Computation of Deformable Image Registration by Meshless Kernel-based Collocation Method

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Abstract

The deformable image registration (DIR) is a class of optimization methods and is commonly used to compute the spatial displacements between different regions of interest. When the process of an anatomical change whose mechanism is not comprehended analytically, the change of such system can be studied through tracing the spatial displacements between the static images and the deformed images. The diffusion of spatial transformation between the profiles of target object between n^{th} treatment and $(n+1)^{th}$ treatment can be estimated. The volumetric analysis and the positional changes of the target object can be monitored and adaptive medical strategies can be applied during the course of treatment.

The present paper aims to set up a kernel-based collocation scheme in order to obtain a possible optimum deformable image registration. The proposed computational scheme will incorporate the meshless reciprocal multiquadric function. Its meshless configuration provides with a simple formulation and also significantly reduces the time used and the cost in setting up the computational algorithm. The computational result of the proposed scheme is verified with a real liver cancer medical imaging dataset. The influence of collocating different number of data points on the performance of the numerical results will also be explored and analyzed in this paper.

Keywords: Deformable image, Meshless RBF, Kernel collocation.

Introduction

Deformable image registration (DIR) is a common process in the medical image analysis. The DIR helps to trace the tumor growth, confine the treatment to a well-defined target region and avoid damage to healthy organs. All these factors are very important for medical treatment decision. For instance, the intended outcome of the medical treatment is to inhibit the tumor growth and minimize the side effects to the patient. In order to optimize the image-guided radiation therapy (IGRT) as discussed in [1], a partition deformable image registration (DIR) is used to delineate the region on the computed tomography (CT) images obtained during the radiation treatment.

The purpose of the deformable image registration is to find out the displacements of the deformed target region. The precision of the measurement of the region of interest depends tremendously on the accuracy of the registered image. Sotira et al. [2] had carried out a detailed survey and reviewed of the recent developments on the field of DIR. Over the last decade, DIR has been recognized a significant influence on the medical field, thus this topic still remains a challenging topic. A number of researchers from different disciplines contributed efforts to find optimal performance of deformable image registration.

The basic idea of the image registration problem is formulated as an optimization model. Suppose the target object of interest has undergone some changes from times t_0 to t_1. Given the static image $s(x, y)$ and deformed image $m(x, y)$ at time t_0 and t_1 respectively, the spatial displacements between the static image $s(x_i, y_i)$ and the deformed image $m(x_i, y_i)$ can be predicted using an appropriate DIR scheme.

The proposed scheme presented in this study is applied to solve the classical Demon's DIR model which is a well established DIR model. The system of Demon's model has taken account of those dynamic and physical effects that occurred from the process of transformation.

The computational scheme is constructed based on the kernel radial basis function collocation method and use it to approximate the spatial displacements between the two deformable images. Radial basis function (RBF) is a novel interpolation method and has been proved to be effective in solving various kinds of differential equations. The meshless configuration of RBF also offers us with an ease and flexible design of the computational algorithm.

The Classical Deformable Image Registration Model

Several forms of deformable image registration models are established in different numerical methods. One of the classical DIR models is called Demon's algorithm developed by [3] in 1996. The basic deformation model are derived from Navier nonlinear elastic model. Let $\mathbf{D}(x, y) = (u_x, u_y)^T$ be the displacements matrix between the static and deformed images. The results of $\mathbf{D}$ can be generated by the differential equations equations

$$\mathbf{D}(x, y) = \frac{(\mathbf{m} - \mathbf{s})\nabla\mathbf{s}(x, y)}{|\nabla\mathbf{s}(\mathbf{x}, \mathbf{y})|^2 + (\mathbf{m} - \mathbf{s})^2}, \tag{1}$$

where $\mathbf{m}$ is moving image and $\mathbf{s}$ is the static image, $(\mathbf{m} - \mathbf{s})$ is the differential forces between the moving image and deformed images. $\nabla\mathbf{s}$ is the gradient operator of the static image defined by

$$\nabla\mathbf{s}(x, y) = \left(\frac{\partial\mathbf{s}(x,y)}{\partial x}, \frac{\partial\mathbf{s}(x,y)}{\partial y}\right).$$

The equation in (1) can be rearranged to a homogeneous partial differential equation as

$$\mathbf{D}\left(|\nabla\mathbf{s}|^2 + (\mathbf{m} - \mathbf{s})^2\right) - (\mathbf{m} - \mathbf{s})\nabla\mathbf{s} = \mathbf{0}$$

subject to the given initial conditions $\mathbf{s}^0 = \mathbf{0}$ and $\mathbf{m}^0 = \mathbf{0}.$

Cachier *et al* [4] in 1999 revised model to improves the registration convergence rate and stability. The improved model is defined by

$$\mathbf{D} = \frac{(\mathbf{m} - \mathbf{s})\nabla\mathbf{s}(x, y)}{|\nabla\mathbf{s}(x, y)|^2 + \xi^2 |(\mathbf{m} - \mathbf{s})|^2} + \frac{(\mathbf{m} - \mathbf{s})\nabla\mathbf{m}(x, y)}{|\nabla\mathbf{s}(x, y)|^2 + \xi^2 |(\mathbf{m} - \mathbf{s})|^2}. \tag{2}$$

This revised model includes the image edge forces of the deformed image. The normalization factor ξ is added to adjust the force strengths. This attempts to normalize the relations between the moving and static image so as to improve the image registration.

Kernel-based Collocation Method

This paper discusses a class of kernel-based approximation methods in the form of radial basis function (RBF). The proposed scheme will be used to solve the deformable image registration model. The kernel-based approximation method has been refined and diversified for facilitating the needs of various types of differential equations. The RBF approximation was originally devised for scattered geographical data interpolation by Hardy [5], who introduced a class of RBF called multiquadric function in the early 1970's.

This study aims to determine an optimum spatial displacements between the deformable images. Let the displacements of DIR model be $\mathbf{D}(x, y) = (u_x, u_y)^T$ between the static and deformed images defined in the equation (1). The basic idea of the kernel-basis RBF interpolation is used to approximate this unknown displacements $\{\mathbf{D}(\mathbf{x}) : \mathbf{x} \in \Omega\}$ by a RBF interpolant, say $\{\mathbf{f}(\mathbf{x}) : \mathbf{x} \in \Omega\}$ at a given set of N distinct points $X = \{\mathbf{x}_i \in \Omega : i = 1, 2, \cdots, N\}$.

The general form of the kernel-based RBF interpolation is a finite linear combination by the following equation

$$\begin{aligned} \mathbf{f}(\mathbf{x}) &= \alpha_1\phi_1(\|\mathbf{x} - \tilde{\mathbf{x}}_1\|) + \cdots + \alpha_N\phi_N(\|\mathbf{x} - \tilde{\mathbf{x}}_N\|) \\ &= \sum_{i=1}^{N} \alpha_i\phi(\|\mathbf{x} - \tilde{\mathbf{x}}_i\|), \ \mathbf{x} \text{ and } \tilde{\mathbf{x}}_i \in \Omega, \end{aligned} \tag{3}$$

where ϕ refers to a specific choice of RBF functions that is solely dependent on the Euclidean distance $r = \|\mathbf{x} - \tilde{\mathbf{x}}_i\|$ between $\mathbf{x}$ and a fixed centre $\tilde{\mathbf{x}}_i \in \Omega$. The unknown coefficients $\{\alpha_i : i = 1, 2, \cdots, N\}$ can be determined by collocating

$$\mathbf{f}(\mathbf{x}_i) = \mathbf{D}(\mathbf{x}_i), \ \text{for } i = 1, 2, \ldots, N, \tag{4}$$

at a set of N distinct data points $\{\mathbf{x}_i \in \Omega, 1, 2, \cdots, N\}$. This yields a system of linear equations which can be expressed in the following matrix form

$$[\mathbf{A}_\phi]\,[\boldsymbol{\alpha}] = [\mathbf{D}]\,, \tag{5}$$

where

$$[\boldsymbol{\alpha}] = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_N \end{bmatrix} \text{ and } [\mathbf{D}] = \begin{bmatrix} D(\mathbf{x_1}) \\ D(\mathbf{x_2}) \\ \vdots \\ D(\mathbf{x_N}) \end{bmatrix}$$

are $N \times 1$ column matrices. The $[\mathbf{A}_\phi] = [\phi(\|\mathbf{x}_i - \mathbf{x}_j\|)]_{i,j=1}^N$ is an $N \times N$ matrix given by

$$[\mathbf{A}_\phi] = \begin{bmatrix} \phi(\|\mathbf{x}_1 - \mathbf{x}_1\|) & \phi(\|\mathbf{x}_1 - \mathbf{x}_2\|) & \cdots & \phi(\|\mathbf{x}_1 - \mathbf{x}_N\|) \\ \phi(\|\mathbf{x}_2 - \mathbf{x}_1\|) & \phi(\|\mathbf{x}_2 - \mathbf{x}_2\|) & \cdots & \phi(\|\mathbf{x}_2 - \mathbf{x}_N\|) \\ \vdots & \vdots & \ddots & \vdots \\ \phi(\|\mathbf{x}_N - \mathbf{x}_1\|) & \phi(\|\mathbf{x}_N - \mathbf{x}_2\|) & \cdots & \phi(\|\mathbf{x}_N - \mathbf{x}_N\|) \end{bmatrix}. \tag{6}$$

Generally, the interpolation points in interior and boundaries are distinct and a specific choice of RBF $\phi(\|\mathbf{x} - \mathbf{x}_j\|)$ is positive definite, the matrix $[\mathbf{A}_\phi]$ is always non-singular, so the linear system in (5) has a unique solution as proved by [6]. The unknown coefficients $[\boldsymbol{\alpha}]$ can then be determined uniquely by solving the system of linear equations by

$$[\boldsymbol{\alpha}] = [\mathbf{A}_\phi]^{-1} [\mathbf{D}] .$$

The approximated displacement matrix $[\mathbf{D}]$ can be evaluated once the unknown coefficients $[\boldsymbol{\alpha}]$ are found.

The most popular types of radial basis functions are listed below:

$$\phi(r_j) = \left\{ \begin{array}{lll} r^3 & \text{Cubic} & (a) \\ (r_j^2)\log r_j, & \text{Thin plate splines in } R^2 & (b) \\ e^{-\sigma r_j^2}, & \text{Gaussian, } \sigma > 0 & (c) \\ (r_j^2 + c^2)^{\frac{1}{2}}, & \text{Multiquadric } \mathbb{R} & (d) \\ (r_j^2 + c^2)^{-\frac{1}{2}}, & \text{Reciprocal multiquadric } \mathbb{R} & (e) \end{array} \right\}, \tag{7}$$

where $\{r_j = \|\mathbf{x} - \mathbf{x}_j\|\,,\ j = 1, 2, \cdots, N\}$ is the Euclidean distance between $\mathbf{x}$ and $\mathbf{x}_j \in \mathbb{R}^d$ and $c^2$is called the shape parameter of the functions in (d) & (e). This shape parameter uses to control the fitting of a smooth surface to the data and could be greatly influence of the intended results. A recent study by Luh [7] in 2012 has developed a concrete function which could help to determine an optimal shape parameter.

The RBF collocation scheme is a well defined efficient scheme, however, there are two crucial issues of RBF always be questioned.

(1) Is there always a unique solution to the system of equations in (5)?

(2) What type of conditions on $\{\phi(r_{i,j}) : i, j = 1, \cdots, N\}$ can guarantee the invertibility of the interpolation matrix $[\mathbf{A}_\phi]$?

Answering these questions is not simple. A full description can involve a number of properties and conditions. In this present study, the kernel-based RBF model is formulated by adding a finite polynomials $\{q_k(\mathbf{x}), k = 1, 2, \cdots, M\}$ into the interpolation system in (3). The purpose is to avoid having singularity in solving the system matrix. The RBFs interpolant in (3) is now extended to the sum of finite series equation as below:

$$\mathbf{f}(\mathbf{x}) = \sum_{i=1}^{N} \alpha_i \phi\left(\|\mathbf{x} - \tilde{\mathbf{x}}_i\|\right) + \sum_{k=1}^{M} b_k q_k(\mathbf{x}), \quad \mathbf{x} \in R^2, \mathbb{R}^2, \quad 0 \le M < N, \tag{8}$$

where $\{\alpha_i\}$ and $\{b_k\}$ are the unknown coefficients to be determined. Given a set of N distinct nodes $X = \{\mathbf{x}_i \in \Omega, i = 1, 2, \cdots, N\} \subseteq \mathbb{R}^d$, the approximation function in (8) will definitely produce a unique solution if the system satisfies the following condition

$$\mathbf{f}(\mathbf{x}_i) = \mathbf{D}(\mathbf{x}_i), \quad i = 1, 2, \cdots, N \tag{9}$$

and the constraints

$$\sum_{i=i}^{N} \alpha_i q_k(\mathbf{x}) = 0, \quad k = 1, 2, \cdots, M \text{ and } i = 1, 2, \cdots, N.$$

The resulting system can be written in matrix form as,

$$\begin{bmatrix} \mathbf{A}_\phi & \mathbf{Q} \\ \mathbf{Q}^T & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha} \\ \mathbf{b} \end{bmatrix} = \begin{bmatrix} \mathbf{D} \\ \mathbf{0} \end{bmatrix}, \tag{10}$$

where $[\mathbf{A}_\phi]$ is the same coefficient matrix and $[\boldsymbol{\alpha}]$ is the unknowns as defined in (5). The matrix $[\mathbf{Q}] = [q_k(\mathbf{x}_i)]$ is an $N \times M$ matrix and $[\mathbf{b}]$ is an $M \times 1$ column matrix for the finite polynomials. They are defined as

$$[\mathbf{Q}] = \begin{bmatrix} q_1(\mathbf{x}_1) & q_2(\mathbf{x}_1) & \cdots & q_M(\mathbf{x}_1) \\ q_1(\mathbf{x}_2) & q_2(\mathbf{x}_2) & \cdots & q_M(\mathbf{x}_2) \\ \vdots & \vdots & \ddots & \vdots \\ q_1(\mathbf{x}_N) & q_2(\mathbf{x}_N) & \cdots & q_M(\mathbf{x}_N) \end{bmatrix}, \quad [\mathbf{b}] = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_M \end{bmatrix}.$$

The interpolation problem in equation (9) is solvable if the matrix of this system is

$$[\tilde{\Phi}] = \begin{bmatrix} \mathbf{A}_\phi & \mathbf{Q} \\ \mathbf{Q}^T & 0 \end{bmatrix}$$

is non-singular. The unknowns $\begin{bmatrix} \boldsymbol{\alpha} \\ \mathbf{b} \end{bmatrix}$ can then be determined by inverting the coefficient matrix as

$$\begin{bmatrix} \boldsymbol{\alpha} \\ \mathbf{b} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_\phi & \mathbf{Q} \\ \mathbf{Q}^T & 0 \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{D} \\ \mathbf{0} \end{bmatrix}.$$

The values of the displacement function ($\mathbf{D}$) can be determined uniquely once the unknown vectors $[\boldsymbol{\alpha}]$ and $[\mathbf{b}]$ are determined.

The Computational Algorithm

The computational scheme of this paper is established according to the basic idea of Demon's algorithm. The equations of Demon's model are discretized by using finite difference scheme. The incremental displacements $\mathbf{D}^j(x,y)$ for the j^{th} iteration is simulated iteratively by the following equation:

$$\mathbf{D}^j(x,y) = \mathbf{D}^{j-1}(x,y) - \frac{\left(\mathbf{m}^{j-1}(x,y) - \mathbf{s}^0(x,y)\right)\nabla\mathbf{s}^0}{\|\nabla\mathbf{s}^0(x,y)\|^2 + \left[\mathbf{m}^{j-1}(x,y) - \mathbf{s}^0(x,y)\right]^2} \tag{11}$$

for $j = 1, 2, \ldots N$. In the present algorithm, where the result of displacement $\mathbf{D}^{j-1}(x,y)$ is generated by using the kernel RBF approximation by

$$\mathbf{D}^{j-1}(x,y) = \left(\sum_{i=1}^{N} \alpha_i^{j-1} \phi(\|\mathbf{r}\|) + \sum_{k=1}^{M} b_k q_k^{j-1}(\mathbf{x}) \right),$$

where $\|\mathbf{r}\| = \|\mathbf{x} - \widetilde{\mathbf{x}}_j\|$ is the Euclidean distance between $\mathbf{x}$ and $\mathbf{x}_j \in \mathbb{R}^2$. The computation requires the input of the following initial values

$$\begin{aligned} \mathbf{D}^0(x,y) &= \mathbf{0}, \\ \mathbf{m}^0(x,y) &= \tilde{\mathbf{m}}^0(x,y), \\ \mathbf{s}^0(x,y) &= \tilde{\mathbf{s}}^0(x,y). \end{aligned}$$

The spatial derivatives $\nabla \mathbf{s}^0 = (\nabla \mathbf{s}_x^0, \nabla \mathbf{s}_y^0)$ of the static image are the distance between two neighbourhood pixels is taken to be 1. Therefore, the spatial gradients $\nabla \mathbf{s}_x^0$ and $\nabla \mathbf{s}_y^0$ can be determined according to the following $n \times n$ matrices.

$$\nabla \mathbf{s}_x^0 = \begin{bmatrix} s_{1,2}^0 - s_{1,1}^0 & s_{1,3}^0 - s_{1,2}^0 & \cdots & s_{1,n}^0 - s_{1,n-1}^0 & s_{1,n}^0 - s_{1,n-1}^0 \\ \vdots & \cdots & \cdots & \vdots & \vdots \\ s_{n,2}^0 - s_{n,1}^0 & s_{n,3}^0 - s_{n,2}^0 & \cdots & s_{n,n}^0 - s_{n,n-1}^0 & s_{n,n}^0 - s_{n,n-1}^0 \end{bmatrix},$$

and

$$\nabla \mathbf{s}_y^0 = \begin{bmatrix} s_{2,1}^0 - s_{1,1}^0 & s_{2,2}^0 - s_{1,2}^0 & \cdots & s_{2,n}^0 - s_{1,n}^0 \\ \vdots & \cdots & \cdots & \vdots \\ s_{n,1}^0 - s_{n-1,1}^0 & s_{n,2}^0 - s_{n-1,2}^0 & \cdots & s_{n,n}^0 - s_{n-1,n}^0 \\ s_{n,1}^0 - s_{n-1,1}^0 & s_{n,2}^0 - s_{n-1,2}^0 & \cdots & s_{n,n}^0 - s_{n-1,n}^0 \end{bmatrix}.$$

The deformed image $\mathbf{m}^{*j}$ at the j^{th} iteration can be updated by substituting $\mathbf{D}^{j-1}$ and $\mathbf{m}^{*j-1}$ into the following forward iterative scheme

$$\begin{aligned} \mathbf{m}^{*j}(x,y) &= \mathbf{m}^{*j-1}(x,y) + \mathbf{D}^{j-1}(x,y)\nabla \mathbf{s}^0, \\ &= \mathbf{m}^{*j-1}(x,y) + \left(\sum_{i=1}^{N} \alpha_i^{j-1}\phi(||\mathbf{r}||) + \sum_{k=1}^{M} b_k q_k^{j-1}(\mathbf{x}) \right) \nabla \mathbf{s}^0. \end{aligned} \qquad (12)$$

In the numerical experiments, a normalized global reciprocal multiquadric function is used. The numerical results could be improved and more stable by adding a weighted factor λ^2. The normalized reciprocal MQ is defined by

$$\phi(||x - x_j||) = \frac{1}{\sqrt{c^2 + \lambda^2 \left[\left(\frac{x-x_j}{511}\right)^2 + \left(\frac{y-y_j}{511}\right)^2 \right]}}.$$

Our previous study reported in paper [8], the chosen of different factor λ^2 had been considered in the numerical experiments, it was found that the optimal result reached at choose $\lambda = 100$.

In the computation, the required stopping criteria is set by

$$\left\| \mathbf{D}^j(x,y) - \mathbf{D}^{j-1}(x,y) \right\| < \varepsilon \leq 0.002,$$

where ε is the preset upper bound of iterative error. The quality of the computed image $\mathbf{m}^*(x,y)$ by RBFs is examined and compared with those results by Thirion and Cachier Demon's algorithm. For comparison, the root mean square errors of the deformed image $\mathbf{m}^{*j}$ are analyzed by the equation

$$RMSE = \sqrt{\frac{1}{n} \sum_{x,y} \left(\frac{\mathbf{m}^*(x,y) - \mathbf{m}(x,y)}{\mathbf{m}(x,y)} \right)^2}. \qquad (13)$$

To illustrate the computational algorithm, the MATLAB code of the deformable image registration kernel-based RBF scheme is outlined below:

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Algorithm:  Demon's algorithm using kernel based RBF
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1. Input data : static image $\mathbf{s}^0$, deformed image $\mathbf{m}$, error tolerance ε
2. Result : $\mathbf{D}^N$, $\mathbf{m}^N$, RMSE
3. Initialization: $\mathbf{D}^j \leftarrow \mathbf{0}$, $\mathbf{m}^0 \leftarrow \mathbf{m}$, $N \leftarrow 1$
4. Compute $\nabla \mathbf{s}^0$

while $\left|\nabla \mathbf{s}^0\right|^2 + (\mathbf{m}^{N-1} - \mathbf{s}^0)^2 \neq \mathbf{0}$ and $\left\|\mathbf{D}^N - \mathbf{D}^{N-1}\right\| \geq \varepsilon$ and $N < 500$ do

$$\mathbf{D}^j := \sum_{i=1}^{n} \left(\alpha_i^j \phi(||\mathbf{r}||)\right) + \sum_{k=1}^{M} b_k q_k^j(\mathbf{x})$$

Compute $\mathbf{D}^N \leftarrow \mathbf{D}^{N-1} - \dfrac{(\mathbf{m}^{N-1} - \mathbf{s}^0)\nabla \mathbf{s}^0}{|\nabla \mathbf{s}^0|^2 + (\mathbf{m}^{N-1} - \mathbf{s}^0)^2}$

Solve α_i^N from $\mathbf{D}^N$

Compute $\mathbf{m}^N \leftarrow \mathbf{m}^{N-1} + \mathbf{D}^{N-1}\left(\nabla \mathbf{s}^0\right)$

$N = N + 1$

END

Case Study: Tracing Liver Cancer Growth

A real-life deformable image registration from a patient with liver cancer is used as a reference case study. One of the original static registered images is depicted in *Figure 1 at time* t_1 and the deformed images are obtained from two different treatment periods *at time* t_2 and t_3.

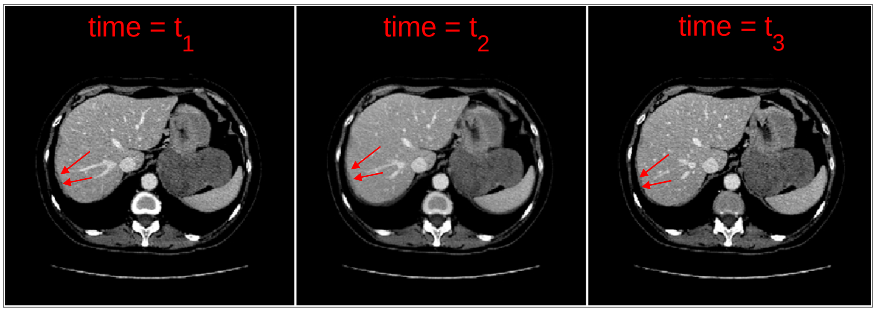

Figure 1 The growth of liver tumor as indicated by an arrow from different time t_1, t_2 and t_3,for $t_1 < t_2 < t_3$

The local ethics committee approval was obtained for a waiver of informed consented for retrospective analysis. The CT images were collected from collaborative hospital in year 2010.

The computational region is set up according to the original registered image. The geometrical structure of the original static image contains 512×512 pixels. The computational region is simplified by removing the insignificant backgrounds. These include the air with intensity equals to 0, and the bone with intensity equals to 1. The remaining valid pixels in the computational region are $82,633$.

The objective of the present study is to use a kernel-based RBF collocation method to measure the tumor changes in time t_3 by using the known information given by the deformable image registration obtained from t_1 and t_2. Similarly, the subsequent changes of tumor can be predicted by using the deformed relationship obtained in t_2 and t_3.

Our model aims to give a optimum estimation of object change so that an appropriate medical treatment can be applied according to the stage of the cancer. For example, the cancer is at early stage, say at stage A when diagnosed, a complete medical treatment may be possible by means of using radiotherapy treatment.

Numerical Results

The effectiveness of RBF scheme is very dependent on the choice of number of collocation points on the studied region. It is generally known that the global RBF scheme will be very computational expensive if a high number of data points is collocated in the study region. In the numerical experiments, the level of accuracy and the efficiency are also considered. The computed image results based on $n = 128$ and on $n = 256$ collocation points are illustrated in *Table 1*. According to the experimental results, the RMSE of using 256 collocation points is clearly lower than that of using 128 collocation points. This indicates that a high degree of accuracy can also be obtained by using an intensive collocation points in the computational region.

Table 1: Reciprocal global MQ function with Finite Polynomial Collocation Scheme

Root Mean Square Errors (RMSE) for the image at time t_4			
Collocation points	Shape parameter:	RMSE	CPU times required
Case 1: 256 points	$c = 0.79$	0.0162	317 seconds
Case 2 : 128 points	$c = 0.65$	0.07273	87 seconds

To compare the performance of the proposed kernel collocation scheme, the Thirion and Cachier Demon's iterative finite difference schemes are set up to simulate the DIR result of the same problem. The analysis of RMSE of the Thirion and Cachier Demon's model is summarized in *Table 2*.

Table 2: Classical Demon's Algorithm with Finite Different Scheme

Root Mean Square Errors (RMSE) for the image at time t_4	
	RMSE
Thirion's Demon Model, 1996	0.0738
Cachier's Demon Model, 1999	0.0733

The rate of convergence of Thirion and Cachier Demon's algorithm are compared graphically in *Figure 2*.

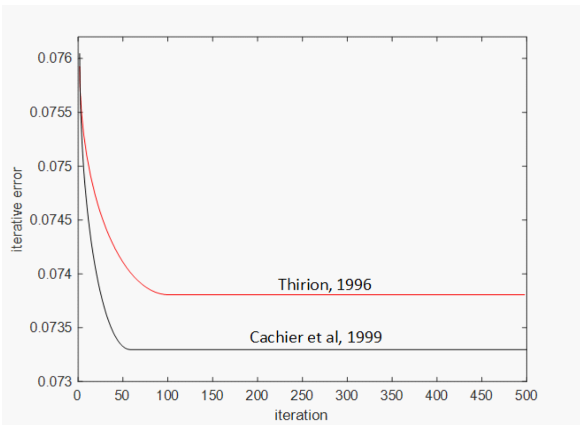

Figure. 2: The convergence of the two Demons algorithms.

The error analysis showed the optimum results of Thirion Demon's algorithm could be achieved in about 100 iterations, while Cachier Demon's algorithm produced an even faster convergent rate, the stopping criteria can be reached by less than 70 iterations. From the comparison of the numerical results, we convinced that the kernel RBF collocation scheme could achieve a higher level of accuracy and stability than the Demon's finite difference scheme.

The simulation for tracing the propagation of tumor, the computed (registered) image at time t_3 and at time t_4 of the region of interest are illustrated in *Figure 3* and *Figure 4*.

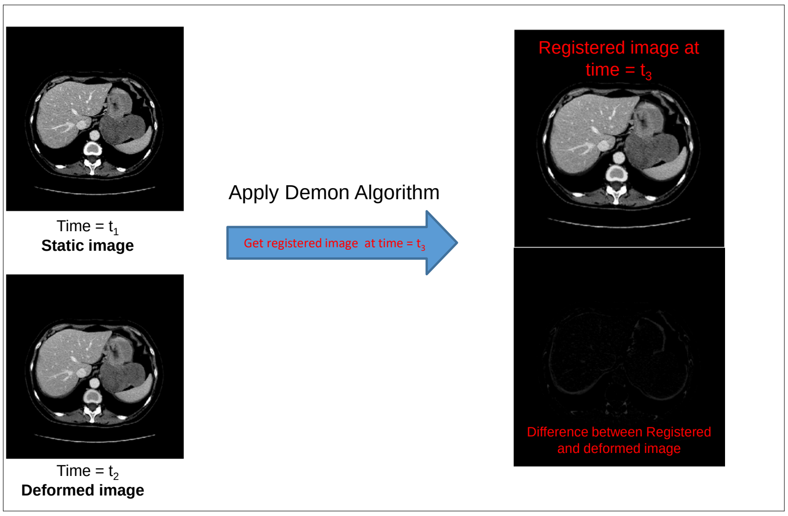

Figure 3 The computed (registered) image at time t_3

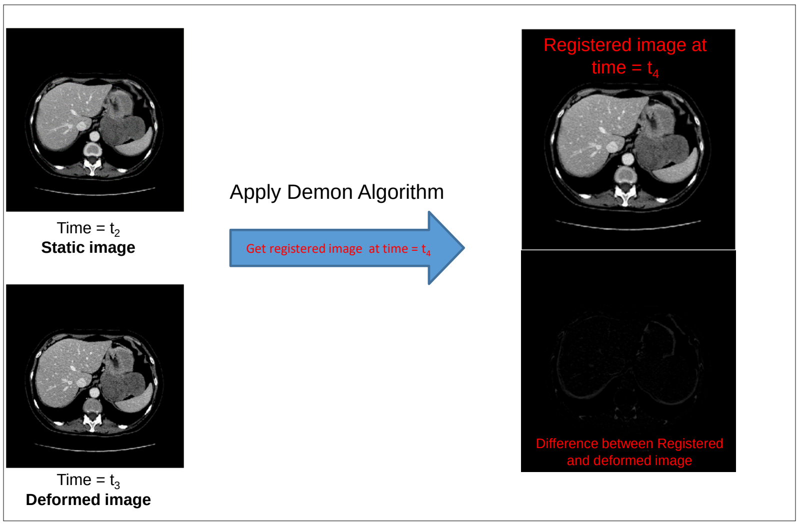

Figure 4 The computed (registered) image at time t_4

The displacements of these images can be identified through the deformation grid as shown in *Figure 4*. The difference between the computed registered image and the deformed CT image is also shown in the above *Figures*. As observed from the diagrams the overall difference is insignificant.

Conclusions

We have developed a kernel RBF collocation scheme to approximate the deformation image registration. In order to prevent the singularity problem, a finite polynomial term has been added into the RBF interpolant. The proposed algorithm based on a global reciprocal multiquadric function is applied to solve the classical Demon's DIR model. The computational region was set up based on a real-life deformable image registration in tracing the growth of a liver tumor. The computational efficiency and accuracy as well as the number of supporting points of the kernel approximation were explored. The computed images were calibrated with the real set of image registration.

In order to compare the capability of the proposed kernel RBF scheme, the two reference DIR models formulated by Thirion's and Cachier's Demon model were set up using finite difference scheme. The error analysis has shown that the proposed kernel RBF scheme produced a high level of accuracy. Regarding the issue of computational efficiency, our experiments showed that the computation of the kernel RBF scheme was clearly less efficient than the classical Demon's finite difference scheme. This was due to the defeat of the global supports of the kernel function setting. Indeed the numerical results lead an important role in the overall simulation of the DIR, thus the trade-off between numerical accuracy of the simulation results and computational efficiency has to be measured when using these schemes.

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