Solving differential variational inequalities in non-smooth dynamics using the preconditioned spectral projected gradient method

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Abstract

The framework of Differential Variational Inequalities (DVIs) provides a mathematical ground for non-smooth contact problems, and can be seen as a generalization of the Measure Differential Inclusion theories pioneered by Jean J. Moreau thirty years ago. This is the result of expressing contact forces, including friction, with set-valued contact forces instead than using regularization; at the cost of the added complexity of the DVI formulation, this allows large time steps yet encompassing discontinuous or impulsive effects such as impacts and stick-slip phenomena. Within this context, the time discretization of the dynamical problem leads to Variational Inequalities (VIs) that must be solved at each time step [1].

We can show that, for the case of 3D rigid shapes in contact with Coulomb friction, the VI becomes a Cone Complementarity Problem (CCP), for whom the LCP problem is a subcase. Our formulation introduces a convex relaxation to the mechanical model, so that the friction becomes associated and the resulting (second-order, convex) CCP is also a QP; we show that when a stabilization term is added to the time stepper, such convexified method converges to the solution of the original DVI for small time steps or small sliding speeds or small friction [2]. Indeed, zero or small sliding speed is a common case in scenarios such as masonry structures, and allows the solution of the CCP by means of an efficient solver for QP with separable convex constraints.

To this end we propose the use of the Spectral Projected Gradient solver (SPG), originally introduced in [3], that we modified in [4] to encompass block-diagonal preconditioning, alternating scaling, and a fallback strategy – hence we named it P-SPG-FB. The method uses a non-monotone Grippo-Lampariello-Lucidi line search. When compared to projected fixed point iterations, the method shows much better convergence, at a similar cost per-iteration.

The time stepper, along with high performance collision detection, has been implemented in the ProjectChrono open-source software.

Keywords: **DVI, CCP, non-smooth contact dynamics, SPG, multibody**

References

[1] Negrut,D., Serban, R. and Tasora, A. (2017) Posing Multibody Dynamics with Friction and Contact as a Differential Complementarity Problem. *J. Comput. Nonlinear Dynam.* **13**(1), 4037415

[2] Tasora, A. and Anitescu, M. (2011) A matrix-free cone complementarity approach for solving large-scale, nonsmooth, rigid body dynamics. *Computer Methods in Applied Mechanics and Engineering* **200**, 439-453

[3] Birgin, E.G., Martinez, J.M. and Raydan, M. (1999) Nonmonotone spectral projected gradient methods on convex sets. *SIAM J. on Optimization* **10**, 1196–1211

[4] Heyn, T., Anitescu, M., Tasora, A. and Negrut, D. (2013) Using Krylov Subspace and Spectral Methods for Solving Complementarity Problems in Many-Body Contact Dynamics Simulation. *Int. Journal For Numerical Methods In Engineering*, **95**(7), 541-561