A sequential method in inverse estimation of the absorption coefficient for the spot laser welding process

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Abstract
In this paper, inverse determination of the absorption coefficient in spot laser welding by using a sequential method is presented. The advantages of this method are that the functional form for the unknown absorption coefficient is not necessary to preselect and nonlinear least-square do not need in the algorithm. Two examples have been fulfilled to demonstrate the proposed method. The obtained results can be concluded that the proposed method is an accurate and stable method to inversely determine the absorption coefficient in the spot laser welding.

Keywords: Inverse Problem, Absorption Coefficient, Spot Laser Welding.

Introduction
In recent years, the rapid development of the laser welding technology has gradually replaced the traditional welding techniques. Comparing the conventional welding process, laser welding was used widely because of its good behaviors such as high efficiency, narrow heat affected zone (HAZ), and high welding speed. Thus, the applications of laser welding have been broadened in modern industries including the aerospace and automotive industries, the microelectronics industry and the medical instrument industry nowadays [1, 2].
As shown in the literature, the absorption coefficient is an important factor in laser welding processing. The absorption coefficient depends on optical material properties, laser wavelength, surface temperature, and surface condition [3, 4]. Nevertheless, numerous studies in laser welding-related problems were assumed that the absorption coefficient is constant [5-7]. The effect of the absorption coefficient on the weld pool shape and on the temperature distribution was investigated by Bannour et al. [8]. The results showed that the peak temperature reaches 1250K for using the constant absorption coefficient and 1300k for the case of the temperature-dependent absorption coefficient. Furthermore, the results in Bannour et al. [8] also evidenced that the molten pool formation and temperature distribution significantly influenced by the absorption coefficient comparing with other parameters such as heat capacity, density and shielding gas. In other words, the use of appropriate absorption coefficients is significant in solving laser welding-related problems, especially the transient laser welding-related problems like spot laser welding and segment laser welding.
In fact, the direct measurement of the absorption coefficient during the laser welding process is not easy. As results, the inverse method is one of the good way to measure this coefficient. Sun et al. [9] used the direct sensitivity coefficient method to inversely estimate the absorptivity by implementing a two-dimensional quasi-static IHCP in laser hardening process. Chen et al. [10] proposed a hybrid technique of the Laplace transform and finite-difference methods to estimate the absorptivity in the laser surface heating process. Wang et al. [11] estimated the surface absorption coefficient in the laser surface hardening by using the

conjugate gradient method with the temperature-dependent thermal properties. However, these studies only deal with in the laser surface hardening in which the maximum temperature of the substrate is less than melting temperature. Thus, this result is no longer correct when the temperature field in the substrate reaches and exceeds the melting temperature in the welding process because the phase change of the laser welding process was not considered in the work. Furthermore, as our knowledge, only a few published papers implement and propose an effective method to determine the absorption coefficient in the spot laser welding until now.

In this paper, a robust and stable method is presented to determine the absorption coefficient in the spot laser welding process. In the proposed method, a modified Newton-Raphson method combined with the concept of the future time is used to solve the problem step by step [12-14]. The estimation of absorption coefficient in spot laser welding process at each time step consists of two phases: the process of direct analysis and the process of inverse analysis. In the process of direct analysis, the absorption coefficient and the boundary conditions are assumed as specified values and then the temperature field is solved by finite element method [15]. In finite element method, the effective heat capacity method [16, 17] are applied to take the latent heat into account due to the phase change in laser welding. Solution from this process are inputted to the sensitivity analysis and integrated with the measured temperature at the sensor's position. Thus, a set of nonlinear equations is formulated for the process of the inverse estimation. In the process of inverse analysis, an iterative method is used to guide the exploring points systematically to obtain the unknown variables. Then, the intermediate values are substituted for the unknown variables for the following analysis. That way, several iterations are performed to achieve the undetermined parameters. The advantage of this inverse method does not adopt the nonlinear least-squares error to formulate the inverse problem, but it is implemented a direct comparison between the measured temperature and the computed melting temperature.

Problem Statement

Considering the three-dimensional cylindrical workpiece, its top surface is heated by an incident laser beam with the laser beam radius of r_b. The rest of workpiece surface is cover by an adiabatic material to avoid the energy lost to the surroundings. The thermocouple is embedded inside the workpiece to capture the temperature history (as Figure 1).

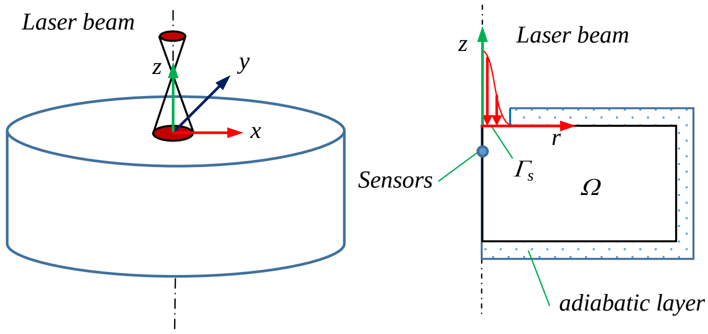

Figure 1: The model of spot laser welding.

The aim of this work is to propose the efficient method to estimate inversely the absortion coefficient in the spot laser welding. To simplify, the heat conduction-based method for this welding problem is thus considered. Due to the symmetry of cylindrical workpiece, the governing equation of transient heat conduction in two-dimensional cylindrical coordinates is given by:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k(T)r\frac{\partial T}{\partial r}\right)+\frac{1}{r}\frac{\partial}{\partial z}\left(k(T)r\frac{\partial T}{\partial z}\right)=\rho(T)C(T)\frac{\partial T}{\partial t} \quad in\ \Omega, \qquad 0\le t\le t_f \tag{1}$$

$$\left.\begin{array}{l} -k(T)\dfrac{\partial T}{\partial z}=q(r) \ \text{ at } \ \Gamma_s \\ -k(T)\dfrac{\partial T}{\partial n}=0 \text{ at otherwise surfaces} \end{array}\right\} , \quad 0\le t\le t_f \tag{2}$$

$$\frac{\partial T}{\partial r}=0 \ \text{ at z-axis} \tag{3}$$

$$T(r,\mathrm{z},0)=T_0 \tag{4}$$

where, k(T), C(T) , $\rho(T)$ are respectively the thermal conductivity, heat capacity, and density; $T(r,z,t)$ is the temperature field; T_0 is the initial temperature; $q(r)$ is the laser heat flux; n is the normal vector. In this work, the heat source model proposed by Friedman [18] is considered and it can expressed as following:

$$q(r)=\frac{3P.\eta}{\pi r_b^2}\exp\left(\frac{-3r^2}{r_b^2}\right) \tag{5}$$

where, P is the laser power; η is the absorption coefficient, r_b is the effective radius of laser beam.

When the absorption coefficient, the boundary conditions and other input parameters are known, the temperature distribution in the domain can be solved numerically by the finite element method [15]. Furthermore, the effective heat capacity method is considered in the finite element method to take account of the latent heat of the phase change in laser welding [16, 17].

The inverse problem is to estimate the absorption coefficient in the process of spot laser welding when the temperature history is measured at $x=x_m$. Thus, a sequential method is proposed in the next section.

Methodology

The proposed method consists of the forward problem, the sensitivity problem, the operational algorithm, and the stopping criterion. The direct problem is implemented to obtain the temperature field, and the sensitivity problem is utilized to find out the search step in the inverse problem. Next, the operational algorithm is used to satisfy the process of the inverse analysis when the solution of both direct and sensitivity problems is available. Finally, the stopping criterion is shown to decide the termination of the iterative process.

Forward problem

The proposed method is based on a sequential algorithm in which the inverse solution is solved at each time step. Accordingly, Eqs. (1-4) are restricted to only one temporal step and the transient problem at $t=t_m$ is governed by the equations as follows:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k\left(T_m\right)r\frac{\partial T_m}{\partial r}\right)+\frac{1}{r}\frac{\partial}{\partial z}\left(k\left(T_m\right)r\frac{\partial T_m}{\partial z}\right)=\rho\left(T_m\right).C\left(T_m\right).\frac{\partial T_m}{\partial t} \quad in\ \Omega\,, t=t_m \tag{6}$$

$$\left.\begin{array}{l} -k(T_m)\dfrac{\partial T_m}{\partial z}=q_m(r) \ \text{ at } \ \Gamma_s \\ -k(T_m)\dfrac{\partial T_m}{\partial n}=0 \text{ at otherwise surfaces} \end{array}\right\} , \ t=t_m \tag{7}$$

$$\frac{\partial T_m}{\partial r}=0 \quad \text{at z-axis} \tag{8}$$

$$T(r,\mathrm{z},t_{m-1})=T_{m-1} \tag{9}$$

$$q_m(r) = \frac{3P.\hat{\eta}_m}{\pi r_b^2}\exp\left(\frac{-3r^2}{r_b^2}\right) \tag{10}$$

where, $\hat{\eta}_m$ is the unknown absorption coefficient at $t = t_m$.

In the present work, the proposed method formulates the problem from the difference between the calculated temperature and the one measured directly. As well, instead of the optimization algorithm, the equation solver solves the inverse problem.

When the estimation is at $t = t_m$, the estimated condition from $t = t_1$ to $t = t_{m-1}$ has been evaluated, and the problem is to estimate the laser heat flux at $t = t_m$. In order to guarantee the stability of estimated results in the inverse algorithm, several future values of the estimation are temporally assumed to be constant or linear relation in the subsequent procedure [19]. Then, the unknown conditions are presented as follows:

$$\hat{\eta}_{m+r}^q = \hat{\eta}_m^q + \xi(\tau-1)(\hat{\eta}_m^q - \hat{\eta}_{m-1}^q) \tag{11}$$

where, τ is the number of the future time; ξ=0 is constant relation and ξ=1 is linear combination.

The forward problem, Eqs. (6-10) are solved in τ steps (from $t = t_m$ to $t = t_{m+\tau}$) and the undetermined absorption coefficient are set by Eq. (11).

Sensitivity problem

In the proposed method, a modified Newton-Raphson method is adapted to solve the inverse problem in which the sensitivity analysis is necessary to achieve the search step in each iteration. The derivative $\partial / \partial\hat{\eta}_m$ is taken at both sides of Eqs. (6-10). Furthermore, because of the small number of future time step and the small temporal increment, we can assume that the thermal properties at the estimating step t_m are constant. Then, we have:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k(T_m)r\frac{\partial X_m}{\partial r}\right) + \frac{1}{r}\frac{\partial}{\partial z}\left(k(T_m)r\frac{\partial X_m}{\partial z}\right) = \rho(T_m)C(T_m)\frac{\partial X_m}{\partial t} \tag{12}$$

$$-k(T_m)\frac{\partial X_m}{\partial z} = \frac{\partial q_m(r)}{\partial \hat{\eta}_m} \tag{13}$$

$$\frac{\partial X}{\partial r} = 0 \tag{14}$$

$$X(r, \mathrm{z}, t_{m-1}) = X_{m-1} = 0 \tag{15}$$

Eqs. (12-15) describe the mathematical equations for sensitivity coefficient, X_m, which can be explicitly solved. These equations are the linear equations and the dependent variable, X_m, with respect to independent variables, x, y, z and t. Therefore, the sensitive solution can be obtained directly through by the finite element method.

Modified Newton-Raphson method

A modified Newton-Raphson method [14] is necessary in the proposed method to deal with the inverse problem with solving a set of nonlinear equations. This set of nonlinear equations is directly formulated the problem from the comparison between the computed temperature and the preselected temperature at the measurement locations. Therefore, the measured temperature Y_{meas}^j and the calculated temperature Y_c^j are evaluated first. Then estimation of the absorption coefficient, $\hat{\eta}_m$, at each time step can be recast as the solution of a set of nonlinear equations:

$$\mathbf{Y} = Y_c^j - Y_{meas}^j = 0 \tag{16}$$

where, $j = m, m+1, ..., m+\tau$ is the number of equations which is equal to the number of the future times τ.
The derivative of $\mathbf{Y}$ with respect to $\hat{\eta}_m$ is solved through Eqs. (12-15) and can be expressed as following:

$$\mathbf{X} = \frac{\partial \mathbf{Y}}{\hat{\eta}_m} \tag{17}$$

where, $\mathbf{X}$ is the sensitivity matrix
With the starting $\hat{\eta}_m^0$ and the above derivations from Eq. (17), we have the following equation:

$$\hat{\eta}_m^{k+1} = \hat{\eta}_m^{k+1} + \Delta^k \tag{18}$$

where, Δ^k is a linear least-squares solution for a set of over-determined linear equations and it can be derived as following:

$$\Delta^k = -\left[\mathbf{X}^T(\hat{\eta}_m^k)\mathbf{X}(\hat{\eta}_m^k)\right]^{-1}\mathbf{X}^T(\hat{\eta}_m^k)\mathbf{Y}(\hat{\eta}_m^k) \tag{19}$$

The preceding derivation is applied at each time step. This method can be carried out in the multi-sensor's measurement. Under this condition, the number of elements in Eq. (16) is based on the number of future time step and the number of measured positions.

The stopping criteria
The modified Newton-Raphson method (Eqs. (16-19)) is used to determine the unknown value of the absorption coefficient at the *m*-th time step, $\hat{\eta}_m$. The step size Δ^k goes from $\hat{\eta}_m^k$ to $\hat{\eta}_m^{k+1}$, and it is determined from Eq. (19). Once Δ^k has been calculated, the iterative to determine $\hat{\eta}_m^{k+1}$ is executed until the stopping criterion is satisfied.
The discrepancy principle [20] is widely used to evaluate the value of the stopping criterion in the inverse technique. Nevertheless, the convergence of the inverse solution is not guaranteed by the stopping criterion created by the discrepancy principle. Therefore, two criteria proposed by Frank and Wolfe [21] are chosen to assure the convergence and to stop iteration:

$$\left\|\hat{\eta}_m^{k+1} - \hat{\eta}_m^k\right\| / \left\|\hat{\eta}_m^{k+1}\right\| \le \delta \tag{20}$$

$$\left\|\mathbf{S}\left(\hat{\eta}_m^{k+1}\right) - \mathbf{S}\left(\hat{\eta}_m^k\right)\right\| / \left\|\mathbf{S}\left(\hat{\eta}_m^{k+1}\right)\right\| \le \varepsilon \tag{21}$$

where

$$\left\|\mathbf{S}\left(\hat{\eta}_m^{k+1}\right)\right\| = \sum_{i=1}^{r}\left[Y_c^i - Y_{meas}^i\right]^2 \tag{22}$$

where, ε and δ are small positive value known as the convergence tolerances.

Computational algorithm

We choose the number of the future time, *r*, the mesh configuration of the problem domain, and the temporal size, Δt first. Given overall convergence tolerance ε and δ, and the initial guess $\hat{\eta}_m^0$. The value of $\hat{\eta}_m^k$ is known at the *k*-th iteration.

Step 1: Let $j = m$ and $T(\mathrm{r}, z, t_{j-1})$ is known.

Step 2: Collect the measured temperature, Y_{meas}^j.

Step 3: Calculate the sensitivity matrix, $\mathbf{X}$, by Eqs. (12-15).

Step 4: Solve the direct problem by Eqs. (6-10), and then obtain the calculated temperature Φ_c^j.

Step 5: Construct $\mathbf{Y}$ by Y_{meas}^j and Y_c^j.

Step 6: Knowing $\mathbf{Y}$ and $\mathbf{X}$, determine the step size Δ^k by Eq. (19).

Step 7: Knowing Δ^k and $\hat{\eta}_m^k$, calculate $\hat{\eta}_m^{k+1}$ through Eq. (18).

Step 9: Terminate the iteration if the stopping criterion (Eqs. (20-21)) is satisfied. Otherwise, return to step 5.

Step 10: Stop the process if the final time step is attached. Otherwise, let *j=m+1* return to step 2.

Results and Discussion

Two simples are presented to demonstrate that the proposed method can estimate accurately the absorption coefficient in spot laser welding. In two examples, the cylindrical substrate has the height of $H = 5[mm]$ and the diameter of $d = 20[mm]$. The material used for these examples is commercial AISI304 which thermal properties are temperature-dependent and are taken from Sabarikanth [22]. The latent heat of fusion is $L = 272[\text{kJ/kg}]$, and the melting temperature range is from solidus temperature $T_s = 1673[\text{K}]$ to liquidus temperature $T_l = 1773[\text{K}]$. A thermocouple is located at $x_m(0, -1[\text{mm}])$. In addition, the measured temperature is generated from Eqs. (1-4) when the input parameters are preselected and it is presumed to have measurement errors. In other work, the random errors of measurement are added to the exact temperature. It can be achieved in the following equation:

$$T^{meas} = T^{exact} + \lambda\sigma \tag{23}$$

where, T^{exact} is the exact temperature, T^{meas} is the measured temperature, λ is random numbers calculated by the IMSL subroutine DRNNOR [23] and chosen over the range $-2.576 \le \lambda \le 2.756$, which presents the 99% confidence bond for the measured temperature.

The mesh in all cases is fine at the incident laser beam with $\Delta x \approx 2.10^{-5}$ [mm] and is coarse at away with $\Delta x \approx 1.5.10^{-3}$ [mm] (as Figure 2). As well, the time increment is $\Delta t = 0.02[s]$.

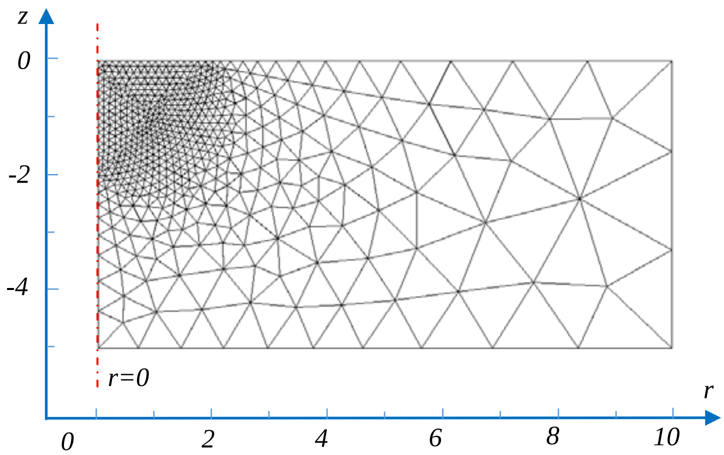

Figure 2: The mesh configuration

To investigate the deviation of the estimated results from the exact solution, the relative average error for the estimated solutions is defined as following:

$$\mu = \frac{1}{N_t}\sum_{i=1}^{N_t}\left|\frac{f - \hat{f}}{\hat{f}}\right| \tag{24}$$

where, f is the estimated results with measurement errors, $\hat{f}$ is the exact results, and N_t is the number of the temporal step. It is declared that a smaller value of μ indicates a better estimation and vice versa.

Example 1: A constant absorption coefficient of 0.3 is assumed in this example. The workpiece is initially at a uniform temperature T_0 = 27[^{0}C], and then is heated by a laser beam with its effective radius of r_b=*0.63*[mm] and power of P=400[W]. In general, the laser heat source can be obtained as follows:

$$q_m(r) = \frac{3\times 400\times 0.3}{\pi\left(0.63.10^{-3}\right)^2}\exp\left(\frac{-3r^2}{\left(0.63.10^{-3}\right)^2}\right) \tag{25}$$

The estimated results of the absorption coefficient in the case of measurement error-free are shown in Figure 3 and Figure 4 shown the exact and computed temperature at the sensor position ($x_m(0,-1[\mathrm{mm}])$). As shown, when $\sigma = 0$, these estimated results are an excellent approximation of the exact solution for both constant and linear type of future time.

In the case of the measurement errors, the estimated results largely diverge from the exact solution (as Figure 4). Table 1 illustrates the relative average errors of the estimated results when the measurement errors are included. In general, the relative average errors are small in all cases. As shown in Table 1, even though the large measurement error of $\sigma = 1.5$, this value for the constant type of future time is 0.01. Furthermore, the relative average errors reduce with the increase of the number of future time step and the decrease of the measurement error. For example, the relative average error moderates from 0.0053 to 0.0022 about 58% as the number of future time steps increase from $\tau = 2$ to $\tau = 4$ for the linear type of future time and reduces from 0.01 to 0.0066 about 66% as the measurement error decrease from $\sigma = 1.5$ to $\sigma = 1$ for the constant assumption of future time.

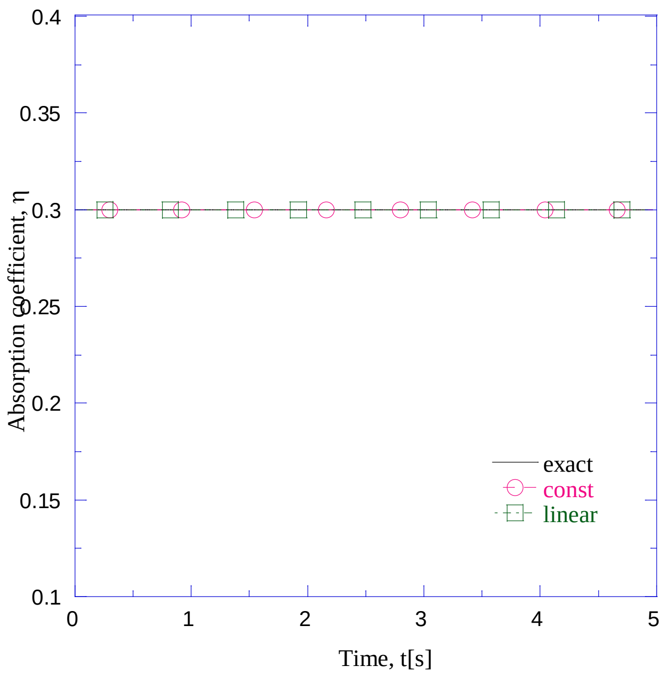

Figure 3: Estimated absorption coefficient in example 1 with $r = 2$ and $\sigma = 0$ with two function kinds of future time

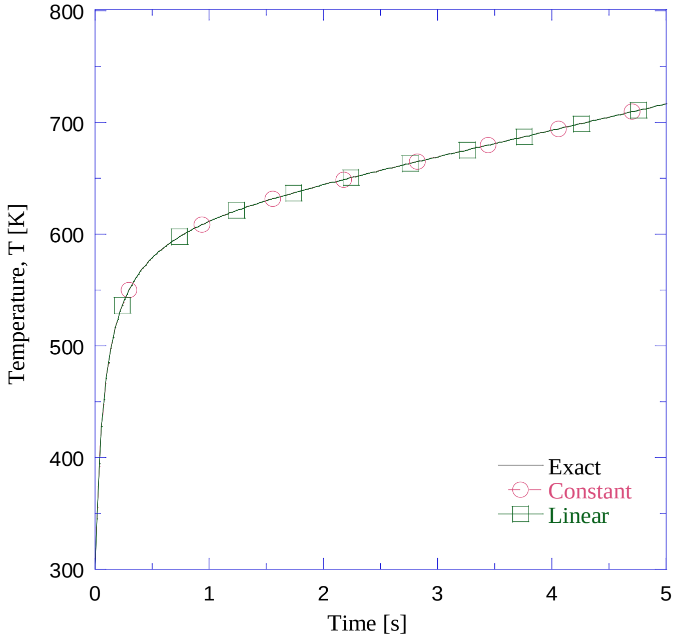

Figure 4: The temperature distribution at the sensor position with $r = 2$ and $\sigma = 0$ with two function kinds of future time

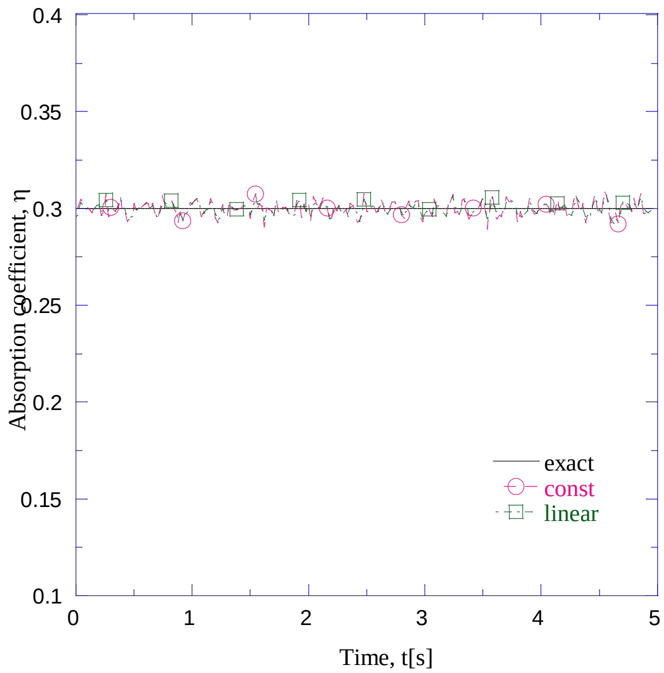

Figure 5: Estimated absorption coefficient in the example 1 with $\tau = 2$ and $\sigma = 1.5$ with two function kinds of future time

Additionally, the effect of the function type of future time on the estimated results is compared. As mentioned above, two kinds of future time description are considered in this work. One is constant type and the other is linear type. The results showed that the accuracy of estimated results for linear function of the future time is better than that for the constant function of the future time (Table 1). The estimated absorption coefficients with $\tau = 2$ and $\sigma = 1.5$ for both constant and linear functions of future time are shown in Figure 5. The results in this profile show that the estimated results for the linear combination of future time can close to the exact solution compared with that for the constant type of future time. In other words, the linear function of future time decreases the relative average error effectively (as Table 1).

Table 1: Relative average errors of example 1

Cases	Future time step	
	τ =2	τ =4
Linear		
σ=1	0.0053	0.0022
σ=1.5	0.008	0.0033
Constant		
σ=1	0.0066	0.0041
σ=1.5	0.01	0.0044

Example 2: In this example, the time variation of the absorption coefficient is assumed as follows:

$$\eta(t) = 0.3(1 - 6.5\times10^{-1}\exp(-(\mathrm{t}-2)/0.75) \qquad (26)$$

The estimated results of the absorption coefficient in the example 2 are shown in Figure 6. Once again, Figure 6 shows that the estimated results have good approximation in the case of measurement free-error.

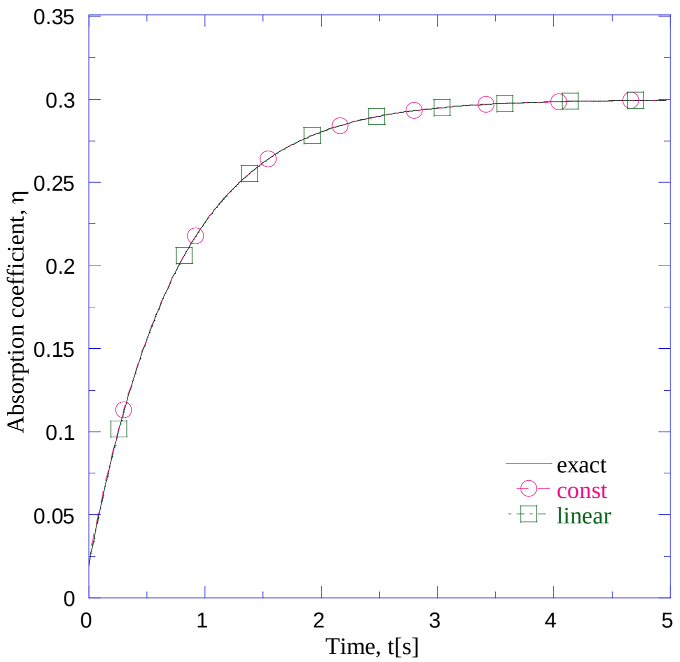

Figure 6: Estimation of laser heat flux in example 2 with $\tau = 2$ and $\sigma = 0$ with two function kinds of future time

Figure 7 and Figure 8 illustrate the estimated results with the measurement errors for the linear assumption of the future time in the cases of $\tau = 2$ and $\tau = 4$, respectively. From Figure 7 and Figure 8, in general, the estimated results have a good approximation to the exact solution with the measurement errors included.

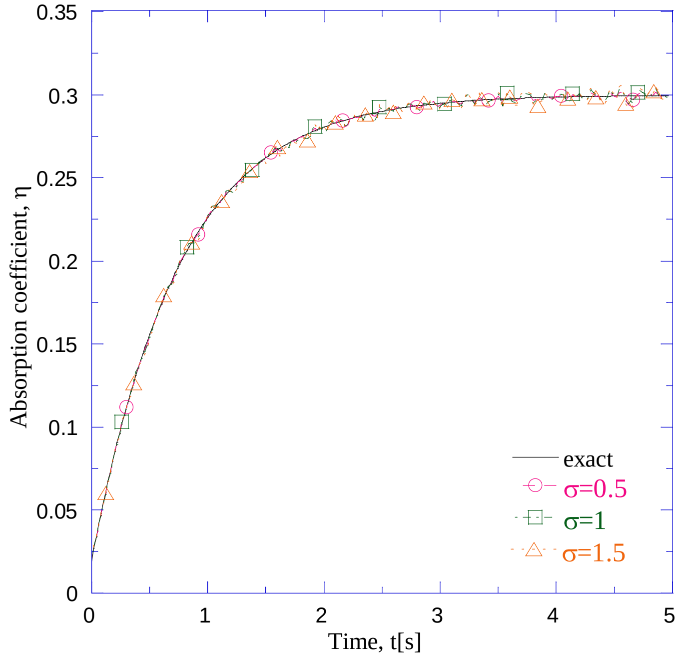

Figure 7: Estimated absorption coefficient in the example 2 with the measurement errors for the linear assumption of future time and τ =2.

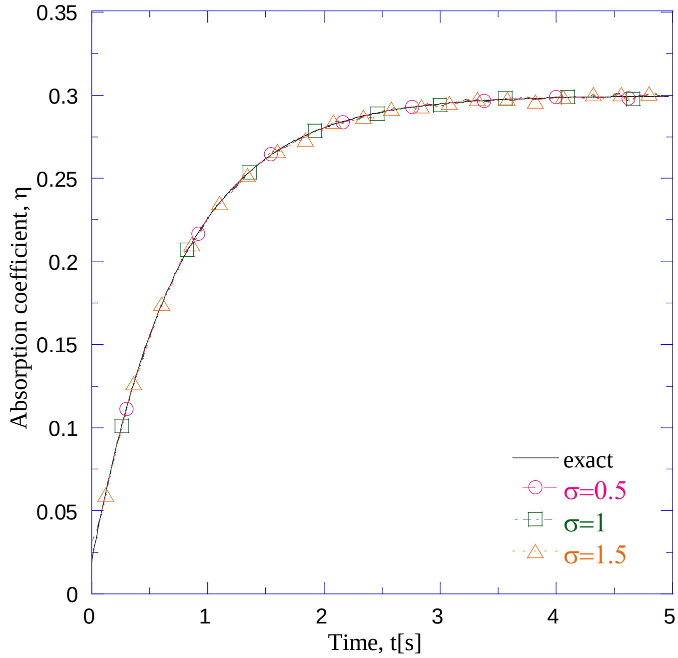

Figure 8: Estimated absorption coefficient in the example 2 with the measurement errors for the linear assumption of future time and τ =4.

Table 2 presents the relative average errors with the different measurement errors and future time steps in the example 2. Table 2 shows that the relative average errors reduce as the measurement errors decrease. It can be noted that, in example 2, the relative average errors do not reduce when the number of the future time step increase. This phenomena is called the "leading error" as has been described by Lin [24, 25]. It appears in these results because of the temporary assumption in the constant and linear types of future time in Eq. (11), as these assumptions might not exactly match the form of unknown absorption coefficient. Furthermore, with the form of undetermined absorption coefficient in the example 2, the linear combination of future time has a better approximation than the constant assumption of future time. Thus, the relative averages errors for the linear combination of future time is less than that for the constant assumption of future time (as Table 2). In general, the relative averages errors in all cases are small. This implies that the proposed method estimates accurately the absorption coefficient in the spot laser welding.

Table 2: Relative average errors of example 2

Cases	Future time step	
	τ =2	τ =4
Linear		
σ=0.5	0.0042	0.0061
σ=1	0.0065	0.0072
σ=1.5	0.0089	0.0094
Constant		
σ=0.5	0.0071	0.0121
σ=1	0.0102	0.0134
σ=1.5	0.0133	0.0148

From the results and discussion above, it can be declared that the proposed method is an effective and stable method to estimate the absorption coefficient in the spot laser welding.

Conclusion

In this paper, the estimation of the absorption coefficient in the spot laser welding was present by using a sequential method. As well, the inverse solution at each time step is solved by a modified Newton-Raphson method. The advantage of this proposed method is that the nonlinear least-squares error is not adopted to formulate the inverse problem, but it is implemented a direct comparison of the measured and calculated temperature. In addition, the special characteristics of this method are that preselected functional form for the unknown absorption coefficient is not necessary. Two examples have been fulfilled to demonstrate the proposed method. The accuracy of the estimated results with the different measurement errors and number of future time steps is investigated. The results show that the accuracy of the estimated results increases when the measurement error decreases and the number of future time step increases. Additionally, two kinds of function of future time are also discussed. In two examples, the results showed that the estimated results with the linear relation of future time is more accurate than that with constant type of future time. In conclusion, from the results in the examples, it can be concluded that the proposed method is an accurate and stable method to determine the absorption coefficient in the spot laser welding.

Reference

[1] Yilbas B.S., Akhtar S., Shuja S.Z., 2013, *Laser Forming and Welding Processes*, Springer International Publishing.

[2] Steen W.M., Mazumder J., (2010) *Laser Material Processing*, Springer.

[3] Wieting T., DeRosa J., (1979) *Effects of surface condition on the infrared absorptivity of 304 stainless steel*, J Appl Phys, 50(2) 1071-1078.

[4] Wieting T., Schriempf J., (1976) *Infrared absorptances of partially ordered alloys at elevated temperatures*, J Appl Phys, 47(9) 4009-4011.

[5] Akbari M., Saedodin S., Toghraie D., Shoja-Razavi R., Kowsari F., (2014) *Experimental and numerical investigation of temperature distribution and melt pool geometry during pulsed laser welding of Ti6Al4V alloy*, Optics & Laser Technology, 59(0) 52-59.

[6] Shanmugam N.S., Buvanashekaran G., Sankaranarayanasamy K., (2013) *Some Studies on Temperature Distribution Modeling of Laser Butt Welding of AISI 304 Stainless Steel Sheets*, World Academy of Science, Engineering and Technology, 7.

[7] Trivedi A., Bag S., De A., (2007) *Three-dimensional transient heat conduction and thermomechanical analysis for laser spot welding using adaptive heat source*, Sci Technol Weld Joi, 12(1) 24-31.
[8] Bannour S., Abderrazak K., Mhiri H., Le Palec G., (2012) *Effects of temperature-dependent material properties and shielding gas on molten pool formation during continuous laser welding of AZ91 magnesium alloy*, Optics & Laser Technology, 44(8) 2459-2468.
[9] Sun Y.S., Weng C.I., Chen T.-C., Li W.-L., (1996) *Estimation of Surface Absorptivity and Surface Temperature in Laser Surface Hardening Process*, Japanese Journal of Applied Physics, 35(6R) 3658.
[10] Chen H.T., Wu X.Y., (2006) *Estimation of surface absorptivity in laser surface heating process with experimental data*, Journal of Physics D: Applied Physics, 39(6) 1141.
[11] Wang J.-T., Weng C.I., Chang J.G., Hwang C.C., (2000) *The influence of temperature and surface conditions on surface absorptivity in laser surface treatment*, J Appl Phys, 87(7) 3245-3253.
[12] Yang C.-Y., (2003) *Estimation of boundary conditions in nonlinear inverse heat conduction problems*, Journal of Thermophysics and Heat Transfer, 17(3) 389-395.
[13] Lin D.T.W., Yang C.-y., (2007) *The estimation of the strength of the heat source in the heat conduction problems*, Applied Mathematical Modelling, 31(12) 2696-2710.
[14] Nguyen Q., Yang C.-Y., (2015) *Inverse determination of laser power on laser welding with a given width penetration by a modified Newton–Raphson method*, International Communications in Heat and Mass Transfer, 65 15-21.
[15] Carnahan B., Luther H.A., Wilkes J.O., 1969, *Applied Numerical Methods*, Wiley, New York.
[16] Huang H.-C., Usmani A.S., 1994, *Finite Element Analysis for Heat Transfer: Theory and Software*, 1 ed., Springer-Verlag London.
[17] Lewis R.W., Ravindran K., (2000) *Finite element simulation of metal casting*, International Journal for Numerical Methods in Engineering, 47(1-3) 29-59.
[18] Friedman E., (1975) *Thermomechanical Analysis of the Welding Process Using the Finite Element Method*, Journal of Pressure Vessel Technology, 97(3) 206-213.
[19] Beck J.V., Blackwell B., St.Clair C.R., (1985) *Inverse heat conduction ill-posed problems*.
[20] Tikhonov A.N., Arsenin V.I.A.k., 1977, *Solutions of ill-posed problems*, Vh Winston.
[21] Frank M., Wolfe P., (1956) *An algorithm for quadratic programming*, Naval research logistics quarterly, 3(1-2) 95 -110.
[22] Sabarikanth R., Sankaranarayanasamy K., Siva Shanmugam N., Buvanashekaran G., (2009) *A study of laser welding modes with varying beam energy levels*, Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, 223(5) 1141-1156.
[23] IMSL, (1987) *Library Edition 10.0, User's Manual: Math Library Version 1.0*, Houston, Tex.
[24] Lin S.-M., Chen C.o.-K., Yang Y.-T., (2004) *A modified sequential approach for solving inverse heat conduction problems*, International Journal of Heat and Mass Transfer, 47(12-13) 2669-2680.
[25] Lin S.-M., (2011) *A sequential algorithm and error sensitivity analysis for the inverse heat conduction problems with multiple heat sources*, Applied Mathematical Modelling, 35(6) 2607-2617.