# Bifurcational analysis of the friction-induced mechanical oscillator with modified LuGre friction model

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## Abstract

The work to be presented focuses on the analysis of the friction-induced mechanical oscillator with cubic nonlinearity and dynamic model of dry friction applied. The studied model is based on the classical LuGre approach to friction force modelling [1]. Modifications to this model are introduced because of observed dynamical responses of friction-induced oscillator [2].

The first attention is paid to the mathematical model of the system. Investigated oscillator consists of mass-spring structure with the mass undergoing to external harmonic forcing while situating on a belt, which moves with constant velocity. Also cubic stiffness term is taken into account. Next authors describe dynamic friction model due to LuGre approach [1] and further the modified one [2]. Authors derive dimensionless equations of motion of the frictional oscillator and integrate them numerically by means of C++ programing language. LuGre and modified LuGre friction models are compared from the point of view of the behaviour nature during pure sliding phase of motion; the most suitable is selected for the further analysis.

The second section presents results of the numerical simulations of the investigated oscillator for the assumed range of parameters. Phase portraits are presented and corresponding friction-velocity figures present different kinds and sizes of hysteretic loops during both stick and slide modes. The crucial influence of internal parameters of friction on the behaviour of the response. Friction hysteresis behaviour can be observed by comparing results of simulation for different parameter sets. Among others it is shown that the oscillations in the sticking phase can by eliminated by decreasing the internal damping of the asperity, the velocity overshoot at the beginning of stick can be decreased by increasing internal stiffness. The following tendency is formulated: the higher the values of internal parameters, the smaller the size of the hysteretic loop in friction force comes out.

The next section contains the analysis of the stability of the oscillator with assumed modified LuGre friction model. Authors start with the introduction of different methods of Lyapunov exponents estimation and discuss recent results in this field of nonlinear dynamics [3-10]. The description of the algorithm of LEs estimation, introduced in [11], is presented. Mentioned method is applicable to investigate the nature of the response of the frictional oscillator to the assumed model [2]. In particular, Poincare sections are necessary to apply this algorithm, because the system under investigation is generally non-smooth. Points on such maps were caught every period of excitation and were presented in the figure for different excitation parameters. Next the Jacobi matrix of a map is found by means of orthogonal perturbation vectors [11]. Changing incrementally the forcing frequency, bifurcation diagram can be obtained along with the corresponding Lyapunov exponents graph. It is shown that oscillator responses different for certain regions of varied parameter. Dominant LE corresponds well to the bifurcation diagram, which is proved in figures. Finally, observations are discussed and conclusions are drawn in the last section of the article.

Keywords: **Dry friction, Hysteretic effects, Stick-slip motion, Modified LuGre model, Lyapunov exponents, Friction-induced vibrations.**

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