Phasor particle swarm optimization of dome structures under limited natural frequency conditions

***Ei Cho Pyone[1], Thu Huynh Van[1], Toan Minh Le[1], Linh Van Hong Bui[2] and †Sawekchai Tangaramvong[1]**

[1]Applied Mechanics and Structures Research Unit, Department of Civil Engineering, Chulalongkorn University, Bangkok 10330, Thailand.
[2]Faculty of Civil Engineering, Ho Chi Minh City Open University, Ho Chi Minh City 700000, Vietnam.

*Presenting author: eicho.ecpyone@gmail.com
†Corresponding author: sawekchai.t@chula.ac.th

Abstract

The paper proposes the novel numerical approach, called phasor particle swarm optimization (PPSO), for the optimal design of structures under natural frequency constraints. The proposed scheme develops the phasor theory in mathematics directly within the PSO algorithm. In essence, a phase angle incorporating the periodic sine and cosine functions is applied during the optimization process to model particle control parameters. This technique enables the fast-learning strategy of the particle velocity and is thus able to capture the optimal sizing distributions of structural members under some specified conditions at modest computing efforts. The application of the proposed method is illustrated for the optimal design of dome structures subjected to the responded natural frequency limits. To demonstrate the effectiveness and robustness of the proposed PPSO algorithm, a simple 3-D dome structure is successfully tested, and then the obtained results are compared with those of the other meta-heuristic algorithms in the literature.

Keywords: Phasor Theory, Particle Swarm Optimization, Dome Structures, Natural Frequency Conditions, Meta-Heuristic Algorithms.

Introduction

The natural frequency is one of the major parameters that indicates the dynamic performance of the structural system. Imposing some appropriate constraints on the frequency avoids undesirable vibrations and resonance under external excitations [1, 2]. The natural frequency constraints generate complexity in the optimal design of structures. The sizing optimization of the dome structure under natural frequency constraints for instance is regarded as the highly implicit nonlinear and/or non-convex problem [1, 3]. To address this problem, the efficient and robust optimization techniques are developed to provide both the most minimum of design structural weight and reasonable computing efforts.

Many metaheuristic algorithms have been employed to systematically capture the weight minima of practical structures without the need of mathematical programming implementations. A wide class of these methods has been studied in the structural optimization with natural frequency conditions. Some of which are genetic algorithm (GA) [4], particle swarm optimization (PSO) [5], democratic PSO (DPSO) [6], colliding-bodies optimization (CBO) [7], harmony search-based mechanism into PSO with aging leader and challengers (HALC-PSO) [8], modified sub-population TLBO (MS-TLBO) [9], colliding-bodies optimization (CBO) [7], vibrating particles system (VPS) [10], improved symbiotic organism search (ISOS) [11], improved differential evolution (IDE) [12] and adaptive hybrid evolutionary firefly (AHEFA) [13].

This paper proposes the sizing optimization method, based on a so-called phasor PSO (PPSO) [14], of dome-like truss structures under the constraints on the associated natural frequency. The implementation of PPSO is simple and adaptive, as it entails only the phase angle incorporating the periodic sine and cosine functions to model particle control parameters. The technique enables the fast-learning strategy of the particle velocity, and quickly captures the structural members under some specified natural frequency conditions.

Sizing optimization problem and formulations

The optimization problem minimizes the total weight (W) of the dome structure, where the design variables read member cross-sectional areas, namely A_d for each d-th member. The constraints consider the intrinsic structural responses and the required natural frequency. This can be mathematically described as follows:

$$\begin{array}{ll} \text{Find} & A_d \ \text{ for } \ \forall d \in \{1,\ldots,n_d\} \\ \text{Minimize} & W = \sum_{d=1}^{n_d} \rho_d A_d L_d \\ \text{Subjected to} & \omega_j \le \omega_j^*, \quad \omega_k \le \omega_k^* \\ & A_{\min} \le A_d \le A_{\max} \end{array} \tag{1}$$

where n_d is the total number of (pin-jointed) truss members, ρ_d the material density, L_d the length of a generic d-th member, ω_j and ω_k the response natural frequencies (i.e., the j-th and k-th modes, respectively), ω_i^* and ω_j^* the natural frequency limits, A_{min} and A_{max} the lower and upper limits on the available sectional areas, respectively.

The problem in Eq. (1) is further reformulated by applying the penalty function f to the objective function (total design weight) W:

$$\left.\begin{array}{l} f = W(1+\varepsilon_1.C)^{\varepsilon_2} \\ C = c_\omega^j + c_\omega^k \end{array}\right\}, \tag{2}$$

$$\text{where } c_\omega^j = \begin{cases} \dfrac{|\omega_j|}{|\omega_j^*|} - 1, & \text{if } \ \omega_j > \omega_j^* \\ 0, & \text{if } \ \omega_j \le \omega_j^* \end{cases}, \quad c_\omega^k = \begin{cases} \dfrac{|\omega_k|}{|\omega_k^*|} - 1, & \text{if } \ \omega_k > \omega_k^* \\ 0, & \text{if } \ \omega_k \le \omega_k^* \end{cases},$$

C is the penalty factor associated with the violation of natural frequency constraints, c_ω^j and c_ω^k the parameters indicating the satisfaction or violation of the natural frequency conditions. The parameters ε_1 and ε_2 are set to 1 and 1.5 (subsequently increase to 2), respectively.

Phasor Particle Swarm Optimization Algorithm

The PPSO method, also known as the nonparametric variant PSO algorithm, is based on the phasor theory in mathematics. The algorithmic procedures are summarized in Fig. 1. The algorithm searches for the global best position of each particle within the design domains. All particles cross over the neighbor particles within d dimensional search spaces using some social cooperation and swarm around the position to optimize (minimize) the objective function. In essence, each generic i-th particle in the swarm randomly generates the information, including the current positions $\mathbf{X}_i \in \Re^{n_d} = [X_{i,1},\ldots,X_{i,n_d}]$, the velocities

$\mathbf{V}_i \in \Re^{n_d} = [V_{i,1},\ldots,V_{i,n_d}]$, the distances between the particle best ($\mathbf{pbest}_i \in \Re^{n_d} = [pbest_{i,1},\ldots, pbest_{i,n_d}]$) and current positions $\mathbf{X}_i$, and the distances between the global best positions $\mathbf{gbest} \in \Re^{n_d} = [gbest_1,\ldots, gbest_{n_d}]$ and $\mathbf{X}_i$. These parameters are iteratively updated to keep the particle converging toward the optimal solutions [14].

In the PPSO algorithm, the phase angle θ lying within a range between 0 and 2π radians is introduced as the particle control parameter that incorporates the periodic sine (i.e., in an interval of $[-1, 1]$) and cosine (in $[0, 1]$) functions. The variations of these periodic functions with phase angles are depicted in Fig. 2. Both functions $\left|\cos\theta_i^t\right|^{2\sin\theta_i^t}$ and $\left|\sin\theta_i^t\right|^{2\cos\theta_i^t}$ construct the adaptive searching characters through the phase angels of individual particles. The velocities $\mathbf{V}_i$ and positions $\mathbf{X}_i$ associated with the i-th particle are updated at the $(t + 1)$-th iteration as follows:

$$\boldsymbol{X}_{i,d}^{t+1} = \boldsymbol{X}_{i,d}^{t} + \boldsymbol{V}_{i,d}^{t+1}, \tag{3}$$

$$\boldsymbol{V}_{i,d}^{t+1} = \left(\frac{\left|\cos\theta_i^t\right|^{2\sin\theta_i^t}}{npop} \boldsymbol{V}_{i,d}^{t} \right) + \left|\cos\theta_i^t\right|^{2\sin\theta_i^t} (\boldsymbol{pbest}_{i,d}^{t} - \boldsymbol{X}_{i,d}^{t}) + \left|\sin\theta_i^t\right|^{2\cos\theta_i^t} (\boldsymbol{gbest}_{d}^{t} - \boldsymbol{X}_{i,d}^{t}), \tag{4}$$

where $\left|\cos\theta_i^t\right|^{2\sin\theta_i^t} / npop$ is an inertia weight factor controlling the balance between the global and local searching abilities, and $\left|\cos\theta_i^t\right|^{2\sin\theta_i^t}$ and $\left|\sin\theta_i^t\right|^{2\cos\theta_i^t}$ the two acceleration control periodic functions. All particles learn from their $\mathbf{pbest}_i$ and $\mathbf{gbest}$ underpinning the swarm to update the corresponding velocities and positions:

$$\mathbf{pbest}_i^{t+1} = \begin{cases} \boldsymbol{pbest}_{i,d}^{t}, & \text{if } f(\boldsymbol{pbest}_{i,d}^{t}) \le f(\boldsymbol{X}_{i,d}^{t+1}) \\ \boldsymbol{X}_{i,d}^{t+1}, & \text{otherwise} \end{cases} \quad \text{for } d \in \{1,\ldots,n_d\}, \tag{5}$$

$$\mathbf{gbest}^{t+1} = \begin{cases} \boldsymbol{pbest}_{i,d}^{t+1}, & \text{if } f(\boldsymbol{pbest}_{i,d}^{t+1}) \le f(\boldsymbol{gbest}_{d}^{t}) \\ \boldsymbol{gbest}_{d}^{t}, & \text{otherwise} \end{cases} \quad \text{for } d \in \{1,\ldots,n_d\}. \tag{6}$$

The phase angle θ_i and maximum velocity $\mathbf{V}_{(\max)i}$ of the i-th particle are updated in the $(t + 1)$-th iteration by [15].

$$\theta_i^{t+1} = \theta_i^t + \left|\cos\theta_i^t + \sin\theta_i^t\right| 2\pi, \tag{7}$$

$$\boldsymbol{V}_{(\max)i,d}^{t+1} = \left|\cos\theta_i^{t+1}\right|^2 (\boldsymbol{X}_{(\max)d} - \boldsymbol{X}_{(\min)d}). \tag{8}$$

This technique enables the particles to overcome the searches in the local optimal region as well as premature solution convergence.

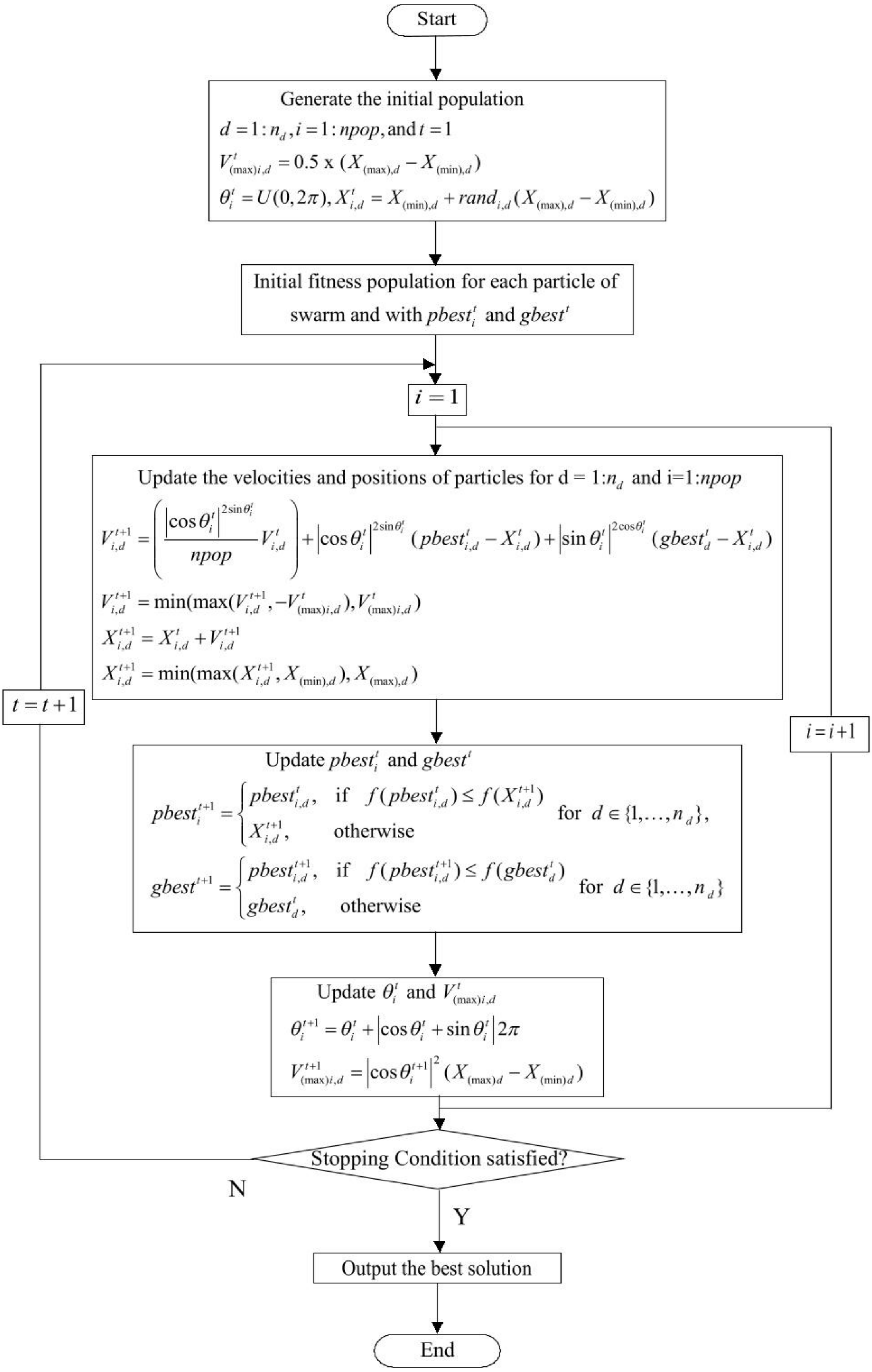

Fig. 1. PPSO procedures.

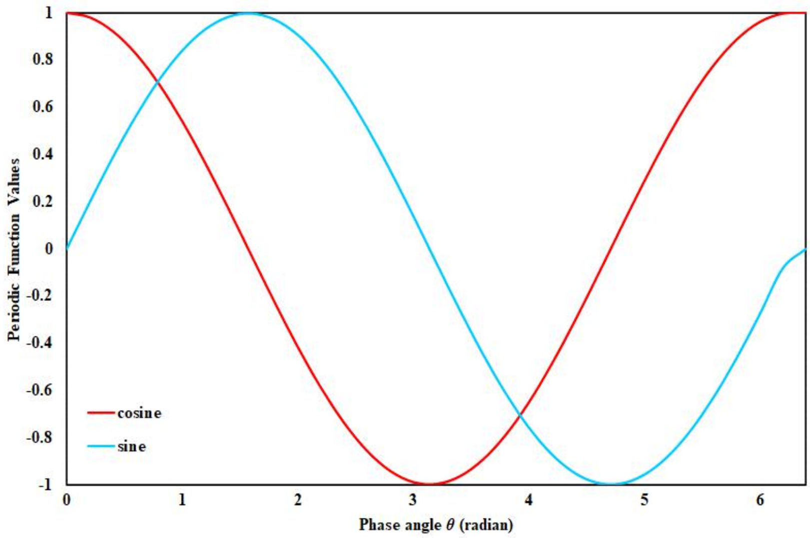

Fig. 2. Variation of periodic functions for different phase angles.

Illustrative Example

The 120-member dome-like truss structure in Fig. 3 was designed for the cost minimization of all member sizes under the limited natural frequency conditions. The lumped mass of m_1 was assigned at node 1, m_2 at nodes 2 to 13, and m_3 for the remaining nodes. All design member areas were categorized into 7 design groups. The material properties and frequency constraints employed are listed in Table 1.

The optimal sizing design of the dome was successfully performed by the proposed PPSO method with the total of 20 independent runs and the population of 30 particles. The optimal solutions, see Fig. 4, converged at the early number of analysis iterations for all the repeating PPSO solves.

The resulting member sizes of all 7 design groups and the total weight of W = 8736.24 kg are reported in Table 2, where the solutions from various analysis methods are also compared. More explicitly, the designed weight values referred are 8896.74 kg by vibrating particles system (VPS) [10], 8890.48 kg by democratic particle swarm optimization (DPSO) [6], 8889.13 kg by colliding-bodies optimization (CBO) [7], 8889.96 kg by harmony search-based mechanism into the particle swarm optimization with an Aging Leader and Challengers (HALC-PSO) [8], 8710.06 kg by improved symbiotic organisms search (ISOS) [11], 8708.73 kg by modified sub-population teaching-learning-based optimization (MS-TLBO) [9], 8707.29 kg by improved differential evolution (IDE) [12], and 8707.26 kg by adaptive hybrid evolutionary firefly (AHEFA) [13]. The optimal design result computed by the present method agrees very well with all benchmarks with the comparable numerical efforts, where the response natural frequencies comply with the imposed limits.

Table 1. Material properties and design parameters.

Parameters	Value
Modulus of elasticity E (N/m^2)	2.1×10^{11}
Material density ρ (kg/m^3)	7971.81
Additional mass (kg)	$m_1 = 3000$; $m_2 = 500$; $m_3 = 100$
Allowable range of cross-section (cm^2)	$1 \leq A \leq 129.3$
Constraints on the first two frequencies (Hz)	$\omega_1 \geq 9$; $\omega_2 \geq 11$

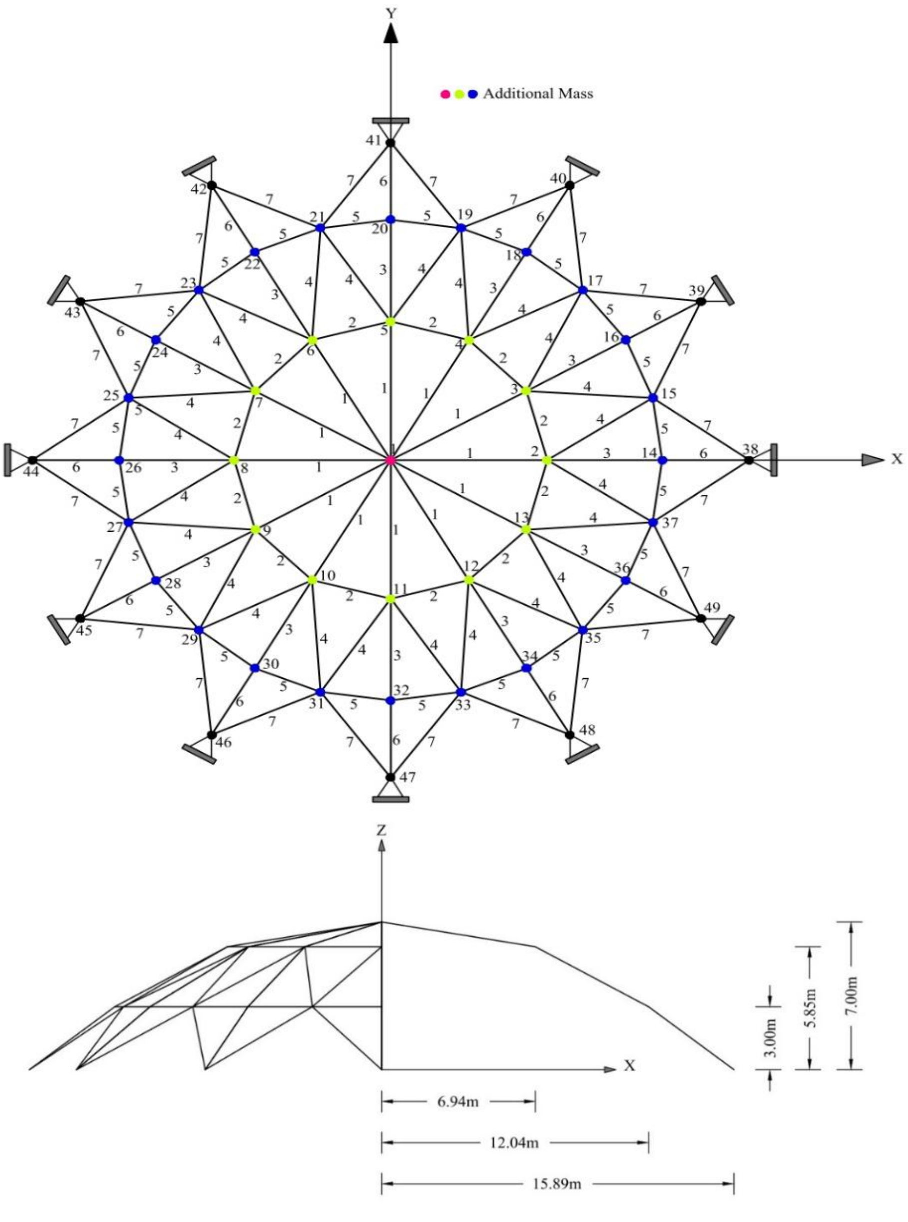

Fig. 3. 120-bar dome truss geometry.

Table 2. Optimal design solutions by variations analysis methods.

Design Variables (Areas) cm^2	Kaveh and Ghazaan	Kaveh, A. and A. Zolghadr	Kaveh, A. and V. Mahdavi Dahoei	A. Kaveh, M. Ilchi Ghazaan	Tejani et al.	Tejani et al.	Ho-Huu et al.	Lieu et al.	Present
	[10]	[6]	[7]	[8]	[11]	[9]	[12]	[13]	
	VPS	DPSO	CBO	HALC-PSO	ISOS	MS-TLBO	IDE	AHEFA	PPSO
A_1	19.6836	19.607	19.6917	19.8905	19.6662	19.4886	19.4670	19.5094	19.4084
A_2	40.9581	41.290	41.1421	40.4045	39.8539	40.3949	40.5004	40.3867	40.1976
A_3	11.3325	11.136	11.1550	11.2057	10.6127	10.6921	10.6136	10.6033	10.7976
A_4	21.5387	21.025	21.3207	21.3768	21.2901	21.3139	21.1073	21.1168	21.0787
A_5	9.8867	10.060	9.8330	9.8669	9.7911	9.8943	9.8417	9.8221	9.8050
A_6	12.7116	12.758	12.8520	12.7200	11.7899	11.7810	11.7735	11.7735	12.1316
A_7	14.9330	15.414	15.1602	15.2236	14.7437	14.5979	14.8269	14.8405	14.9168
Best Weight (kg)	8888.74	8890.48	8889.1303	8889.96	8710.062	8708.729	8707.2898	8707.2559	8736.242
Number of analyses	30000	6000	6000	17000	4000	4000	4060	3560	30000
Mean Weight(kg)	8896.04	8895.99	8891.254	8900.39	8728.5951	8734.7450	8707.8147	8707.5580	8737.6056
f_1 (Hz)	9.000	9.0001	9.000	9.000	9.001	9.0002	9.000	9.000	9.000
f_2 (Hz)	11.000	11.0007	11.0000	11.0000	10.998	11.0000	11.0000	11.0000	11.0000

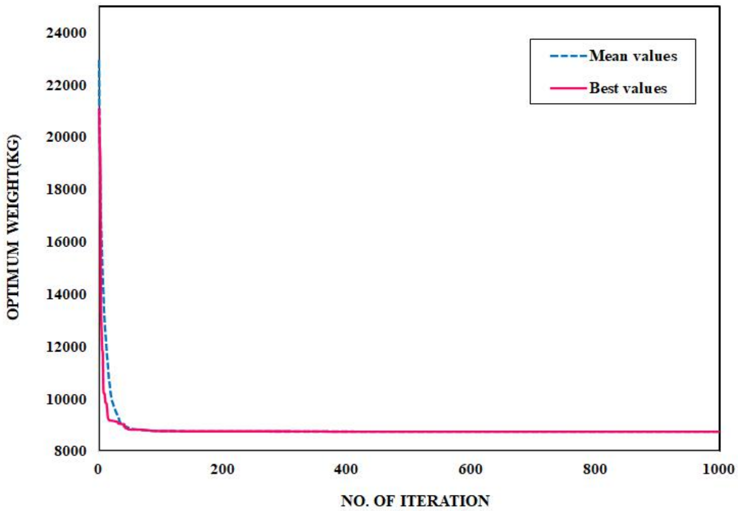

Fig. 4. Solution convergence by PPSO method.

Concluding remarks

The paper presents the novel PPSO method for the sizing optimization of dome-like truss structure subjected to the constraints on its natural frequencies. The approach mathematically adopts the phasor theory (namely the periodic sine and cosine functions) directly to the PSO that enhances its fast solution searching schemes and more importantly overcomes the likelihood of local optimal (premature) convergence as would be expected in standard PSO techniques. Its applications have been illustrated through the optimal sizing design of the modest-size dome truss structure, where its accuracy and robustness are evidenced by the good comparisons with various available benchmarks. An ongoing extension of the present work is the development of an effective PPSO algorithm with the capability to incorporate the time-dependent dynamic (seismic) constraints.

Acknowledgements

This research has been supported under the Thailand Research Fund (Grant No. RSA6280089). The financial support provided by Chulalongkorn University under Ratchadaphiseksomphot Endowment Fund is acknowledged.

References

[1] Grandhi, R., Structural optimization with frequency constraints-a review. AIAA journal, 1993. 31(12): p. 2296-2303.

[2] Bellagamba, L. and T. Yang, Minimum-mass truss structures with constraints on fundamental natural frequency. AIAA Journal, 1981. 19(11): p. 1452-1458.

[3] Sedaghati, R., A. Suleman, and B. Tabarrok, Structural optimization with frequency constraints using the finite element force method. AIAA journal, 2002. 40(2): p. 382-388.

[4] Lingyun, W., et al., Truss optimization on shape and sizing with frequency constraints based on genetic algorithm. Computational Mechanics, 2005. 35(5): p. 361-368.

[5] Gomes, H.M., Truss optimization with dynamic constraints using a particle swarm algorithm. 2011. 38(1 %J Expert Syst. Appl.): p. 957–968.

[6] Kaveh, A. and A. Zolghadr, Democratic PSO for truss layout and size optimization with frequency constraints. Computers & Structures, 2014. 130: p. 10-21.

[7] Kaveh, A. and V. Mahdavi Dahoei, Colliding-Bodies Optimization for Truss Optimization with Multiple Frequency Constraints. Journal of Computing in Civil Engineering, 2015. 29.

[8] Kaveh, A. and M. Ilchi Ghazaan, Hybridized optimization algorithms for design of trusses with multiple natural frequency constraints. Advances in Engineering Software, 2015. 79: p. 137-147.

[9] Tejani, G.G., V.J. Savsani, and V.K. Patel, Modified sub-population teaching-learning-based optimization for design of truss structures with natural frequency constraints. Mechanics Based Design of Structures and Machines, 2016. 44(4): p. 495-513.

[10] Kaveh, A. and M. Ilchi Ghazaan, Vibrating particles system algorithm for truss optimization with multiple natural frequency constraints. Acta Mechanica, 2017. 228(1): p. 307-322.

[11] Tejani, G.G., et al., Truss optimization with natural frequency bounds using improved symbiotic organisms search. Knowledge-Based Systems, 2018. 143: p. 162-178.

[12] Ho-Huu, V., et al., Optimal design of truss structures with frequency constraints using improved differential evolution algorithm based on an adaptive mutation scheme. Automation in Construction, 2016. 68: p. 81-94.

[13] Lieu, Q.X., D.T.T. Do, and J. Lee, An adaptive hybrid evolutionary firefly algorithm for shape and size optimization of truss structures with frequency constraints. Computers & Structures, 2018. 195: p. 99-112.

[14] Camp, C.V., Design of space trusses using big bang-big crunch optimization. 2007. 133(7): p. 999-1008.

[15] Ghasemi, M., et al., Phasor particle swarm optimization: a simple and efficient variant of PSO. Soft Computing, 2019. 23(19): p. 9701-9718.