Analytical formulation of dynamic response bounds for structures based on interval process model

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Abstract

Recently the authors developed a kind of non-probabilistic analysis method called 'non-random vibration analysis method' to deal with the important random vibration problems, in which the excitation and response are both given in the form of interval process rather than stochastic process. In this paper, some significant improvements were made for the non-random vibration analysis method, making it not only more rigorous in theory but also more practical in engineering. Firstly, the definitions and relevant conceptions of interval process model are further standardized and improved, and in addition some important conceptions such as the interval process vector and the cross-covariance function matrix are complemented. Secondly, the analytic formulation of dynamic response bounds is deduced for both of the linear single degree of freedom (SDOF) vibration system and the multiple degree of freedom (MDOF) vibration system, providing an important theoretical basis for non-random vibration analysis. Thirdly, this paper also gives the formulation and corresponding numerical methods of structural dynamic response bounds based on finite element method (FEM) for complex continuum problems, effectively enhancing the applicability of non-random vibration analysis method in engineering.

Keywords: **Random vibration; Non-probabilistic analysis; Interval process; Dynamic response bounds; Vibration system; Continuum structure**

Introduction

In a lot of practical engineering problems, the dynamic excitation on a structure not only shows time-variant characteristics, but also exhibits obvious uncertainties, such as ground motion acceleration on building basement in the earthquake, which often causes random vibration of the structure. Traditionally, the stochastic process and random vibration methods are adopted for quantification of a time-varying parameter and corresponding random vibration analysis [1]. However, great number of time-history testing samples are required to obtain the precise probability distribution characteristics of the stochastic process, which is often difficult or even impossible to obtain because of restrictions in experimental condition or cost.

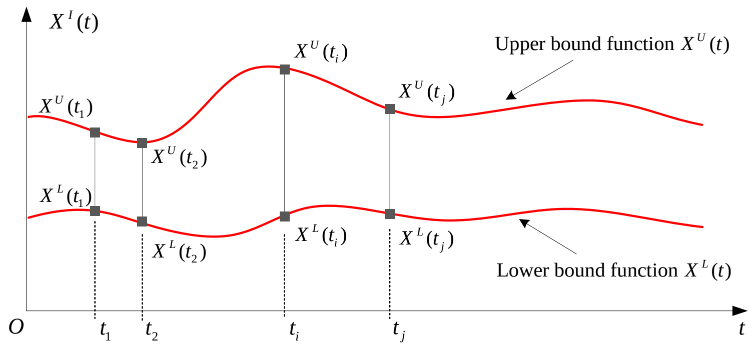

Figure 1. The interval process [2]

In our recent work, therefore, the interval method is successfully extended to time-variant problems, and a new model called interval process model [2, 3] was proposed to deal with the time-variant or dynamic uncertainties. As shown in Figure 1, the interval process model proposed by the authors [2, 3] employs a bounded and closed interval for quantification of the parametric uncertainty at arbitrary time point. Since then, by combining the interval process model with the traditional vibration theory, the authors further proposed a kind of non-probabilistic analysis method to deal with the random vibration problems, which is called the 'non-random vibration analysis method' [3-5]. In non-random vibration analysis, the interval process model rather than traditional stochastic process model is used to describe the uncertain excitation. The excitation and response are both given in the form of the time-history bounds, which avoids the introduction of probability characteristics. Solving the response bounds can then provide important reference data for the safety evaluation and reliability design of the practical vibration systems. This paper deduces the analytic formulation of dynamic response bounds for the linear single degree of freedom (SDOF) and multiple degree of freedom (MDOF) vibration systems and gives the formulation and corresponding numerical methods of structural dynamic response bounds based on FEM for complex continuum problems.

Results and discussions

Figure 2 shows a vehicle door structure, and point A and B are measurement and load point, respectively. We consider two cases of radius function of the load, namely, 20N (case 1) and 40N (case 2). For both of the cases, the auto-correlation coefficient function of the load is set as $\rho(t_i, t_j) = e^{-|t_i - t_j|}\cos(t_i - t_j)$. By applying the proposed method, the dynamic displacement response bounds at point A are obtained as shown in Figure 3. Firstly, it can be found that the dynamic displacement response of the vehicle door structure at point A also shows significant uncertainty because of the load uncertainty. In addition, for this problem, only 1 FEM computation was carried out in the whole non-random vibration analysis, which also reflects a very high computational efficiency of the proposed method.

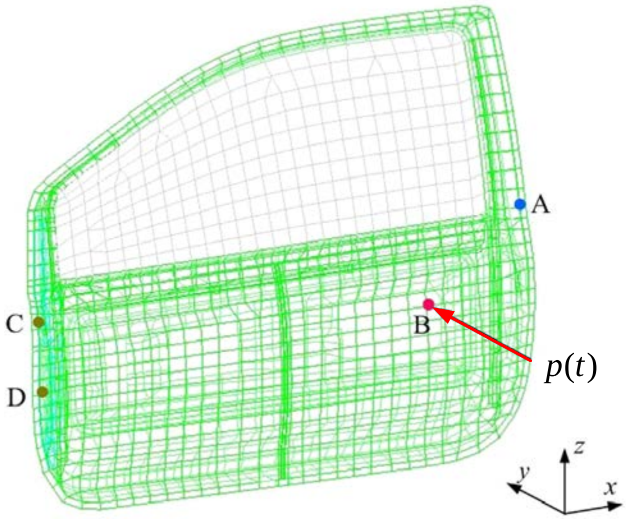

Figure 2. A vehicle door structure and its FEM model [6]

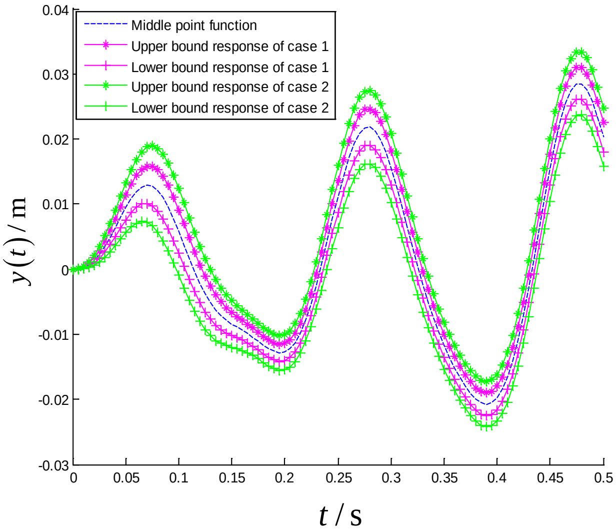

Figure 3. Displacement response bounds at the measurement point A [5]

Conclusions

Based on our previous work, this paper made some significant improvements for the non-random vibration analysis method. For the linear SDOF and MDOF vibration systems, an analytical formulation of the dynamic response bounds under uncertain excitation is derived by introducing the Duhamel's integral, providing an important theoretical basis for non-random vibration analysis. Additionally, for complex continuum structures, the formulation and corresponding numerical methods of the structural dynamic response bounds are also given through introducing the Green's kernel function technique, effectively enhancing the applicability of non-random vibration analysis method in engineering. As an effective supplement to the traditional random vibration theory, we hope to extend the non-random vibration analysis method to some relevant fields in the future, such as nonlinear vibration, structural reliability analysis, fatigue analysis of structures, etc.

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