Variation bounds analysis for structures under uncertain excitations

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Abstract

Recently, the authors extended the interval method into the time domain and proposed a new mathematical model for time-varying uncertainty quantification, namely, the "interval process model". Interval process uses a lower bound and an upper to describe the imprecision of a time-variant parameter at any time point rather than the precise probability distribution, and hence compared with the traditional stochastic process it shows some advantages in uncertainty quantification such as easy to understand, convenient to use, small dependence on samples, etc. This paper firstly gives the conceptions of limit and continuity of interval process, based on which the differential and integral of interval process are defined. Secondly, the middle point function, auto-covariance function and cross-covariance function of the differential and integral of interval process are deduced, which thus enriches the theory of interval process model. Thirdly, the above conceptions are applied to the vibration analysis of structures under uncertain excitations. The variation bounds of the velocity and acceleration responses are deduced for both of the linear single degree of freedom (SDOF) vibration system and the multiple degree of freedom (MDOF) vibration system under uncertain excitations.

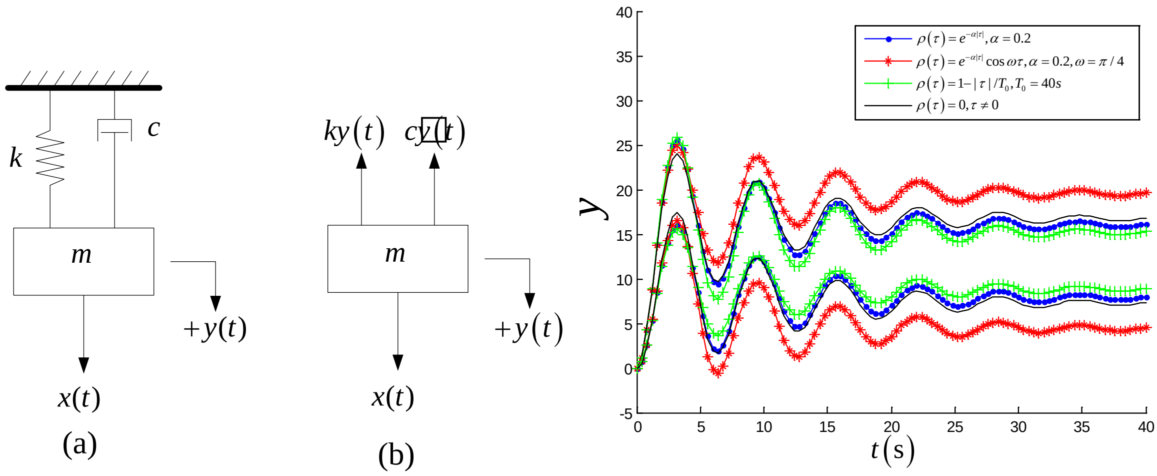
Keywords: **Interval process; Differential and integral of interval process; Time-variant uncertainty; Non-random vibration analysis; Response bounds**

Introduction

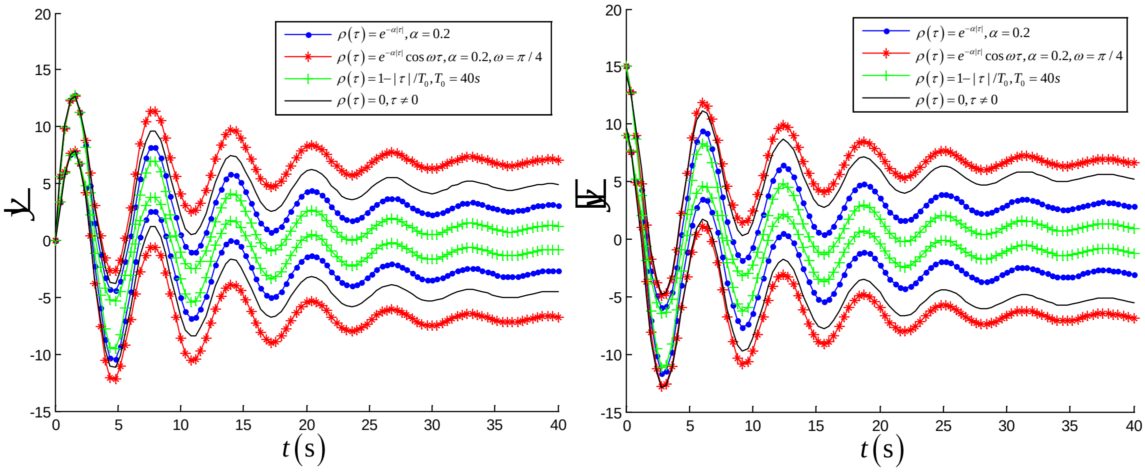
In the existing researches, the convex model approach [1-3] is primarily used to solve the time-invariant problems, in which the uncertainty of the involved parameters does not change with time. Nevertheless, for many practical problems, there exist some parameters not only exhibiting uncertainty, but also having time-varying or dynamic characteristics, such as the degrading material property with time, random dynamic loads applied to structures, etc. Recently, the authors thus extended the convex model approach to the time domain, and proposed a new kind of mathematical model to quantify the time-varying or dynamic uncertainty, namely, the "interval process model" [4, 5]. In the interval process model, an interval rather than a precise probability distribution is used to describe the parametric uncertainty at each time point and two boundary curves are then employed to depict the whole time-varying uncertainty of the parameter. Compared with the traditional stochastic process, the interval process theoretically has some attractive advantages, such as easy to understand, convenient to use, small dependence on samples, etc. Thus we hope that it can be a useful supplement to the classical stochastic process model in the future.

This paper firstly proposes the conceptions of differential and integral of interval process, which enriches the theory of interval process model. Secondly, the differential and integral of interval process are applied to the vibration analysis of mechanical systems under uncertain excitations. Not only the variation bounds of the displacement response for the vibration systems under uncertain excitation can be obtained, but also the variation bounds of the velocity and acceleration responses for the vibration systems can be derived.

Results and discussions



(a) The SDOF spring-mass-damper vibration system (b) Displacement responses



(c) Velocity responses (d) Acceleration responses

Figure 1. Displacement, velocity and acceleration response bounds of the damped SDOF system in different auto-correlation coefficient functions [6]

Figure 1 shows a SDOF spring-mass-damper vibration system.We consider four kinds of auto-correlation coefficient functions as follows: $\rho(\tau)=\mathrm{e}^{-\alpha|\tau|}$ (case a); $\rho(\tau)=\mathrm{e}^{-\alpha|\tau|}\cos\omega\tau$ (case b); $\rho(\tau)=1-|\tau|/T_0$ (case c); $\rho(\tau)=0,\tau\neq 0$ (case d). Herein the parameters are set as $\alpha=0.2$, $\omega=\pi/4$, $T=40\mathrm{s}$, and τ indicates the temporal distance. By using the proposed method, the variation bounds of the displacement, velocity and acceleration responses for the SDOF system can be obtained and plotted, as shown in Figure 1. From Figure 1, it can be seen that the bounds of the displacement, velocity and acceleration responses for the SDOF system will change when the auto-correlation coefficient function of the external excitation changes. Under four types of correlation cases, the above three response bounds can be roughly divided into two stages, i.e., the transient response stage and the steady-state response stage. In the transient response stage, the three response bounds exhibit obvious oscillation, and their middle point and radius functions both fluctuate with time. This kind of oscillation tends to be slighter with time, and a steady state can be reached. In this stage, both the middle points and radiuses of the three responses keep almost constants.

Conclusions

Based on our previous work, this paper firstly gives the conceptions of the limit, continuity, differential and integral of interval process and the corresponding properties, which enriches the interval process model theory. Secondly, the variation bounds of velocity and acceleration responses for the linear SDOF and MDOF vibration systems under uncertain excitations are derived based on the differential and integral of interval process, effectively enhancing the analytical ability of the non-random vibration analysis method. As the extension of the interval process model, the relevant conceptions of the differential and integral of interval process can be extended into some other fields, such as time-variant reliability analysis, finance engineering, etc.

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