The influence of non-linear parameters on the model of reconstructed middle ear

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Abstract

The paper presents numerical research of a reconstructed middle ear system using the element made of shape memory alloy via changing the various nonlinear parameters of the model. Shape memory alloy is modeled based on hysteretic nonlinear theory. The dynamic response of the reconstructed middle ear model is investigated. This work presents bifurcation analysis of the system to obtain different kind of solutions starting from regular and ending with chaotic vibrations.

Keywords: **Middle ear mechanics, Nonlinear dynamics**

Introduction

Shape memory alloys (SMAs) are plastically deformed materials, which belong to a group of structural materials and are used in many engineering fields (as elements of jet engines or micro-electromechanical systems) as well as in biomedical industry (as stent grafts or dental braces).The main feature of SMAs is a reversible thermo-elastic transformation between two phases: martensite and austenite. This phenomenon is called as shape memory effect (SME). The phase changes can be generated either by thermal actions or by mechanical stress. To SMEs can be two types: one-way SMA where material keeps a deformed state after cooling. The two-way SMA exists when the material can remember its shape at high and low temperature [3]. Furthermore, the SMAs can exsist in two different phases with three different crystalline lattice structures: twinned martensite, detwinned martensite and austenite. In these configurations six possible transformations can occur. The austenite structure is stable at high temperatures, and the martensite structure is stable at lower temperatures. The deformation depends on the temperatures: when material is heated, it starts the transition from the martensite into the austenite phase. In order to regain from the structure of austenite to twinned martensite the SMA should be cooled to the temperature, at which the phase of martensite is stable. The thermo-mechanical features of SMAs can be modeled at different scales [5]. The main and most popular class of models are based on an assumed phase transformation kinetics. In this case, the certain mathematical functions can be used to describe the phase transformation behavior of the material. This approach was presented by Tanaka and Nagaki [10]. The other scientists presents solution based on modified transformation kinetics laws [4]. The next step in scientific community were an analysis of the models which was based on Devonshire's theory: for a one-dimensional stress state proposed by Falk [1] and Falk and Konopka [2]. A similar model was developed by Fremond [3] and many others [7][8][9][10].

Model of a reconstructed middle ear

A dynamic model of middle ear is presented as a two degrees of freedom oscillator with shape memory (SMA) element (Figure 1). The SMA element (spring) is mathematically describe as a force F_{SMA} and damper c_{SMA}.

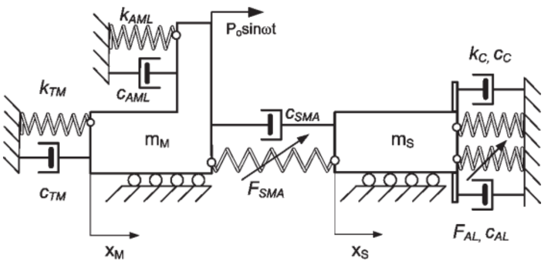

Figure 1. Two degree of freedom model with SMA element

The force F_{SMA} in nonlinear function of displacement (spring elongation) and velocity as follows:

$$F_{SMA} = \beta_2 (x_1 - x_2)^2 + \beta_3 (x_1 - x_2)^3 + \delta_3 (\dot{x}_1 - \dot{x}_2)^3 \tag{1}$$

The differential equation of the system motion (1) is expressed by non-dimensional form:

$$\begin{aligned} &\ddot{x}_1 + \alpha_{11} x_1 - \alpha_2 x_2 + \delta_{11} \dot{x}_1 - \delta\ \dot{x}_2 + \alpha_{13} x_1^3 + F_{SMA} - q\sin(\omega\tau) = 0 \\ &m\ddot{x}_2 - \alpha_2 x_1 + \alpha_{21} x_2 - \delta\ \dot{x}_1 + \delta_{21} \dot{x}_2 + \alpha_{22} x_2^2 + \alpha_{23} x_2^3 - F_{SMA} = 0 \end{aligned} \tag{2}$$

This nonlinear model of SMA oscillator is analyzed under the angle of nonlinearities and their effect on system behavior.

Results and discussion

Eq. (2) is solved numerically using Matlab Simulink applying parameters: $\alpha_{11} = 143.80$, $\alpha_2 = 142.80$, $\alpha_{13} = 3.0$, $\alpha_{21} = 143.5482$, $\alpha_{22} = 0.2727$, $\alpha_{23} = 2.0455$, $\delta = 0.0033$, $\delta_{11} = 0.00145$, $\delta_{21} = 0.003422$, $m = 0.0712$ and $\omega = 1$. In the 2 DOF model both non-chaotic dynamic system response, and chaotic behavior is observed, which depends on nonlinear parameters of SMA and external excitation q. An example of bifurcation diagram is presented in the Figure 2, where some symptoms of chaotic and irregular response is observed.

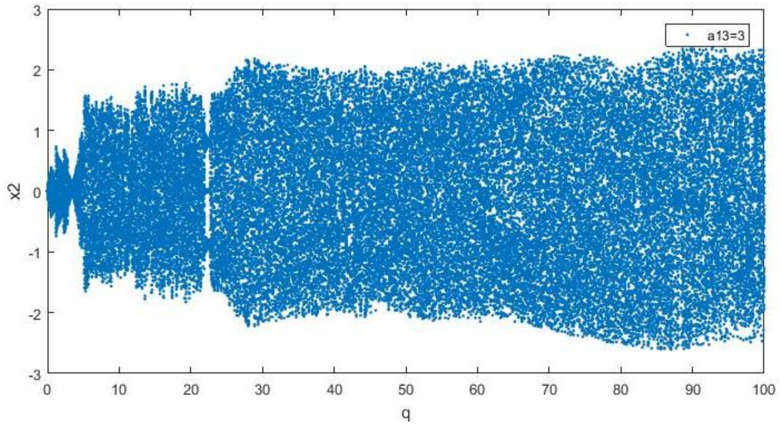

Figure 2. Bifurcation diagram for x_2 vibration for $a_{13} = 3$

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