Simulations of Thermal-Hydraulics Two-Phase Flows using Mixture Formulations

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ABSTRACT

This short work presents an assessment of the capabilities of a non-equilibrium flow model to solve two-phase flows evolving spatial and temporal discontinuities. The model is based on mixture parameters of state and feature the relative velocity resolution between the two phase systems. It demonstrate an important link between mixture formulations and two-phase flow thermal-hydraulics. Numerical simulations of the two-phase flows are performed using highly accurate and efficient Godunov-type finitevolume solvers. The computational efficiency and ability of both the model equations and the solvers are validated by test cases available in the open literature.

Keywords: Two-phase flow, Non-equilibrium, Mixture model, Godunov methods, Simulation

Introduction and Equations

The various thermal-hydraulics two-phase flow models available in literature are mainly based on either a mixture model, a volume of fluid model or a two-fluid model [3, 5]. The latter is mostly employed in computer codes such as RELAP-5, TRACE and WAHA for the design and safety assessment of nuclear reactors. See for example [7] and references therein. Within individual thermal-hydraulics phenomena where two-phase flow does occur the system of partial differential equations (PDEs) describing such flows is a non-hyperbolic and cannot be written in a conservative form. However, hyperbolicity property can be examined under certain physical restrictions yet the system of equations remains non-conservative [3]. In addition to that, the governing equations cannot handle the relative motion between the two phase system without interphase exchange. Hyperbolic nature of two-phase flow equations is an advantageous property since it allows discontinuities in the solutions related to nuclear thermal-hydraulic systems. These solutions entails the use of different numerical methods of interest within the currently used two-phase flow models context. In this paper, we present recent results of the application of a hyperbolic and conservative system of PDEs to the simulation of thermal-hydraulic mixture of gas and liquid phases [1, 6]. This system enables a fairly straightforward consideration for the relative velocity between the different phases and able to capture shock and expansion waves in two-phase mixture flows. The set of equations is based on three balance equations for mixture mass (ρ), mixture momentum (ρu) and mixture energy (E) as well as a relative velocity (u_r) balance law written in process without dissipation as [6]:

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x}(\rho u) = 0, \tag{1}$$

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u^2 + P + \rho c(1-c)u_r^2) = 0, \tag{2}$$

$$\frac{\partial}{\partial t}(u_r) + \frac{\partial}{\partial x}\Big(u u_r + (1-2c)\frac{u_r^2}{2} + \psi(P)\Big) = 0, \tag{3}$$

$$\frac{\partial}{\partial t}\Big(\rho E\Big) + \frac{\partial}{\partial x}\Big(\rho u E + P u + \rho c(1-c)u_r\Big(u u_r + (1-2c)\frac{u_r^2}{2} + \psi(P)\Big)\Big) = 0, \tag{4}$$

and supplemented by the following gas void fraction (α) and gas mass void fraction (c) balance laws:

$$\frac{\partial}{\partial t}(\rho\alpha) + \frac{\partial}{\partial x}(\rho u\alpha) = 0, \tag{5}$$

$$\frac{\partial}{\partial t}(\rho c) + \frac{\partial}{\partial x}(\rho u c + \rho c(1-c)u_r) = 0. \tag{6}$$

Here P is the mixture pressure defined as

$$P = \alpha P_2 + (1-\alpha)P_1,$$

where the relationship between the gas and liquid volumes is shown as follows

$$\alpha + (1-\alpha) = 1.$$

In addition to that, the ideal compressible equation of state is employed for the gas phase and stiffened gas equation of state is used for the liquid phase. This mixture pressure along with the two different equations of state defines the mixture equation of state. The function $\psi(P)$ is an expression that describes the relationship between the gas and liquid phases through the momentum equations. This is given by

$$\psi(P) = e_2 + \frac{P_2}{\rho_2} - e_1 - \frac{P_1}{\rho_1},$$

where the indices 2 and 1 refer to the gas and liquid phases, respectively.
The properties of system (1)-(4) along with (5)-(6) were studied earlier (see, e.g. [6]). Hyperbolicity of the system of equations were also examined numerically over an entire range of parameters typical of practical applications.

Godunov-Type Resolutions

Finite volume Godunov methods are used to discretise the model equations, (1) to (6), in the form

$$\mathbb{U}_i^{n+1} = \mathbb{U}_i^n - \frac{\Delta t}{\Delta x}\Big(\mathbb{F}^n_{i+\frac{1}{2}} - \mathbb{F}^n_{i-\frac{1}{2}}\Big), \tag{7}$$

where the interface fluxes, $\mathbb{F}^n_{i+\frac{1}{2}}$ and $\mathbb{F}^n_{i-\frac{1}{2}}$, are calculated using approximate solution of the following Riemann problem

$$\frac{\partial \mathbb{U}}{\partial t} + \frac{\partial \mathbb{F}(\mathbb{U})}{\partial x} = 0, \tag{8}$$

with the initial data defined as

$$\mathbb{U} = \begin{cases} \mathbb{U}_L, & x \le x_0, \\ \mathbb{U}_R, & x > x_0, \end{cases}$$

where $\mathbb{U}_L$ and $\mathbb{U}_R$ represent the values of gas and liquid properties on a two-phase shock tube at the left and at the right from an interface between the two states at $x = x_0$. Godunov methods of centred-type are considered for the resolution of equation (8). This is due to the large number of unknown variables for the individual phases and the two-phase mixture. Further, the numerical fluxes in (7) are approximated using the Slope-Limited Centered (SLIC) scheme where the solution of the Riemann problem is fully numerical. The SLIC scheme is a second-order in time and space and Total Variation Diminishing (TVD) using any limiter of interest. We refer the reader to [4] for more details of this second-order scheme.

To demonstrate the capability of the equations of the mixture model, the SLIC scheme is employed for the resolution of a well-documented air-water shock-tube problem presented recently in [2]. This problem deals with large physical variations between the two phases where discontinuities appear clearly within thermal-hydraulics, in particular in many types of reactor cores. A tube of $10\,m$ with a diaphragm in the middle which separate the following left and right states are defined as follows at $t = 5$ ms, see [2]:

$$\begin{aligned} \big(\alpha, \rho_2, u_2, \rho_1, u_1\big)_L &= \big(0.25, 57.941, 0.0, 1003.1, 0.0\big) \quad \text{if} \quad x \le x_0, \\ \big(\alpha, \rho_2, u_2, \rho_1, u_1\big)_R &= \big(0.1\ \ , 25.527, 0.0, 999.85, 0.0\big) \quad \text{if} \quad x > x_0. \end{aligned}$$

Results are shown in figures 1 at $t = 5$ ms without any source terms consideration. The key reason for employing no source terms effect in the current work is the mixture formulation of the current model equations. Furthermore, unlike other work reported in literature, the results presented in this work are based on compressible liquid where the gas is air. Interestingly, the solution for both phases and mixture flow variables consists of a left rarefaction wave, a contact discontinuity and a right shock wave. The simulation results shown in figure 1 are similar to those presented in [2] except for the middle discontinuities. The relative velocity also is not presented in [2].

Concluding Remarks

A non-equilibrium flow model based on mixture formulation has been proposed for the simulation of two-phase flow thermal-hydraulics. The mixture model is successfully applied to the simulations of a two-phase shock tube with large difference between the gas and liquid flows. It is concluded that this mixture model is capable of solving such two-phase flows without differential closure laws or specific equations of state. The model also provide significant numerical resolutions without any numerical conditions.

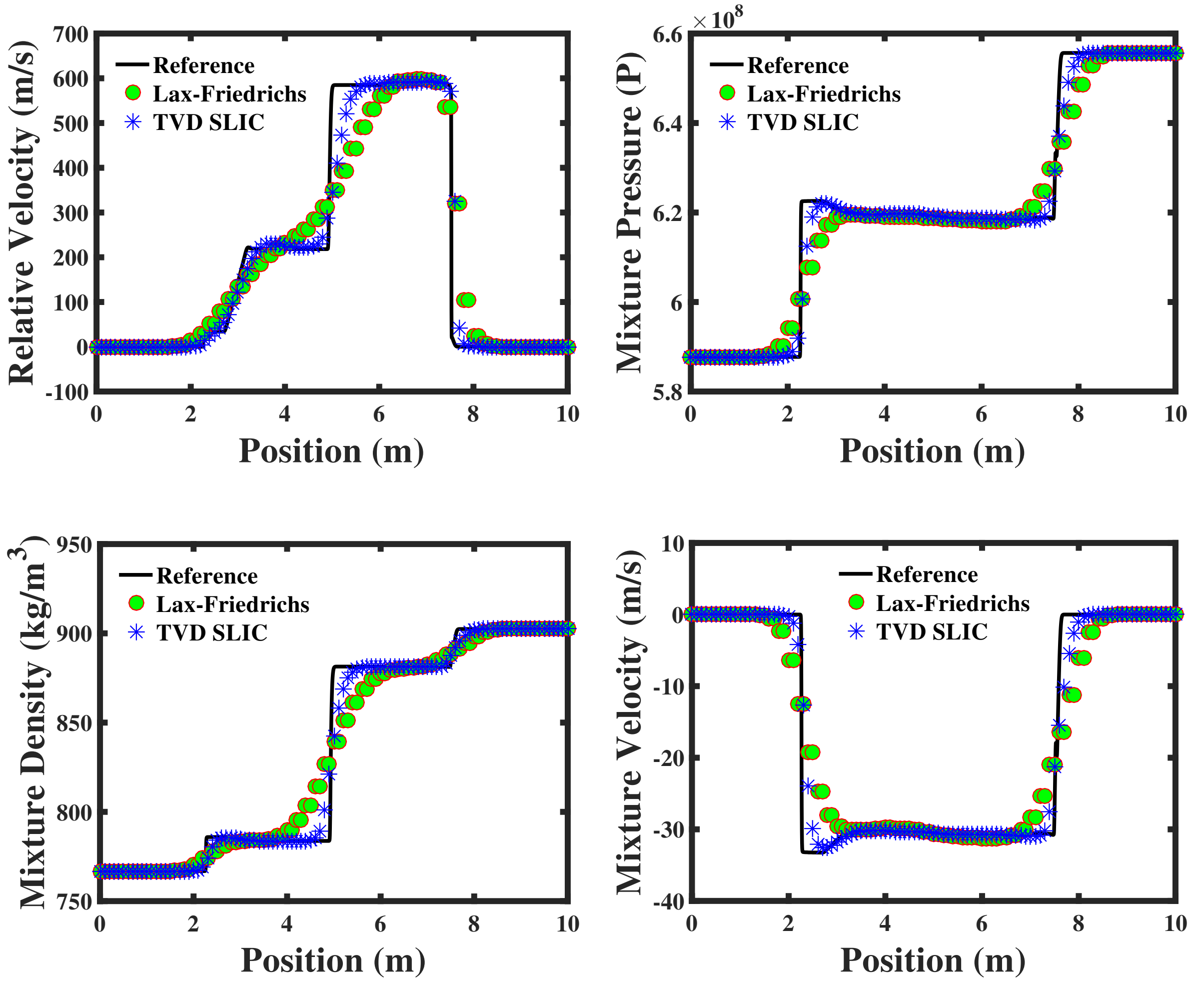

Figure 1: Test 1: Air-water shock-tube problem of [2] at time $t = 5$ ms. The TVD SLIC and first-order Lax-Friedrichs methods are compared with the reference solution results. Coarse meshes, symbols, are provided on 100 cells and very fine meshes of 10000 cells for the solid lines. The waves seen from left to right, repeated left rarefaction and repeated right shock waves separated by a multiple contact discontinuity.

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