible cylinder in sheared flow. And they chose to solve modal amplitudes through the least-square sense with the existence of some modal amplitudes that are not physical with regard to the frequency content. Willden and Graham[13] and Yamamoto et al[16] carried out numerical studies of VIV of a flexible cylinder in uniform flow adopting strip method. Numerical results were in good agreements with experiments and the strip method was appropriate for solving VIV problems.

In order to study the vibration features of a circular cylinder around oscillatory flow, comprehensive researches have been carried out by Bearman[1][2], Kozakiewicz et al[8], Sarpkaya[11][12], Williamson[15], and Zhao et al[17]-[19]. Williamson[15] and Sarpkaya[11][12] conducted a series of experiments to investigate motions of vortices around a single cylinder in relative oscillatory flow. And several vortex regimes were identified within particular ranges of Keulegan-Carpenter (KC) Numbers: the attached vortices regime

(0<KC<7), where no major vortices shed during a cycle; the single pair regime (7<KC<15); the double pairs regime (15<KC<24); the three pairs regime (24<KC<32) and ect. For further KC regimes, the number of vortices pairs shed in each oscillating period would be increased by one each time the KC regime changed to a higher one. Kozakiewicz et al[8] and Zhao et al[17] carried out experiments and numerical simulations of a cylinder exposed to oscillatory flow for two KC numbers of 10 and 20 respectively. Kozakiewicz et al[8] found that the cross-flow vibration of the cylinder changed the vortex shedding trail and the number of vortices generated over one oscillating cycle comparing with the fixed cylinder. Zhao et al[17] found that the reduced velocity had significant effects to the XY- trajectory mode of the cylinder and the VIV frequency decreased with the increase of reduced velocity. And when the reduced velocity was extremely large, the vibration amplitude in the cross-flow direction was negligible smaller than that of the inline direction. Basing on the previous simulations, Zhao et al[19] carried out simulations of a circular cylinder experiencing combined oscillatory flow and steady flow at KC=10. They found that the lock-in regime was widened due to the combination of oscillatory and steady flow and the widest lock-in regime were twice as wide as that in the pure oscillatory or pure steady flow. For flexible cylinder condition, Fu[7] carried out a series of experiments of a flexible cylinder in relative oscillatory flow at KC=178. They proposed the VIV development process of “Build up—Lock in—Die out” in each half oscillating period. And Moreau and Huang[10] conducted experiments of cross-flow vortex-induced vibration in combined in-line current and oscillatory flow, including 12 different combinations of flow and cylinder conditions in total. He found that the VIV amplitude response was much reduced in the combined flow comparing with the pure steady flow at a given reduced velocity.

In this paper, VIV of a flexible cylinder experiencing combined oscillatory and uniform flow is investigated. All numerical simulations are carried out by the in-house CFD code viv-FOAM-SJTU, which is developed basing on the strip theory method and the pimpleDyMFOAM solver attached to the open source code OpenFOAM. The effect of flow ratio is concerned and the numerical model refers to the experiments of Fu et al[7]. The flow ratio α is defined as the proportion of the uniform flow velocity in the total velocity. The flow ratio varies from 0 to 1 with an interval of 0.2. Firstly, the validation is conducted at the pure oscillatory flow condition comparing results of cross-flow vibration history and dominant vibration frequency with Fu et al[7]. Then modal analysis and wavelet analysis methods are used to study the effect of flow ratio to VIV response.

This paper is organized as follows: the first section gives a brief introduction to the referenced experiments and the numerical methodology. The second section presents the results and the final section concludes the paper.

Method

Hydrodynamics Governing Equations

The flow field is supposed to be incompressible, with constant dynamic viscosity μ and constant density ρ. The Reynolds-averaged Navier-Stokes equations are used as the hydrodynamics governing equations as follow:

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0 \qquad (1)$$

$$\frac{\partial}{\partial t}(\rho\bar{u}_i)+\frac{\partial}{\partial x_j}(\rho\bar{u}_i\bar{u}_j)=-\frac{\partial\bar{p}}{\partial x_i}+\frac{\partial}{\partial x_j}\left(2\mu\bar{S}_{ij}-\rho\overline{u_j^{'}u_i^{'}}\right) \tag{2}$$

where $\bar{S}_{ij}=\frac{1}{2}\left(\frac{\partial\bar{u}_i}{\partial x_j}+\frac{\partial\bar{u}_j}{\partial x_i}\right)$ is the mean rate of strain tensor, $-\rho\overline{u_j^{'}u_i^{'}}$ is referred as Reynolds stress τ_{ij} computed by $\tau_{ij}=-\rho\overline{u_j{'}u_i{'}}=2\mu_t\overline{S_{ij}}-\frac{2}{3}\rho k\delta_{ij}$, where μ_t is the turbulent viscosity and $k=(1/2)\overline{u_i{'}u_i{'}}$ is the turbulent energy, computing from the fluctuating velocity field.

Structural Dynamic Governing Equations

In order to form the relatively oscillatory flow, the supporting frame is forced to oscillate harmonically during the investigations. The oscillation can be expressed as:

$$x_s = A_m\cdot\sin(\frac{2\pi}{T}t) \tag{3}$$

$$U_s=\frac{2\pi\cdot A_m}{T}\cdot\cos(\frac{2\pi}{T}t)=U_m\cdot\cos(\frac{2\pi}{T}t) \tag{4}$$

$$KC=\frac{2\pi\cdot A_m}{D}=\frac{U_m\cdot T}{D} \tag{5}$$

where A is the oscillating amplitude，T is the oscillating period, x_s is the oscillating displacement，U_s is the oscillating velocity ，U_m is the amplitude of the oscillating velocity ，D is the diameter of the cylinder。

Fu[6] uses the support excitation method combined with the Bernoulli–Euler bending beam theory to obtain the structural response of the cylinder. The in-line displacement of the cylinder is the sum of support frame motion and the relative in-line vibration of the cylinder:

$$x_t = x_s + x \tag{6}$$

where x_t is the in-line displacement, x_s is the support displacement and x is the relative in-line displacement.

The equilibrium of forces for this system can be written as follow:

$$f_I + f_D + f_S = f_H \tag{7}$$

where f_I, f_D, f_S, f_H are the inertial, the damping, the spring, and the hydrodynamic force respectively.

Then the equilibrium of forces for the system can be written as:

$$m\ddot{x}_t + c\dot{x} + kx = f_H \tag{8}$$

$$m\ddot{x} + c\dot{x} + kx = f_H - m\ddot{x}_s \tag{9}$$

where m, c, k are the mass, the damping and the stiffness of the system.

Adopting the finite element method(FEM), the equations can be discretized as:

$$\mathbf{M\ddot{x}+C\dot{x}+Kx=F_{Hx}-M\ddot{x}_s} \tag{10}$$

$$\mathbf{M\ddot{y}+C\dot{y}+Ky=F_{Hy}} \tag{11}$$

where **M**, **C**, **K** are the mass, the damping and the stiffness matrices, while $\mathbf{x}$, $\mathbf{x_s}$ and $\mathbf{y}$ are the relative in-line, the support and the cross-flow nodal displacement vectors. $\mathbf{F_{Hx}}$ and $\mathbf{F_{Hy}}$ are the hydrodynamic force in the in-line and cross-flow direction respectively.

Problem Description

The numerical model used in this paper follows experiments of Fu[7] and the layout of the experiments is shown in Fig. 1. Detailed information about main parameters of the cylinder is shown in Table 1. 20 strips located equidistantly along the cylinder totally. Fig. 2 shows the distribution of flow field strips along the span of the cylinder and the entire computational domain and meshes of strips.

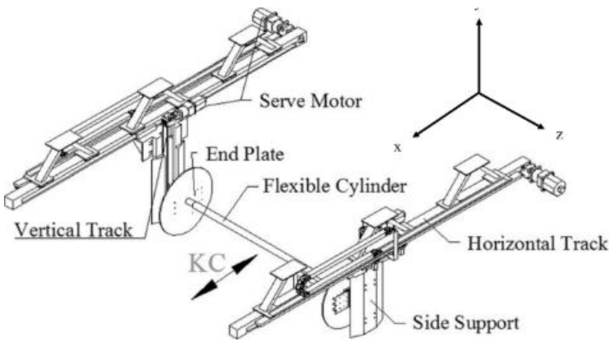
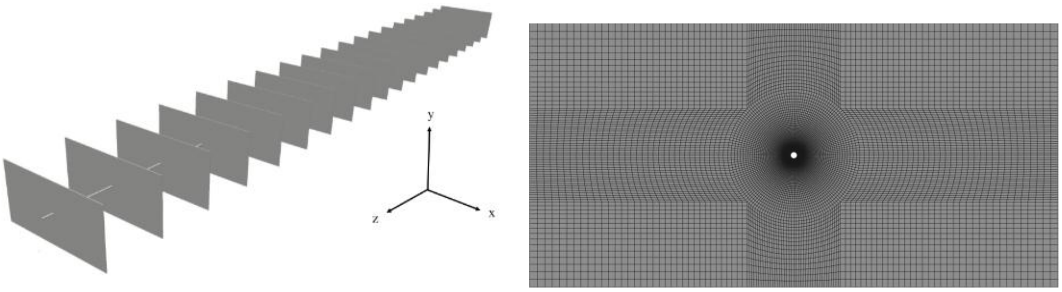


Figure 1. Layout of the experiments of Fu et al

Table 1: Main parameters of the cylinder

	Symbols	Values	Units
Mass ratio	m^*	1.53	−
Diameter	D	0.024	m
Length	L	4	m
Bending stiffness	EI	10.5	$N \cdot m^2$
Top tension	T_t	500	N
First natural frequency	f_n^1	2.68	Hz
Second natural frequency	f_n^2	5.46	Hz



(a) (b)

Figure 2. Illustration of multi-strip model and computational domain of a strip

In this paper, VIV of a cylinder in combined uniform and oscillatory flow are investigated. The flow ratio α represents the proportion of uniform flow velocity in the total flow velocity. According to equations (3) and (4), the total velocity and the flow ratio can be written as equations (12) and (13). Detailed computational conditions are shown in Table 2.

$$U_c(t) = U_s + U_m \cos(\frac{2\pi}{T} t) = U_s + A_m \frac{2\pi}{T} \cos(\frac{2\pi}{T} t) \tag{12}$$

$$\alpha = \frac{U_s}{U_s + U_m} = 1 - \frac{A_m}{U_c} \cdot \frac{2\pi}{T} \tag{13}$$

where U_s is the uniform flow velocity, U_c is the total velocity, A_m is the amplitude of the oscillation, T is the oscillating period.

Table 2 Computational conditions

	U_c	α	U_s	A_m	T	KC
Case1	0.2589	0	0	0.68	16.5	178
Case2	0.2589	0.2	0.05178	0.68	20.6	178
Case3	0.2589	0.4	0.10356	0.68	27.5	178
Case4	0.2589	0.6	0.15534	0.68	41.3	178
Case5	0.2589	0.8	0.20712	0.68	82.5	178
Case6	0.2589	1	0.2589	0.68	-	178

Strip Theory

In this paper, numerical investigations are carried out by the viv-FOAM-SJTU solver basing on the strip method and the pimpDyMFOAM solver attached to the open source code OpenFOAM. The strip method is very appropriate for solving CFD investigations of supramaximal computational domain. It owns high computational efficiency and the computational accuracy is reliable, The reliability of the viv-FOAM-SJTU solver has been testified by Duan[2]' in which the benchmark case has been verified in detail.

For a long flexible cylinder, the direct computation of the three dimensional flow field will cost too much resources. Instead of this, we simplify CFD model and obtain the two dimensional flow field on strips distributed equably along the cylinder. The hydrodynamic force is obtained from each strip, which is then applied to the structural field. The structural displacements of all nodes are interpolated to get the boundary motion of dynamic mesh of flow field. The strip theory is shown as Fig. 1.

During the numerical investigations, the RANS equations and SST k-ω turbulence model are adopted to solve the flow field in each strip, while the whole structure filed is solved through Bernoulli–Euler bending beam theory with the finite element method. The fluid-structure interaction is carried out by loose coupling strategy.

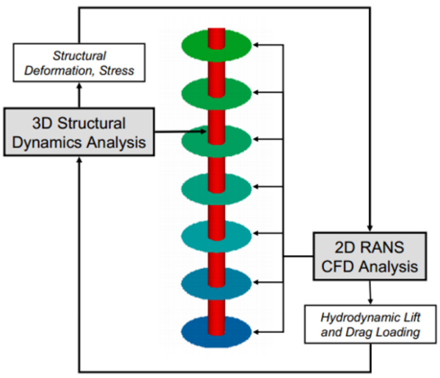


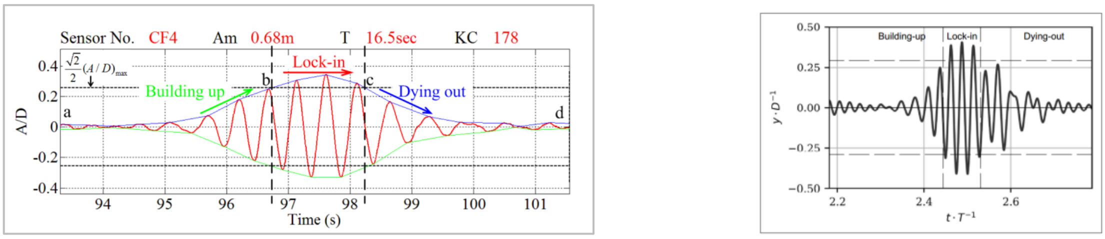
Figure. 3 Schematic diagram of strip theory

Results

Validation

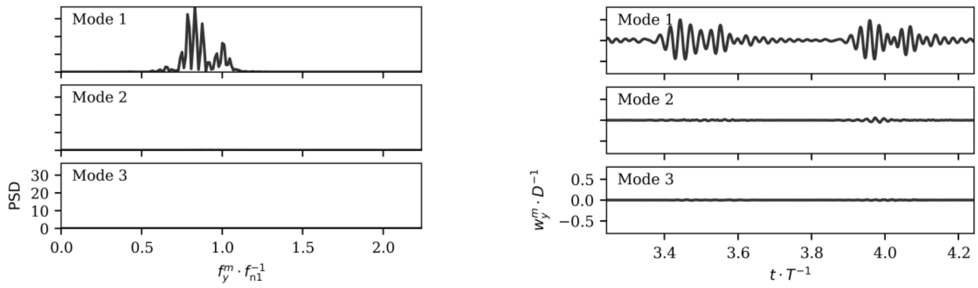
Fig. 4 shows subplots of non-dimensional cross-flow amplitude of the intermediate node of the cylinder between experiment and simulation. From comparison, it can be concluded: (i)the development process of “Building-up—Lock-in—Dying-out” of vortex-induced vibration is observed in both experiment and numerical simulation; (ii)the lock-in region is 17.3% of the half oscillating period in numerical simulation, which is close to the experiment result of 17%; (iii)the non-dimensional cross-flow amplitude is 0.37D in half oscillating period, which is close to the experiment result of 0.36D.

Fig. 5 are subplots of power spectral density and modal weight of each vibration mode in an oscillating period respectively. From these figures, it can be known that the dominant vibration frequency is 2.2Hz, which is close to the result of experiments of 2.1Hz. While the dominant vibration mode of the cylinder is the 1st mode.



(a)Result of Fu et al. (b)The present simulation

Figure 4. Non-dimensional cross-flow vibration amplitude of the intermediate node in half an oscillating period



(a) (b)

Figure 5. Cross-flow power spectral density and modal weight of each vibration mode of the intermediate node: (a) power spectral density; (b) modal weight

Modal Analysis

Fig. 6 are subplots of non-dimensional cross-flow vibration amplitude of the intermediate node of the cylinder ranging from α=0 to α=1.0 in an oscillating period. Two VIV development process of “Building-up—Lock-in—Dying-out” can be observed both in Fig. 6(a) and 6(b), which shows that the oscillatory flow plays the dominant role in the VIV phenomenon of the cylinder. There are two obvious lock-in region in Fig. 7(b) at α=0.2 in an oscillating period. When flow velocities are in the same direction, the vibration amplitude is 0.27D and the lock-in region is 31.4% of the half oscillating period. When flow velocities are in the opposite direction, the vibration amplitude is 0.05D and the lock-in region is 13.6% of the half oscillating period. As shown in Fig. 6(c) and 6(d), the obvious VIV phenomenon is observed in the half oscillating period where the oscillatory flow velocity and the uniform flow velocity are in the same direction. While no obvious VIV phenomenon happens when two flow velocities are in opposite direction. Both oscillatory flow and uniform flow have non-negligible influence to the VIV of the cylinder. With the increase of flow ratio, the proportion of uniform flow velocity in the total flow velocity increases and the dominant effect of the uniform flow to VIV of the cylinder becomes obvious. From Fig. 6(e) and 6(f), it can be seen that the obvious VIV phenomenon is observed in the whole oscillating period. It can be concluded that the vibration feature of the cylinder in combined flow is similar to that in pure oscillatory flow (α=0) when flow ration $\alpha \leq 0.2$ and similar to that in pure uniform flow (α=1.0) when $\alpha \geq 0.8$.

Comparing Fig. 6(b) with 6(c), it can be known that the dominant effect of oscillatory flow becomes weak with the increase of flow ratio. When flow velocities are in the same direction at α=0.4, the reduced velocity is large enough to generate VIV phenomenon in the whole half period. When oscillatory flow velocity reverses, the reduced velocity decreases and no VIV phenomenon generated in the whole half period. When flow velocity increases to α=0.6 as shown in Fig. 7(d), VIV phenomenon occurs in the whole region when flow velocities are in the same direction as that of α=0.4 and in the preliminary stage and final stage of the half period when flow velocities are in the opposite direction. In these two stages, the oscillatory flow velocity is small and the uniform flow still owns dominant effect to the vibration of the cylinder. However in the intermediate stage, the increase of oscillatory flow velocity leads to the decrease of total flow velocity, then VIV phenomenon becomes weaker and disappears finally. When flow velocity increases to α=0.6 as shown in Fig. 7(e), it is found that the uniform flow plays dominant role in the cross-flow vibration of the cylinder. And the cross-flow vibration amplitude reaches its peak or valley value when the oscillatory velocity reaches its peak value.

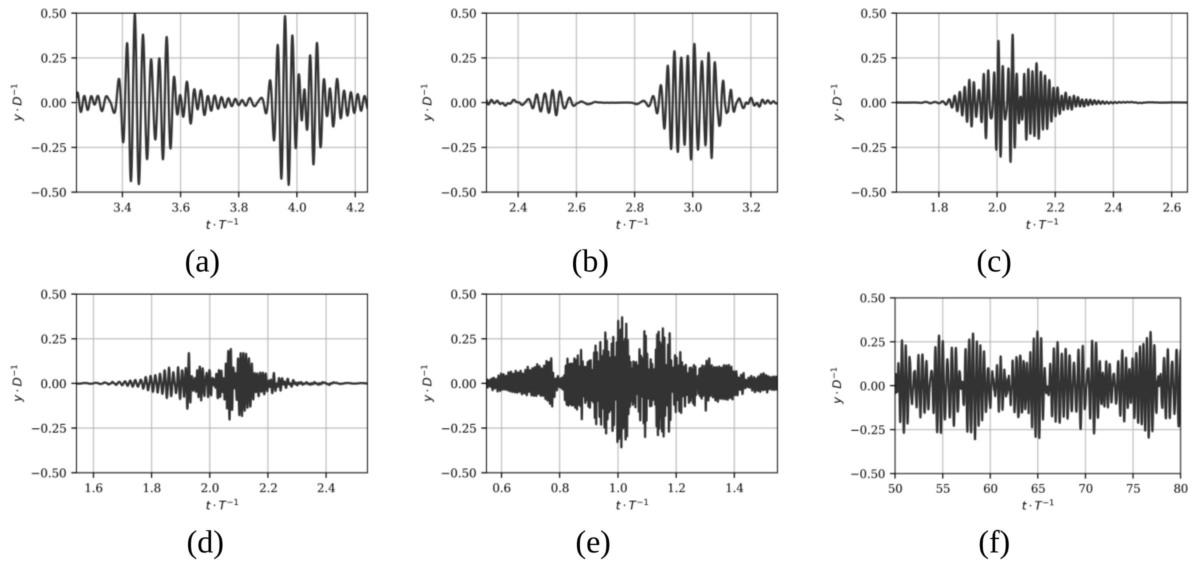
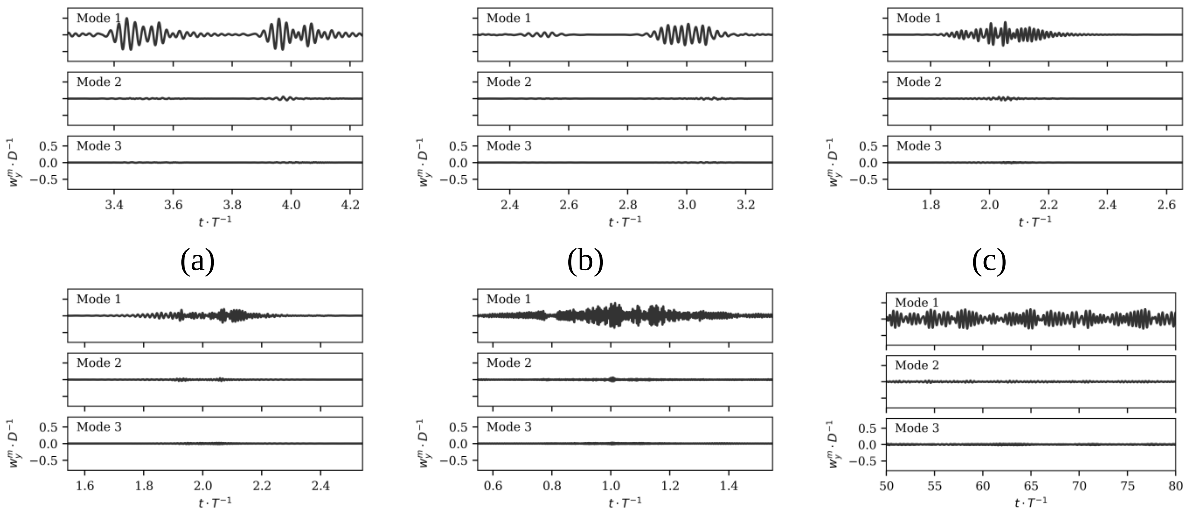


Figure 6. Non-dimensional cross-flow vibration amplitude of the intermediate node in an oscillating period: (a) α=0; (b) α=0.2; (c) α=0.4; (d) α=0.6; (e) α=0.8; (f) α=1.0

Fig. 7 are subplots of cross-flow modal weight of each vibration mode of the intermediate node of the cylinder ranging from α=0 to α=1.0 in an oscillating period. It can be found that the first mode is the dominant vibration mode when obvious VIV phenomenon occurs. And the disturbance of second mode is too small to change the dominant vibration mode of and only can be observed in pure oscillatory flow and combined flow when flow velocities are in the same direction.



(a) (b) (c) (d) (e) (f)

Figure 7. Cross-flow modal weight of each vibration mode of the intermediate node in an oscillating period: (a) α=0; (b) α=0.2; (c) α=0.4; (d) α=0.6; (e) α=0.8; (f) α=1.0

Wavelet Analysis

The wavelet analysis method is used to obtain the dominant cross-flow vibration frequency of the cylinder along the time. Fig. 8 shows cross-flow vibration wavelet of different nodes along the span of the cylinder. These subplots can be divided into three groups: (A) Fig. 8(a) and 8(b); (B) Fig. 8(c) and 8(d); (C) Fig. 8(e) and 8(f). The periodical variation of the dominant vibration frequency is obvious from Fig. 8(a) to 8(d).

In Fig. 8(a) and 8(b), the dominant vibration frequency is close to the first natural frequency of the cylinder in most of the oscillating period. In the flow ratio range of group A, the oscillatory flow has significant effect to the cross-flow vibration frequency. With the increase of flow ratio, the proportion of the cross-flow vibration frequency close to the first natural frequency decreases. As shown in Fig. 8(c) and 8(d), we can see that the increasing and decreasing process of the dominant vibration frequency is similar to the shape of sinusoidal function. During the half process of flow velocities in the same direction, the cross-flow vibration of the cylinder is drastic which leads to the generation of high vibration frequency region. During the half process of flow velocities in the opposite direction, the cross-flow vibration of the cylinder is relatively weak which leads to the generation of the low vibration frequency region. In the flow ratio range of group B, both oscillatory flow and uniform flow influence the cross-flow vibration frequency. From Fig. 8(e) and 8(f), it can be known that the variation of dominant vibration frequency is relatively small comparing with other cases. And the uniform flow plays the dominant role in this flow ratio range.

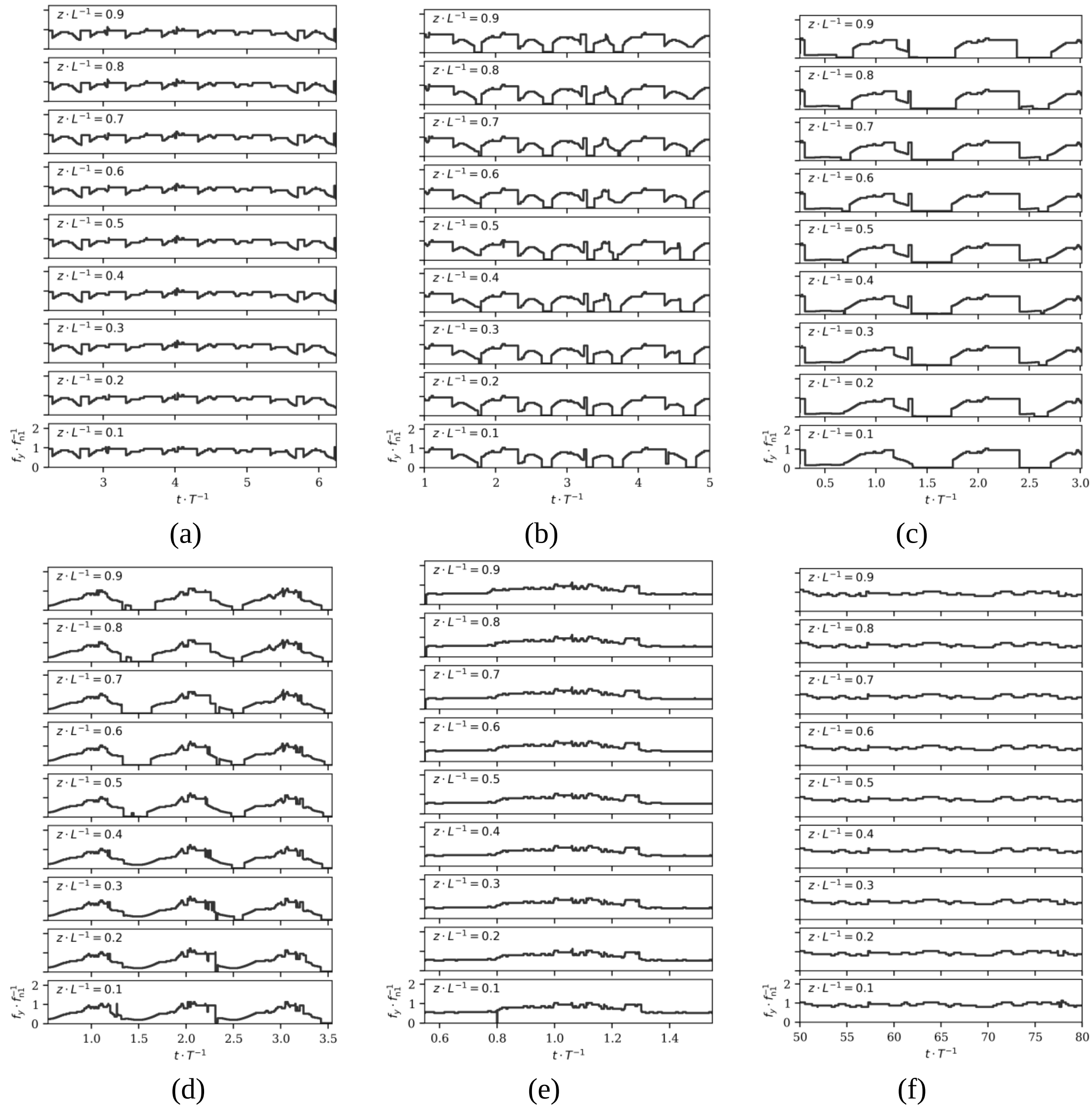


Figure 8. Cross-flow vibration wavelet of different nodes along the span of the cylinder: (a) α=0; (b) α=0.2; (c) α=0.4; (d) α=0.6; (e) α=0.8; (f) α=1.0

Cross-flow Vibration Trajectory

Fig. 9 are subplots of cross-flow vibration trajectory of three nodes along the span of the cylinder. With the increase of flow ratio, the trajectory shape of the intermediate node of the cylinder changes from the "H" type to the "1" type. For the pure oscillatory flow condition (Fig. 9(a)), the cross-flow vibration and the in-line deformation reaches its peak value when the cylinder moves across the center, which generate the two sides of the "H". During the preliminary and the final stages, the oscillatory flow velocity reaches its valley value that leads to the small cross-flow vibration and in-line deformation of the cylinder, which generates the short transverse line of the "H" shape. When the flow ratio increases to 0.2, the flow velocity decreases during the process of oscillatory flow and uniform flow velocity in opposite direction. And the right side of the trajectory is generated due to the low cross-flow vibration amplitude and in-line deformation when the cylinder moves across the center during the process. With the flow ratio increasing, the total flow velocity in the process of opposite flow velocities keeps decreasing. Then the cross-flow vibration amplitude and in-line deformation becomes smaller than those in flow ratio of 0.2. So the trajectory in the region of $x \geq 0$ is very small as shown in Fig. 9(c) and 9(d). Meanwhile, the proportion of the total flow velocity that generate relatively larger vibration amplitude increases in the process of the same flow velocities direction, which lead to the change of trajectory shape from triangle to rectangle in the region of $x \leq 0$. From Fig. 9(e), it can be concluded that the cross-flow vibration and in-line deformation of the cylinder are extremely small in the region of $x \geq 0$ and the trajectory shape is similar to that in Fig. 9(f), which means that the uniform flow plays the dominant role when flow ratio α=0.8.

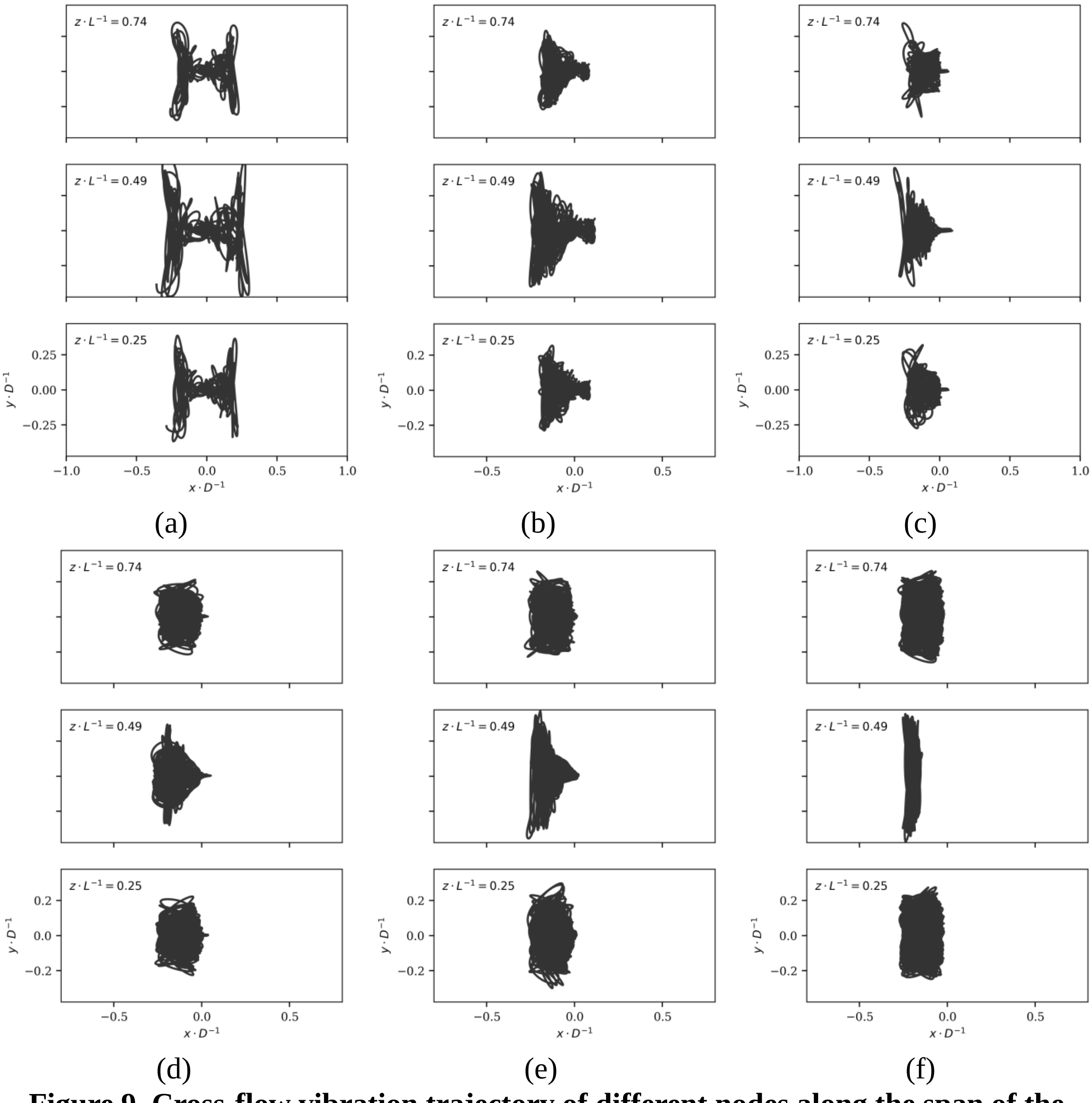


Figure 9. Cross-flow vibration trajectory of different nodes along the span of the cylinder: (a) α=0; (b) α=0.2; (c) α=0.4; (d) α=0.6; (e) α=0.8; (f) α=1.0

Conclusion

In this paper, numerical simulations of vortex-induced vibrations of a flexible cylinder experiencing combined oscillatory and uniform flow are carried out by the in-house CFD code viv-FOAM-SJTU solver. Results of cross-flow displacement history, modal weights, wavelet and vibration trajectory are analyzed in detail.

Results of cross-flow displacement history and modal weights show that the first mode is the dominant vibration mode of the cylinder in all flow conditions. The dominant vibration frequency of the cylinder is approximately near the first natural frequency through results of cross-flow wavelet. With the flow ratio increasing, the vibration trajectory of the cylinder changes from the "H" type to the "1" type. The vibration responses of the cylinder in combined oscillatory and uniform flow are similar to that in pure oscillatory flow (α=0) when flow ration $\alpha \leqslant 0.2$ and similar to that in pure uniform flow (α=1.0) when $\alpha \geqslant 0.8$.

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