## Application of BEM to defect detection by means of topology optimization with strain objective functional

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### Abstract

In defect identification inverse analyses using boundary measurement data, a sort of cost functional is defined as a type of boundary integral of a norm consisting of difference between computed data and measurement data. The location and shape of the defects are parameterized and the sensitivity of the objective functional with respect to these parameters are used for searching the optimum locations and shapes minimizing the value of the objective functional. Topological derivative, which is the sensitivity of the objective functional with respect to an infinitesimal change in topology, has been successfully applied to the detection of cavities in an elastic medium using BEM analysis [1]. This may work because the change of the objective functional will be large if a cavity is generated at a point where the defect is located. Another approach is detection of locations and shapes of defects simultaneously using a scheme of topology optimization [2]-[4]. The boundary element method (BEM) is useful for calculating the values of objective functional and the topological derivatives because the analysis has to be repeated both for direct physical system and its adjoint system by changing the shape of the defects in the process of minimizing the objective functional. Therefore, although the meshing cost is low, reduction in cost of BEM calculation is still important.

In the present study, an objective functional defined as a boundary integral of strain is considered. In the standard formulation of the above topology optimization, a topological derivative is derived by appropriately defining an adjoint problem, which requires that the objective functional is defined as a boundary integral of either displacement or tranction, or both. However, in the actual engineering application, the measurement data may not necessarily be displacements nor tractions. Thus, an adjoint problem is derived in this study when the objective functional is defined as a boundary integral of strain. Because the strain components are related as gradients of displacements, appropriate shape functions are needed to derive the boundary condition of the adjoint problem.

In Figure 1 is shown a result of defect detection using the present topology optimization scheme using the BEM.

# Keywords: Topology optimization, defect detection, boundary element method, adjoint problem

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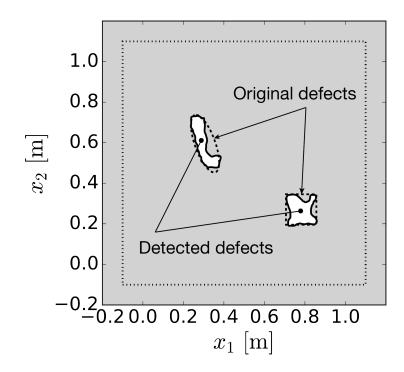


Figure 1. Results for detected defects of ellipse and square shape using the topology optimization scheme and BEM.