Identification of Neumann boundary condition assuring the destruction of target region of biological tissue

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Abstract

An axially symmetrical domain of biological tissue exposed to an external heat source is analyzed. The temperature field is described using the dual-phase lag equation supplemented by Neumann boundary conditions and initial conditions. At first, the direct problem is solved using the implicit scheme of finite difference method. Based on the determined temperatures, the Arrhenius integral is calculated. Next, the inverse problem related to the identification of Neumann boundary condition assuring the destruction of target region of biological tissue is considered. The inverse problem is solved using the gradient method. In the final part of the paper the results of computations and conclusions are presented.

Keywords: **Bioheat transfer, Dual-phase lag model, Arrhenius integral, Inverse problem**

Introduction

Controlled or uncontrolled heating of biological tissues can lead to their destruction. One of the mathematical methods for assessing the degree of tissue destruction is the so-called Arrhenius integral [1]-[3]. The estimation of its value requires the knowledge of temperature history at the selected set of points (measuring points) from the domain considered. To determine the temperature distribution, the different mathematical models can be used e.g. Pennes equation [4]-[9], Cattaneo-Vernotte model [10]-[12], dual-phase lag model [13]-[18] or generalized dual phase lag model [19]-[22]. These equations should be supplemented by appropriate boundary and initial conditions. The solution obtained using the selected model and the suitable numerical method allows one to determine the temperature distribution in the biological tissue and then the values of the Arrhenius integral.

In the inverse problem considered here, the knowledge of the Arrhenius integral at the set of measuring points of the domain is assumed and on this basis the parameters of the Neumann boundary condition are identified. This procedure can be helpful in planning artificial hyperthermia treatment, because it allows to predict the amount of necessary heat delivered to the tissue that ensures the destruction of the target region.

Direct problem

An axially symmetrical domain of biological tissue exposed to an external heat source is considered. Thermal processes can be described by dual-phase lag model [14], [18], [21]

$$(r,z)\in\Omega:\quad c\rho\left[\frac{\partial T(r,z,t)}{\partial t}+\tau_q\frac{\partial^2 T(r,z,t)}{\partial t^2}\right]=\lambda\left[\nabla^2 T(r,z,t)+\tau_T\frac{\partial}{\partial t}\nabla^2 T(r,z,t)\right]+ wc_b\rho_b\left[T_b-T(r,z,t)\right]+Q_m \tag{1}$$

where c, c_b are the specific heat of tissue and blood, respectively, ρ, ρ_b are the mass density of tissue and blood, λ is the thermal conductivity of tissue, τ_q is the relaxation time, τ_T is the thermalization time, w is the perfusion coefficient, T_b is the arterial blood temperature, T is the tissue temperature, Q_m is the metabolic heat source, r, z, t denote the spatial coordinates and time.
On the upper surface of domain, the Neumann condition is assumed

$$q_b(r,0,t)=\begin{cases} q_0\dfrac{t}{t_e}\left(1-\dfrac{t}{t_e}\right)\exp\left(-\dfrac{r^2}{r_D^2}\right), & t\le t_e \\ 0, & t>0 \end{cases} \tag{2}$$

where q_0 is the constant value and t_e is the exposure time, while $r_D \le R$ where R is the radius of cylinder.
On the remaining boundaries the no-heat flux conditions can be accepted.
The initial conditions are also known

$$t=0: \quad T(r,\,z,\,0)=T_0, \quad \left.\frac{\partial T(r,\,z,\,t)}{\partial t}\right|_{t=0}=u(r,z) \tag{3}$$

where T_0 is the constant initial temperature of tissue and $u(r, z)$ is the initial heating rate.

The thermal damage parameter can be evaluated according to the Arrhenius integral [1]-[3], [23]

$$A\left(r,z,t^f\right)=P\int_0^{t^f}\exp\left(-\frac{E}{R_gT(r,z,t)}\right)\mathrm{d}t \tag{4}$$

where P [1/s] is the pre-exponential factor, E [J/mole] is the activation energy, R_g [J/(mole K)] is the universal gas constant, $T(r, z, t)$ [K] is the tissue temperature and $[0, t^f]$ is the time interval under consideration.
A value of damage integral $A(r, z, t^f) = 1$ corresponds to a 63% probability of cell death at a specific point, while $A(r, z, t^f) = 4.6$ corresponds to 99% probability of cell death at this point.

Inverse problem

The inverse problem formulated here concerns the estimation of the boundary heat flux (2), more specifically, the values q_0 and t_e, which ensure the destruction of target region of biological tissue. Thus, the following criterion is formulated

$$S(q_0,t_e)=\sum_{f=1}^{F}\sum_{i=1}^{M}\left[A(r_i,z_i,t^f,q_0,t_e)-A_m(r_i,z_i,t^f)\right]^2 \tag{5}$$

where $A_m(r_i, z_i, t^f)$ is the 'measured' Arrhenius integral. $A(r_i, z_i, t^f, q_0, t_e)$ is the calculated Arrhenius integral obtained from the direct problem solution with the current estimation of the unknown parameters q_0 and t_e, while M is the number of points and F is the number of time steps.

In the case of typical gradient method application [24], [25] the criterion (5) is differentiated with respect to the unknown parameters q_0, t_e and next the necessary condition of optimum is used. Finally, one obtains the following system of equations

$$\begin{aligned}\frac{\partial S(q_0,t_e)}{\partial q_0} &= 2\sum_{f=1}^{F}\sum_{i=1}^{M}\left(A_i^f - A_{m,i}^f\right)\frac{\partial A(r_i,z_i,t^f,q_0,t_e)}{\partial q_0} = 0\\ \frac{\partial S(q_0,t_e)}{\partial t_e} &= 2\sum_{f=1}^{F}\sum_{i=1}^{M}\left(A_i^f - A_{m,i}^f\right)\frac{\partial A(r_i,z_i,t^f,q_0,t_e)}{\partial t_e} = 0\end{aligned} \tag{6}$$

where (c.f. equation (4))

$$R_{1,i}^f = \frac{\partial A(r_i,z_i,t^f,q_0,t_e)}{\partial q_0} = P\int_0^{t^f}\frac{E}{R_g T^2(r_i,z_i,t)}\exp\left(-\frac{E}{R_g T(r_i,z_i,t)}\right)\frac{\partial T(r_i,z_i,t)}{\partial q_0}\,\mathrm{d}t \tag{7}$$

$$R_{2,i}^f = \frac{\partial A(r_i,z_i,t^f,q_0,t_e)}{\partial t_e} = P\int_0^{t^f}\frac{E}{R_g T^2(r_i,z_i,t)}\exp\left(-\frac{E}{R_g T(r_i,z_i,t)}\right)\frac{\partial T(r,z,t)}{\partial t_e}\,\mathrm{d}t \tag{8}$$

and $A_i^f = A\left(r_i,z_i,t^f,q_0,t_e\right)$, $A_{m,i}^f = A\left(r_i,z_i,t^f\right)$.

Function A_i^f is expanded into a Taylor series for the known values of q_0^k and t_e^k, this means

$$A_i^f = \left(A_i^f\right)^k + \left(R_{1,i}^f\right)^k\left(q_0^{k+1} - q_0^k\right) + \left(R_{2,i}^f\right)^k\left(t_e^{k+1} - t_e^k\right) \tag{9}$$

where k is the number of iteration, q_0^k and t_e^k for $k = 0$ are the arbitrary assumed values of q_0 and t_e, while for $k > 0$ q_0^k and t_e^k result from the previous iteration.

Introducing formula (9) to equations (6) one obtains

$$\begin{aligned}&\sum_{f=1}^{F}\sum_{i=1}^{M}\left[\left(A_i^f\right)^k + \left(R_{1,i}^f\right)^k\left(q_0^{k+1} - q_0^k\right) + \left(R_{2,i}^f\right)^k\left(t_e^{k+1} - t_e^k\right) - A_{m,i}^f\right]\left(R_{1,i}^f\right)^k = 0\\ &\sum_{f=1}^{F}\sum_{i=1}^{M}\left[\left(A_i^f\right)^k + \left(R_{1,i}^f\right)^k\left(q_0^{k+1} - q_0^k\right) + \left(R_{2,i}^f\right)^k\left(t_e^{k+1} - t_e^k\right) - A_{m,i}^f\right]\left(R_{2,i}^f\right)^k = 0\end{aligned} \tag{10}$$

it means

$$\begin{aligned}&\left(q_0^{k+1} - q_0^k\right)\sum_{f=1}^{F}\sum_{i=1}^{M}\left[\left(R_{1,i}^f\right)^k\right]^2 + \left(t_e^{k+1} - t_e^k\right)\sum_{f=1}^{F}\sum_{i=1}^{M}\left(R_{1,i}^f\right)^k\left(R_{2,i}^f\right)^k = \sum_{f=1}^{F}\sum_{i=1}^{M}\left[A_{m,i}^f - \left(A_i^f\right)^k\right]\left(R_{1,i}^f\right)^k\\ &\left(q_0^{k+1} - q_0^k\right)\sum_{f=1}^{F}\sum_{i=1}^{M}\left(R_{1,i}^f\right)^k\left(R_{2,i}^f\right)^k + \left(t_e^{k+1} - t_e^k\right)\sum_{f=1}^{F}\sum_{i=1}^{M}\left[\left(R_{2,i}^f\right)^k\right]^2 = \sum_{f=1}^{F}\sum_{i=1}^{M}\left[A_{m,i}^f - \left(A_i^f\right)^k\right]\left(R_{2,i}^f\right)^k\end{aligned} \tag{11}$$

or in the matrix form

$$\begin{bmatrix} \sum_{f=1}^{F}\sum_{i=1}^{M}\left[\left(R_{1,i}^{f}\right)^{k}\right]^{2} & \sum_{f=1}^{F}\sum_{i=1}^{M}\left(R_{1,i}^{f}\right)^{k}\left(R_{2,i}^{f}\right)^{k} \\ \sum_{f=1}^{F}\sum_{i=1}^{M}\left(R_{1,i}^{f}\right)^{k}\left(R_{2,i}^{f}\right)^{k} & \sum_{f=1}^{F}\sum_{i=1}^{M}\left[\left(R_{2,i}^{f}\right)^{k}\right]^{2} \end{bmatrix} \begin{bmatrix} \Delta q_0^k \\ \Delta t_e^k \end{bmatrix} = \begin{bmatrix} \sum_{f=1}^{F}\sum_{i=1}^{M}\left[A_{m,i}^{f}-\left(A_i^{f}\right)^{k}\right]\left(R_{1,i}^{f}\right)^{k} \\ \sum_{f=1}^{F}\sum_{i=1}^{M}\left[A_{m,i}^{f}-\left(A_i^{f}\right)^{k}\right]\left(R_{2,i}^{f}\right)^{k} \end{bmatrix} \tag{12}$$

After solving the system of equations (12), the new values of identified parameters are determined using the formulas

$$\begin{aligned} q_0^{k+1} &= q_0^k + \Delta q_0^k \\ t_e^{k+1} &= t_e^k + \Delta t_e^k \end{aligned} \tag{13}$$

The iterative process is continued until the assumed number K of iterations is achieved.

Sensitivity analysis

To solve the inverse problem, the sensitivity functions, it means the partial derivatives of Arrhenius integral and tissue temperature with respect to the parameters q_0 and t_e (c.f. equations (7), (8)) should be determined. For this purpose the direct approach of sensitivity analysis [26]-[29] can be used. Thus, the governing equations are differentiated with respect to the parameter p_s , s = 1, 2, where $p_1 = q_0$, $p_2 = t_e$. The differentiation of equation (1) gives

$$\begin{aligned} &c\rho\left\{\frac{\partial}{\partial p_s}\left[\frac{\partial T\left(r,z,t\right)}{\partial t}\right]+\tau_q\frac{\partial}{\partial p_s}\left[\frac{\partial^2 T\left(r,z,t\right)}{\partial t^2}\right]\right\}= \\ &\lambda\left\{\frac{\partial}{\partial p_s}\left[\nabla^2 T\left(r,z,t\right)\right]+\tau_T\frac{\partial}{\partial p_s}\left[\frac{\partial}{\partial t}\nabla^2 T\left(r,z,t\right)\right]\right\}-wc_b\rho_b\frac{\partial T\left(r,z,t\right)}{\partial p_s} \end{aligned} \tag{14}$$

or

$$\begin{aligned} &c\rho\left[\frac{\partial U_s(r,z,t)}{\partial t}+\tau_q\frac{\partial^2 U_s(r,z,t)}{\partial t}\right]= \\ &\lambda\left\{\nabla^2 U_s\left(r,z,t\right)+\tau_T\frac{\partial}{\partial t}\left[\nabla^2 U_s\left(r,z,t\right)\right]\right\}-wc_b\rho_b U_s\left(r,z,t\right) \end{aligned} \tag{15}$$

where

$$U_s(r,z,t)=\frac{\partial T\left(r,z,t\right)}{\partial p_s} \tag{16}$$

are the sensitivity functions.

The boundary condition (2) and initial conditions (3) are also differentiated. Thus

$$\frac{\partial q_b(r,0,t)}{\partial q_0} = \begin{cases} \dfrac{t}{t_e}\left(1-\dfrac{t}{t_e}\right)\exp\left(-\dfrac{r^2}{r_D^2}\right), & t \le t_e \\ 0, & t > 0 \end{cases} \tag{17}$$

$$\frac{\partial q_b(r,0,t)}{\partial t_e} = \begin{cases} q_0\dfrac{t}{t_e^2}\left(2\dfrac{t}{t_e}-1\right)\exp\left(-\dfrac{r^2}{r_D^2}\right), & t \le t_e \\ 0, & t > 0 \end{cases} \tag{18}$$

and

$$t = 0: \quad U_s\left(r,\, z,\, 0\right) = 0, \quad \left.\frac{\partial U_s\left(r,\, z,\, t\right)}{\partial t}\right|_{t=0} = 0 \tag{19}$$

It should be noted that in the dual-phase lag model the Neumann condition should be formulated in a different way than in the macroscopic Fourier model, namely [16], [17]

$$\left(r,\, z\right) \in \Gamma: \quad -\lambda\left[\mathbf{n}\cdot\nabla T\left(r,\, z,\, t\right) + \tau_T \frac{\partial\left[\mathbf{n}\cdot\nabla T\left(r,\, z,\, t\right)\right]}{\partial t}\right] = q_b\left(r,\, z,\, t\right) + \tau_q \frac{\partial q_b\left(r,\, z,\, t\right)}{\partial t} \tag{20}$$

where **n** is the normal outward vector.
Thus, the differentiation of boundary condition (20) gives

$$\begin{aligned} &-\lambda\left\{\mathbf{n}\cdot\frac{\partial}{\partial p_s}\left[\nabla T\left(r,\, z,\, t\right)\right] + \tau_T \frac{\partial}{\partial p_s}\left[\frac{\partial\left[\mathbf{n}\cdot\nabla T\left(r,\, z,\, t\right)\right]}{\partial t}\right]\right\} = \\ &\frac{\partial q_b\left(r,\, z,\, t\right)}{\partial p_s} + \tau_q \frac{\partial}{\partial p_s}\left[\frac{\partial q_b\left(r,\, z,\, t\right)}{\partial t}\right] \end{aligned} \tag{21}$$

or

$$-\lambda\left\{\mathbf{n}\cdot\nabla U_s(r,z,t) + \tau_T \frac{\partial}{\partial t}\left[\mathbf{n}\cdot U_s(r,z,t)\right]\right\} = \frac{\partial q_b\left(r,\, z,\, t\right)}{\partial p_s} + \tau_q \frac{\partial}{\partial t}\left[\frac{\partial q_b\left(r,\, z,\, t\right)}{\partial p_s}\right] \tag{22}$$

Finally, the Arrhenius integral (4) is differentiated with respect to the parameter p_s

$$R_s^f(r,z,t^f) = \frac{\partial A(r,z,t^f,p_s)}{\partial p_s} = P\int_0^{t^f} \frac{E}{R_g T^2(r,z,t)} \exp\left(-\frac{E}{R_g T(r,z,t)}\right) U_s(r,z,t)\mathrm{d}t \tag{23}$$

Method of solution

A structure of equations (1) and (15) is similar, therefore they can be written in the form

$$c\rho\left[\frac{\partial Z_s(r,z,t)}{\partial t}+\tau_q\frac{\partial^2 Z_s(r,z,t)}{\partial t}\right]=\lambda\left\{\nabla^2 Z_s\left(r,z,t\right)+\tau_T\frac{\partial}{\partial t}\left[\nabla^2 Z_s\left(r,z,t\right)\right]\right\}- \\ wc_b\rho_b Z_s\left(r,z,t\right)+W_s(r,z,t) \tag{24}$$

where $Z_0(r, z, t) = T(r, z, t)$, $Z_s(r, z, t) = U_s(r, z, t)$, $s = 1,2$ and

$$W_s(r,z,t)=\begin{cases} wc_b\rho_b T_b+Q_m, & s=0 \\ 0, & s=1,2 \end{cases} \tag{25}$$

In a similar way the boundary conditions (20) and (22) can be expressed as

$$\left(r,\ z\right)\in\Gamma:\quad -\lambda\left\{\mathbf{n}\cdot\nabla Z_s(r,z,t)+\tau_T\frac{\partial}{\partial t}\left[\mathbf{n}\cdot Z_s(r,z,t)\right]\right\}=V_s(r,z,t) \tag{26}$$

where

$$V_s(r,z,t)=\begin{cases} q_b\left(r,\ z,\ t\right)+\tau_q\dfrac{\partial q_b\left(r,\ z,\ t\right)}{\partial t}, & s=0 \\ \dfrac{\partial q_b\left(r,\ z,\ t\right)}{\partial p_s}+\tau_q\dfrac{\partial}{\partial t}\left[\dfrac{\partial q_b\left(r,\ z,\ t\right)}{\partial p_s}\right], & s=1,2 \end{cases} \tag{27}$$

The equations (24) – (27) are supplemented by the initial conditions (3) and (19).
The basic problem and additional ones connected with the sensitivity functions are solved using the implicit scheme of finite difference method.
The following approximation of equation (24) is proposed

$$c\rho\left(\frac{Z_{i,j}^{f}-Z_{i,j}^{f-1}}{\Delta t}+\tau_q\frac{Z_{i,j}^{f}-2Z_{i,j}^{f-1}+Z_{i,j}^{f-2}}{\left(\Delta t\right)^2}\right)= \\ \lambda\left(\nabla^2 Z_{i,j}^{f}+\tau_T\frac{\nabla^2 Z_{i,j}^{f}-\nabla^2 Z_{i,j}^{f-1}}{\Delta t}\right)-wc_b\,\rho_b Z_{i,j}^{f}+W_{i,j}^{f} \tag{28}$$

where

$$\nabla^2 Z_{i,j}^{f}=\frac{Z_{i,j-1}^{f}-2Z_{i,j}^{f}+Z_{i,j+1}^{f}}{h^2}+\frac{1}{r_{i,j}}\frac{Z_{i,j+1}^{f}-Z_{i,j-1}^{f}}{2h}+\frac{Z_{i-1,j}^{f}-2Z_{i,j}^{f}+Z_{i+1,j}^{f}}{h^2} \\ \nabla^2 Z_{i,j}^{f-1}=\frac{Z_{i,j-1}^{f-1}-2Z_{i,j}^{f-1}+Z_{i,j+1}^{f-1}}{h^2}+\frac{1}{r_{i,j}}\frac{Z_{i,j+1}^{f-1}-Z_{i,j-1}^{f-1}}{2h}+\frac{Z_{i-1,j}^{f-1}-2Z_{i,j}^{f-1}+Z_{i+1,j}^{f-1}}{h^2} \tag{29}$$

In the above equations, index s is omitted for simplification.

After mathematical manipulations one has

$$Z_{i,j}^{f}=\frac{\lambda\Delta t\left(\Delta t+\tau_T\right)}{M}\left(Z_{i-1,j}^{f}+Z_{i+1,j}^{f}+\frac{2r_{i,j}-h}{2r_{i,j}}Z_{i,j-1}^{f}+\frac{2r_{i,j}+h}{2r_{i,j}}Z_{i,j+1}^{f}\right)+$$
$$\frac{c\rho h^2\left(\Delta t+2\tau_q\right)+4\lambda\Delta t\tau_T}{M}Z_{i,j}^{f-1}-\frac{\lambda\Delta t\,\tau_T}{M}\left(Z_{i-1,j}^{f-1}+Z_{i+1,j}^{f-1}+\frac{2r_{i,j}-h}{2r_{i,j}}Z_{i,j-1}^{f-1}+\frac{2r_{i,j}+h}{2r_{i,j}}Z_{i,j+1}^{f-1}\right)- \quad (30)$$
$$\frac{c\rho h^2\tau_q}{M}Z_{i,j}^{f-2}+\frac{h^2\left(\Delta t\right)^2}{M}W_{i,j}^{f}$$

where

$$M=h^2\left[c\rho\left(\Delta t+\tau_q\right)+wc_b\rho_b\left(\Delta t\right)^2\right]+4\lambda\Delta t\left(\Delta t+\tau_T\right) \quad (31)$$

The approximation of the boundary conditions (26) is the following

$$-\lambda\left\{\mathbf{n}\cdot\nabla Z_s(r,z,t^f)+\frac{\tau_T}{\Delta t}\left[\mathbf{n}\cdot Z_s(r,z,t^f)-\mathbf{n}\cdot Z_s(r,z,t^{f-1})\right]\right\}=V_s(r,z,t^f) \quad (32)$$

Thus, one obtains the formulas

- for j=1, 2, ..., n–1

$$Z_{0,j}^{f}=Z_{1,j}^{f}-\frac{\tau_T}{\Delta t+\tau_T}\left(Z_{0,j}^{f-1}-Z_{1,j}^{f-1}\right)+\frac{h\Delta t}{\lambda\left(\Delta t+\tau_T\right)}V_{0,j}^{f} \quad (33)$$

$$Z_{n,j}^{f}=Z_{n-1,j}^{f}+\frac{\tau_T}{\Delta t+\tau_T}\left(Z_{n,j}^{f-1}-Z_{n-1,j}^{f-1}\right)-\frac{h\Delta t}{\lambda\left(\Delta t+\tau_T\right)}V_{n,j}^{f} \quad (34)$$

- for i=1, 2, ..., n–1

$$Z_{i,0}^{f}=Z_{i,1}^{f}-\frac{\tau_T}{\Delta t+\tau_T}\left(Z_{i,1}^{f-1}-Z_{i,0}^{f-1}\right)-\frac{r_{i,0}h\Delta t}{\lambda\left(\Delta t+\tau_T\right)}V_{i,0}^{f} \quad (35)$$

$$Z_{i,n}^{f}=Z_{i,n-1}^{f}+\frac{\tau_T}{\Delta t+\tau_T}\left(Z_{i,n}^{f-1}-Z_{i,n-1}^{f-1}\right)-\frac{r_{i,n}h\Delta t}{\lambda\left(\Delta t+\tau_T\right)}V_{i,n}^{f} \quad (36)$$

It should be noted that the implicit scheme of finite difference method is unconditionally stable [30].

Results of computations

As axially symmetrical domain of biological tissue is considered ($R = Z = 0.015$ mm). The following values of parameters are assumed [31], [32]: specific heat of tissue $c = 4000$ [J/(kg K)], specific heat of blood $c_b = 3770$ [J/(kg K)], tissue density $\rho = 1000$ [kg/m^3], blood density $\rho_b = 1060$ [kg/m^3], thermal conductivity of tissue $\lambda = 0.5$ [W/(m K)], blood temperature $T_b = 37$ [°C], blood perfusion rate $w = 0.0005$ [1/s], relaxation and thermalization times $\tau_q = \tau_T = 0.5$ [s], metabolic heat source $Q_m = 250$ [W/m^3]. Initial temperature of tissue is equal to $T_0 = 37$ [°C] and initial heating rate equals $u = 0$. Parameters for Arrhenius's integral [33]: $P = 1.98 \cdot 10^{106}$ [1/s] and $E = 6.67 \cdot 10^5$ [J/mol].

At first, the direct problem is solved under the assumption that $t_e = 200$ s and $q_0 = 15\,000$ [W/m^2] (c.f. equation (2)) and $r_D = R / 4$. The spatial grid step equals $h = 0.0003$ [m] and the time step $\Delta t = 0.1$ [s].

In Figure 1 the temperature histories at the points P_1(0.3mm; 0.3mm) and P_2(2.7mm; 2.7mm) are shown. As can be seen maximum temperature at point P_1 (the most heated node of domain interior) is almost 52 °C. The maximum temperature appears after 120 s, while the exposure time is 100 s. This is due to the relaxation time and the thermalization time that occur in the considered model.

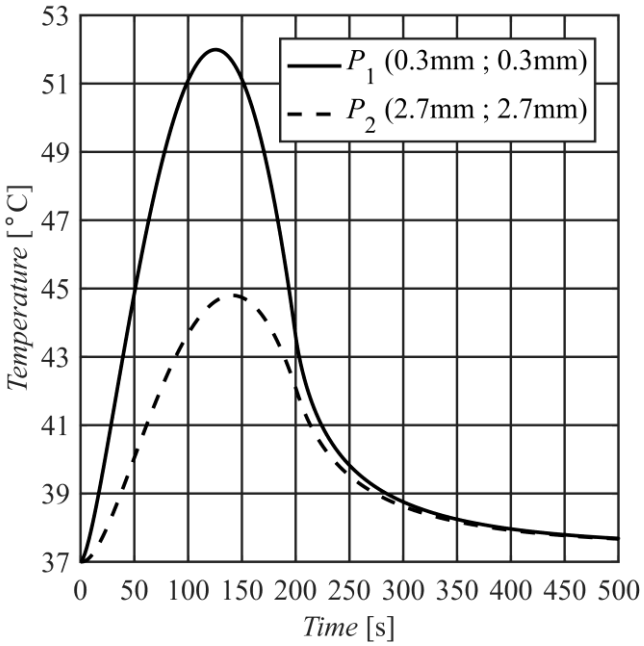

Figure 1. Temperature history at selected points - direct problem

The distribution of Arrhenius integral in the domain considered is presented in Figure 2. It should be noted that Arrhenius integral at the point P_1 is above 4.6, while at the point P_2 is lower than 1.

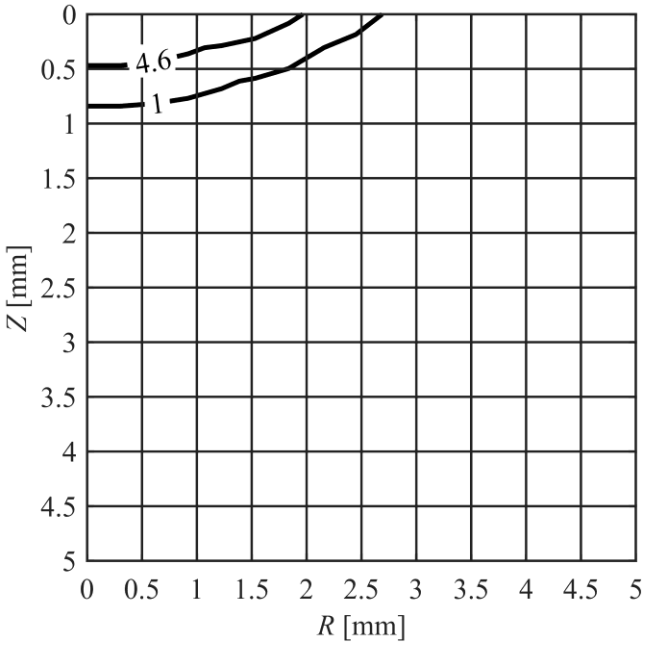

Figure 2. Arrhenius integral distribution

As the measuring points, the fifty points uniformly distributed in the range R: 0 – 2.7 mm and Z: 0 – 2.7 mm were selected. At those points the history of Arrhenius integral (A_m) was saved during solving the direct problem.

Next, three variants of starting values have been assumed:

- L_{V1}: q_0 = 50 000 W/m^2, t_e = 500 s
- L_{V2}: q_0 = 13 000 W/m^2, t_e = 150 s
- L_{V3}: q_0 = 50 000 W/m^2, t_e = 110 s

In Figures 3 – 5 the convergence of the algorithm is presented. It can be seen that for all three variants the concurrence is obtained after different numbers of iterations. When the starting values are close to the values used in direct problem, the algorithm needs fewer numbers of iterations, what can be seen in Figure 4.

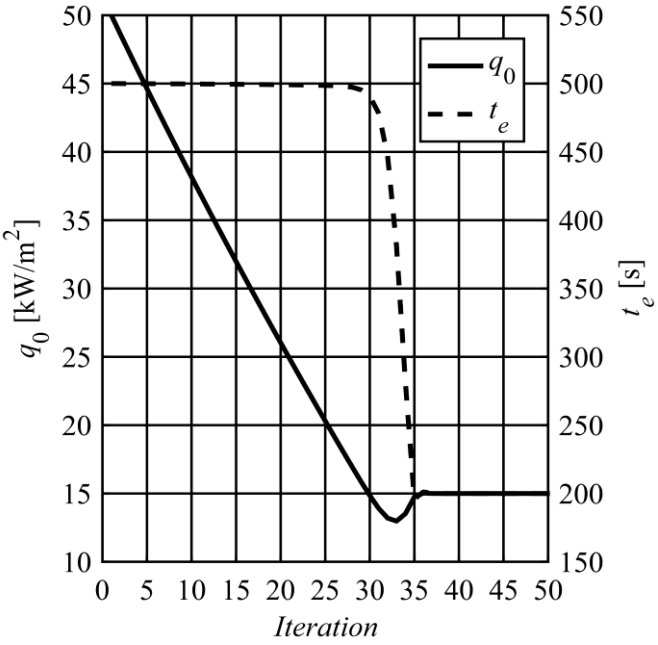

Figure 3. Convergence of the algorithm for the starting values L_{V1}

For the starting values L_{V1} and L_{V3} (q_0 = 50 kW/m^2) the value of q_0 straightforward goes to the desired value, while the value of t_e for several iterations remains constant, and then quickly changes to the desired value (c.f. Figures 1 and 3).
In Figure 4 the values of both variables in first two iterations increase significantly, while after tenth iteration they are almost concurrent.

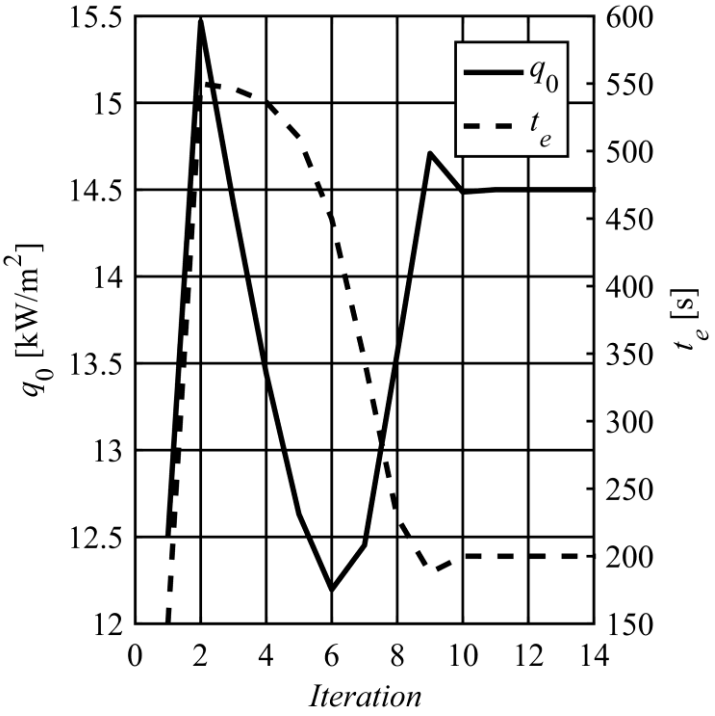

Figure 4. Convergence of the algorithm for the starting values L_{V2}

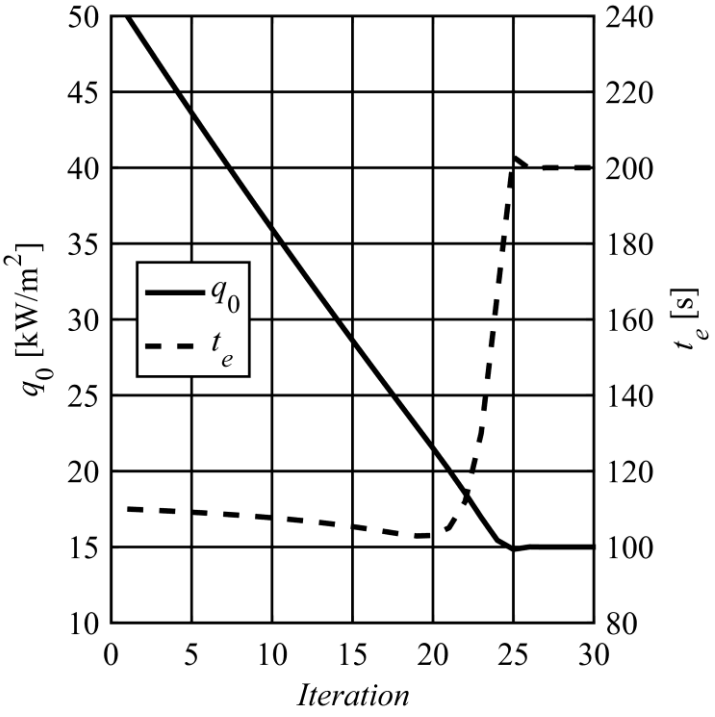

Figure 5. Convergence of the algorithm for the starting values L_{V3}

Conclusions

Thermal processes proceeding in axially symmetrical domain of heated tissue are considered. The inverse problem concerns the estimation of the boundary condition parameters. The mathematical model of the required sensitivity functions is also discussed.
The inverse problem was solved on the basis of the results of direct problem solution. The convergence of algorithm is presented for three different starting sets of values.
The presented approach can be efficiently used for determining the parameters of the artificial hyperthermia treatment.

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